STRUCTURAL ELEMENTS DESIGN MANUAL

WORKING WITH EUROCODES

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Greek letters

Greek letter	Letter name	Pronounced	To rhyme with
α	alpha	alfa	
β	beta	beater	
γ	gamma		hammer
δ	delta		shelter
η	eta	eater	
ε	epsilon	ep-silon	
λ	lambda	lam-da	
ρ	rho	row	hoe
σ	sigma		stigma
τ	tau	tore	more
Φ, ϕ	phi	fi	pie
χ	chi	As at the start of *ch*ild	
ψ	psi	As in the middle of cam*ps*ite	

Chapter 1 and generally

Symbol	Meaning
g_k, G_k	Characteristic permanent action, e.g. Dead Load
q_k, Q_k	Characteristic variable action, e.g. Imposed Load
SLS	Serviceability limit state
ULS	Ultimate limit state
w	Vertical deflection
γ_f	Partial factor of safety for load
γ_m	Partial factor of safety for material strength
ψ	Combination factor
μ_1	Undrifted snow load coefficient

Chapter 2 Timber

Symbol	Meaning
A	Area of cross section
b	Breadth of section
d	Diameter of fastener
E	Elastic modulus of timber
f	Strength of timber
G	Shear modulus of timber
h	Depth of section
i	Radius of gyration of section
I	Second moment of area of section
k_c	Factor for the effect of buckling on axial compressive strength
$k_{c,90}$	Factor giving enhanced strength at bearings
k_{def}	Factor for the effect of service class on long term deflection

Chapter 2 Timber (continued)

Symbol	Meaning
k_h	Depth factor, giving enhanced bending strength for members smaller than a stated size
k_m	Factor for combined bending
k_{mod}	Factor for the effect of load duration and service class on strength
k_{sys}	System strength factor, used when several members can share a load
k_v	Factor reducing shear strength at notched ends
w	Vertical deflection of a beam
W	Elastic section modulus
λ	Slenderness ratio
ρ	Density of timber
σ	Stress in timber
ψ	Combination factors from EC0

Chapter 3 Reinforced Concrete

Symbol	Meaning
A_s	Area of reinforcement
A_{sw}	Cross-section area of shear reinforcement
b	Breadth of section
b_c	Breadth of the compression face of a beam
b_v	Breadth of beam used to calculate the shear stress
d	Effective depth to reinforcement
f_{ck}	Characteristic cylinder strength of concrete
f_{yk}	Characteristic yield strength of reinforcement
h	Overall depth of section
K	Coefficient obtained from the design formula for rectangular beams
K_{lim}	K_{lim} is the upper limit on K if compression reinforcement is not to be provided
s	Spacing of shear links
V	Shear force
v	Shear stress
v_E	Shear stress due to loads
v_R	Shear resistance
x	Depth to neutral axis
z	Lever arm
ρ_l	Reinforcement ratio for longitudinal reinforcement
ρ_w	Reinforcement ratio for shear reinforcement
ϕ	Diameter of reinforcing bar

Chapter 4 Masonry

Symbol	Meaning
A	horizontal cross-section area of a wall
e	Eccentricity of load on a wall leaf
h_{eff}	Effective height of a wall
f_d	Design compressive strength of masonry
f_k	Characteristic compressive strength of masonry
f_m	Characteristic compressive strength of mortar
f_b	Normalised mean compressive strength of masonry unit
h	Height of wall panel
h_{ef}	Effective height of wall
t	Thickness of a wall
t_1, t_2	Thicknesses of the leaves of a cavity wall
t_{eff}	Effective thickness of a wall, or of one leaf of a cavity wall
α	Load ratio
δ	Shape factor for a masonry unit
η	Moment reduction factor
λ	Slenderness ratio h_{ef}/t_{ef}
Φ_A	Capacity reduction factor for walls of area less than 0.1 m^2
Φ_s	Capacity reduction factor for slenderness
ρ	Effective height reduction factor
ρ_t	Effective thickness coefficient for a wall stiffened by piers

Chapter 5 Steel

Symbol	Meaning
A	Area of section
b	Breadth of section
C_1	Factor for shape of bending moment diagram
c	Outstand of a flange
d	Depth of straight portion of a web
E	Modulus of elasticity of steel
f	Enhancement factor for LTB
f_y	Yield strength of steel
G	Shear modulus of steel
h	Depth of section
h_w	Clear height of web between flanges
I_y, I_z	Second moment of area of section
I_T	Torsion constant
I_w	Warping constant
i	Radius of gyration of section
k	Interaction factor
k_c	Factor for stanchion baseplate outstand
k_t	Factor for stanchion baseplate thickness
LTB, LT	Lateral-torsional buckling
$M_{b,Rd}$	Design buckling resistance moment
M_{cr}	Elastic critical buckling moment
$M_{c,Rd}$	Design bending resistance
$M_{pl,Rd}$	Plastic bending resistance
r	Radius of root fillet
t_f	Flange thickness
t_w	Web thickness
V	Shear force
V_{Ed}	Design shear force
$V_{pl,Rd}$	Design plastic shear resistance
v	Shear stress
W_{el}	Elastic section modulus
W_{pl}	Plastic section modulus
w	Vertical deflection
α	Imperfection factor
β	Buckling correction factor
ε	Coefficient depending on f_y for determining section class
λ	Slenderness ratio
$\bar{\lambda}$	Non-dimensional slenderness ratio
$\bar{\lambda}_0$	Limiting slenderness ratio
χ	Strength reduction factor for buckling
σ	Stress
ψ	Ratio of moments in a segment of a beam
Φ	Buckling parameter

Structural Elements Design Manual Working with Eurocodes

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Second edition

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Preface

When the Eurocode edition of this book was conceived, our main aim was to retain the ethos of the original British Standards version that had proved so popular for over 18 years. That basically was a reader-friendly one-stop introduction to the structural Eurocodes.

We were both very pleased to work together producing this Eurocode edition of the manual. Peter had found the first edition to be an excellent source of advice and examples, and had used it throughout his time as a lecturer in structural design. He recommended it to his students as a good supplement to lecture notes and as a useful summary, in a single volume, of information from a range of design standards.

We soon decided that the new edition should look forward to the Eurocodes not back to UK codes, so we have not included any retrospective material. We hope that engineers starting on their education and careers can use the new edition without being overburdened by references to methods formerly used, and that established engineers can make their own connections to existing knowledge as required.

We commend the Eurocodes to structural engineers. The work has given us the opportunity to study in detail four of the main codes, and we have found them to be thorough, rational and comprehensive. To those who feel that 20th century structural design methods need no updating we offer some advice from Václav Havel:

> 'Keep the company of those who seek the truth. Run away from those who have found it'.

Tackling the Eurocodes was a challenging task, not because they are unclear in themselves but because the profession has not yet had time to build up a body of compliant supporting methods. Instead we have devised our own ways of summarising, tabulating and simplifying the Eurocodes in a way that will, we hope, prove useful for practicing engineers. We have concentrated on the behaviour and practical design of the main elements that comprise a building structure and have included plenty of worked examples.

We are grateful to Peter's Compaq laptop and to Microsoft Office 2000 for providing a bench on which we have worked and re-worked the material. Our thanks go to Mary Brettle at the

Steel Construction Institute for her invaluable help with some knotty problems, and especially to our wives Ann and Jendy for their forbearance and support.

Trevor Draycott and Peter Bullman

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CHAPTER 1

General Matters

Contents

1.1 Introduction

Eurocode 0 'Basis of Structural Design' states:

> *'A structure is an organised combination of connected parts designed to carry loads and provide adequate rigidity.'*

Structural engineering is the study of how the various components of a building or other structure act together to transmit forces down to the foundations. Stages in the process are:

Structural planning stage: When a structural scheme is devised to suit both the purpose of the building and the site conditions.

Structural analysis stage: When the loads on the structure are determined and the way that the loads disperse through the structure is analysed using the principles of structural mechanics.

Structural element design stage: When the size and properties of each member are determined.

Structural detailing stage: When detail drawings are produced to illustrate how the structure is to be constructed on site.

Structural specification stage: When specification clauses are compiled to define the standard of materials and workmanship to be used.

Construction stage: When the structure is built, with appropriate supervision, inspection and testing to ensure that it complies with the drawings and the specification.

This book is primarily about the third of these stages, although some aspects of the other stages are also covered.

Although structural design should be carried out by structural engineers, other construction professionals require some knowledge of structural behaviour. Architects, quantity surveyors, building control officers, clerks of works and site staff will all benefit from studying the design of structural elements in different building materials.

1.2 Codes and Standards

Guidelines for the use of structural materials are found in many published codes, and this manual is mostly based on a set of codes produced for use throughout the European Union – the Eurocodes. Each country in the European Union defines how each code is to be used by publishing a National Annex for each code, and this manual is based on the UK National Annexes.

The various Eurocodes make extensive use of Greek letters, details of which are given in the respective sections of this manual.

Designs which accord with the Eurocodes are both safe and economic, and are deemed to satisfy UK Building Regulations. The codes do not inhibit the exercise of engineering judgement or the production of exciting structures, but they set out sound and well understood limits to which various materials can be taken.

In relation to structural design, the codes and standards are in two groups:

(a) Those relating to materials and components

(b) Those relating to loading and to the design of structures.

Tables 1.1 and 1.2 list a selection of these codes and standards.

Extracts from the codes and standards in this manual are reproduced by permission of the British Standards Institution. Complete copies can be obtained from BSI, 389 Chiswick High Road, London W4 4AL. Information regarding online access can be found at www.bsi-global.com.

While the principles of structural mechanics and materials behaviour do not change, codes and standards are often revised. Designers should ensure that they are working to current editions and latest amendments of any documents.

Table 1.1: Codes relating to materials and components

Material	BSI reference	Title
Timber	BS EN 336	Structural timber – sizes, permitted deviations
	BS EN 338	Structural timber strength classes
	BS EN 1912	Structural timber – strength classes – assignment of visual grades and species
Reinforced concrete	BS 4449	Steel for the reinforcement of concrete
	BS 4483	Steel fabric for the reinforcement of concrete
	BS 8500	Complementary British Standard to BS EN 206-1
	BS 8666	Scheduling, dimensioning, bending and cutting of steel reinforcement for concrete
	BS EN 206	Concrete – specification, performance, production and conformity
Masonry	BS EN 771	Specification for masonry units
	BS EN 772	Methods of test for masonry units
	BS EN 998	Mortar for masonry
	BS EN 1052	Methods of test for mortar
Structural steel	BS 4	Structural steel sections
	BS EN 10025	Hot rolled products of structural steels

Table 1.2: Codes relating to loading and to the design of structures*

	BSI reference	Title
Eurocode 0 (EC0)	**BS EN 1990**	**Basis of structural design**
Eurocode 1 (EC1)	**BS EN 1991**	**Actions on structures**
Eurocode 2 (EC2)	**BS EN 1992**	**Design of concrete structures**
Eurocode 3 (EC3)	**BS EN 1993**	**Design of steel structures**
Eurocode 4 (EC4)	BS EN 1994	Design of composite steel and concrete structures
Eurocode 5 (EC5)	**BS EN 1995**	**Design of timber structures**
Eurocode 6 (EC6)	**BS EN 1996**	**Design of masonry structures**
Eurocode 7 (EC7)	BS EN 1997	Geotechnical design
Eurocode 8 (EC8)	BS EN 1998	Design of structures for earthquake resistance
Eurocode 9 (EC9)	BS EN 1999	Design of aluminium alloy structures

*Each code should be read with the appropriate National Annex. Titles shown in bold are relevant to this manual.

Trade and professional organisations

Further useful guidance on structural design is available from many trade and professional organisations. Table 1.3 lists some organisations relevant to structural engineering generally and to the structural materials covered in this manual.

Design working life

EC0 gives indicative values of design working life, as shown in Table 1.4. This manual is about the design of class 4 structures.

Table 1.3: Trade and professional organizations, useful websites

Organization	Abbreviation	Website
Institution of Structural Engineers	IStructE	www.istructe.org.uk
Steel Construction Institute	SCI	www.steel-sci.org
The Concrete Centre		www.concretecentre.com
British Precast Concrete Federation	BPCF	www.britishprecast.org
Brick Development Association	BDA	www.brick.org.uk
Timber Research and Development Association	TRADA	www.trada.co.uk
Eurocodes Expert		www.eurocodes.co.uk

Table 1.4: Design working life of structures

Class	Indicative design working life (years)	
1	10	Temporary Structures*
2	10 to 30	Replaceable Parts, e.g. gantry girders, bearings
3	15 to 25	Agricultural and similar structures
4	**50**	**Building structures and other common structures**
5	120	Monumental building structures, highway and railway bridges, and other civil engineering structures

*Structures or parts of structures that can be dismantled with a view to being re-used should not be considered as temporary.
Source: Table NA 2.1 of EC0.

1.3 Loads and Actions

Once a structural scheme has been chosen, the next step is the determination of the loads and actions that the structure should be expected to sustain in the normal course of events. Eurocodes use the word *action* to describe an effect which may cause distress to the structure.

- Direct actions, or *loads,* are forces applied to the structure.
- Indirect actions are movements caused by, for example, earthquakes, temperature changes, moisture variation or uneven foundation settlement.

Actions may be permanent or variable. The self-weight of a building is a permanent action; loads from occupation, stored materials, vehicles or wind are variable actions.

Tables 1.5 and 1.6 give design values for loads imposed by the occupation of buildings.

The self-weight or dead-weight of a building or component is a permanent action G_k, or g_k if expressed as a distributed load. Values in EC1, as shown in Table 1.7, are given in kN (kilonewtons) and can be used directly. Sometimes the information may be given in kg (kilograms), which are units of mass, and must be converted to kN before it can be used. Conversion is based on the value of gravity at the surface of the earth which is 9.8 m/s^2.

Table 1.5: Design values q_k and Q_k for imposed loads on floors (from EC1)

Category in EC1	Description	Design values of imposed loads	
		Uniformly distributed load q_k (kN/m^2)	Point load Q_k kN, applied over a 50 mm × 50 mm square
A1	Self-contained dwellings	1.5	2.0
B1	Offices generally other than B2	2.5	2.7
B2	Offices at or below ground level	3.0	2.7
C1	Areas where people may congregate: areas with tables	3.0	4.0
C2	Areas where people may congregate: areas with fixed seats	4.0	4.0
C5	Areas where people may congregate: areas susceptible to large crowds	5.0	4.5
C12	Reading rooms with no book storage	2.5	4.0
C13	Classrooms	3.0	3.0
C21	Assembly areas with fixed seats	4.0	3.6
C31	Corridors in institutional buildings	3.0	4.5
C51	Assembly areas without fixed seating, concert halls, bars and places of worship	5.0	3.6
D1/D2	Shops	4.0	3.6
E12	Reading rooms with book storage (e.g. libraries)	4.0	4.5
E13	General storage	2.4 per m of storage height	7.0
E19	Cold storage	5.0 per m of storage height but with a minimum of 15.0	9.0
E2	Industrial use	See PD 6688	
F	Garages and vehicle traffic areas, gross vehicle weight not more than 30 kN	2.5	10.0
G	Garages and vehicle traffic areas, gross vehicle weight between 30 kN and 160 kN	5.0	To be determined
Roof areas not accessible except for normal maintenance and repair, roof slope a			
	α less than 30°	0.6	0.9
	α between 30° and 60°	$0.6(60 - \alpha)/30$	0.9
	α more than 60°	Zero	0.9

The weight of a 1 kg mass is

$$9.8\,\text{N} = 0.0098/\,\text{kN}$$

As an example, tables of steel section sizes show the mass of the section in kg/m. A steel section with a tabulated mass of 73 kg/m has a weight of $73 \times 0.0098 = 0.715$ kN/m.

Table 1.6: Design loads q_k on floors from movable partitions (from EC1)

Self weight of partition wall (kN/m)	q_k (kN/m^2)
Not more than 1.0	0.5
Not more than 2.0	0.8
Not more than 3.0	1.2

Table 1.7: Nominal density of materials

Material	Unit weight (kN/m^3)
Construction materials	
Unreinforced concrete – normal weight concrete	24.0
Reinforced concrete – normal weight concrete	25.0
Brickwork – clay bricks	22.0*
Brickwork – concrete bricks	23.0*
Blockwork – normal weight concrete blocks	22.0*
Blockwork – lightweight concrete blocks	6.0*
Timber – softwood	3.5 to 5.0
Timber – hardwood	6.4 to 10.8
Timber – strength class C14	3.5
Timber – strength class C16	3.7
Timber – strength class C18	3.8
Timber – glulam (glued laminated timber)	3.5 to 4.4
Plywood – softwood	5.0
Plywood – birch	7.0
Laminboard and blockboard	4.5
Chipboard	8.0
Oriented strand board	7.0
Fibreboard – medium density (MDF)	8.0
Aluminium	27.0
Steel	77.0 to 78.5
Lead	112.0 to 114.0
Glass	25.0
Acrylic sheet	12.0
Natural stone	20.0 to 31.0
Slate	28.0
Mastic asphalt	18.0 to 22.0
Stored liquids	
Water	10.0
Diesel oil	8.3
Petrol	7.4

Source: most of the values are taken from EC1.
*Values marked are not in EC1 but are commonly accepted.

Snow load s_k on roofs

The characteristic ground snow load s_k (kN/m^2) is given by

$$s_k = [0.20 + 0.1Z] + \left(\frac{A - 100}{525}\right)$$

where Z is the zone number from the map in Figure NA1 of BS EN 1991-1-3:2003 (see Figure 1.1) and A is the site altitude in metres above sea level. If the site altitude is not more than 100 m then a simplified formula is usually used:

$$s_k = [0.20 + 0.1Z]$$

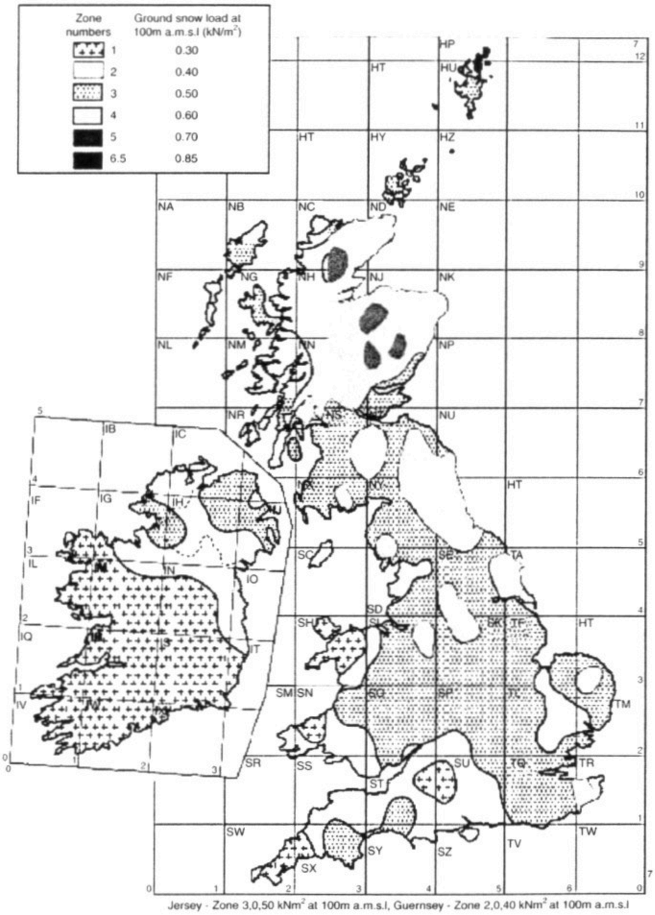


Figure 1.1: Snow loads: Figure NA1 of UK National Annex to BS EN 1991-1-3:2003

The design snow load on a roof is $\mu_1 s_k$. Table 1.8 gives values of μ_1 for flat and simple pitched roofs for undrifted snow. Higher values of snow load should be used where parapets, valleys or abrupt changes in roof level may cause drifting of snow (see EC1 Part 1–3 for guidance).

Table 1.8: Coefficient μ_1 for undrifted snow load on pitched roofs, roof slope α_1°

Roof slope α_1°	**Coefficient μ_1**
Flat roofs, roofs with α_1 less than 30°	0.80
α_1 = 35°	0.67
α_1 = 40°	0.53
α_1 = 45°	0.40
α_1 = 50°	0.27
α_1 = 55°	0.13
α_1 = 60° or more	Zero

Source: Table 5.2 of EC1.

Wind loads

This may be defined as all the loads acting on a building that are induced either by wind pressure or by wind suction. Wind loads are determined from statistical data and it is found that small areas of cladding, especially near edges, are likely to receive higher wind pressures or suctions than large areas or whole buildings. BS EN 1991 Part 1.4 gives wind speeds to be adopted for the design of buildings relative to their geographical location in the UK and elevation above sea level. It also gives methods of calculating wind pressures or suctions on the various parts of the building, such as roofs and walls, in relation to its size and shape.

1.4 Limit State Design Philosophy

The basic requirement for a structure is stated in EC0 as follows.

'A structure shall be designed and executed in such a way that it will, during its intended life, with appropriate degrees of reliability and in an economical way:

- sustain all actions and influences likely to occur during execution and use, and

- remain fit for the use for which it is required'

This is achieved by verifying that the structure does not go beyond one or more limit states. Two categories of limit states are defined:

- Ultimate limit states (ULS), which concern the safety of people and the safety of the structure. These include loss of equilibrium (overturning), failure through excessive deformation, fracture and rupture.
- Serviceability limit states (SLS), which concern the function of the structure under normal use, the comfort of people or the appearance of the structure.

Structural safety is assured by applying appropriate partial safety factors γ to the values of loads and material properties used in the design.

- Partial safety factors for loads, denoted by γ_f, are intended to allow for some unforeseen increases in loads. The factors of safety for loads which are difficult to predict, such as imposed loads, are higher than those for loads which are easier to predict, such as self-weights. Table 1.9gives the partial factors of safety for loads which apply to the simple structures considered in this manual.
- Partial safety factors for materials, denoted by γ_m, make some allowance for sub-standard materials or for the deterioration of materials during the life of the structure. The factors of safety used for materials which are inherently variable, such as concrete and masonry, are higher than those for more consistent materials, such as steel. Partial safety factors for materials are covered in the chapters of this manual which relate to specific materials.

The factors of safety used for verifying an ultimate limit state are higher than those used for serviceability limit states, because the consequences of exceeding an ultimate limit state are more severe. Thus the following load combinations are used.

Table 1.9: Partial safety factors for loads in serviceability and ultimate limits states

Limit state	Partial safety factor for permanent actions G_k (dead loads)	Partial safety factor for variable actions Q_k (imposed and snow loads)
ULS	$\gamma_G = 1.35$	$\gamma_Q = 1.50$
SLS	$\gamma_G = 1.00$	$\gamma_Q = 1.00$

Source: EC1.

For verifying ultimate limit states:

$$\gamma_G G_k + \gamma_Q Q_k = 1.35G_k + 1.50Q_k$$

For verifying serviceability limit states:

$$\gamma_G G_k + \gamma_Q Q_k = 1.00G_k + 1.00Q_k$$

Combined variable loads at ULS

When more than one variable load is to be considered, for instance on a roof which is to be designed for both snow load and access load, it is very unlikely that all the variable loads will have their maximum value at the same time. For this reason EC0 allows the use of combination factors ψ which reduce the design values of variable loads when they act together. Values of ψ_0 are given in Table 1.10, and the use of ψ_1 and ψ_2 is explained later in the text.

Table 1.10: Values of ψ factors for buildings

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings			
Category A: domestic, residential areas	0.7	0.5	0.3
Category B: office areas	0.7	0.5	0.3
Category C: congregation areas	0.7	0.7	0.6
Category D: shopping areas	0.7	0.7	0.6
Category E: storage areas	1.0	0.9	0.8
Category F: traffic area, vehicle weight $= <30$ kN	0.7	0.7	0.6
Category G: traffic area, vehicle weight between 30 kN and 160 kN	0.7	0.5	0.3
Category H: roofs	0.7	0	0
Snow loads on buildings			
- for sites at altitude $H = <1000$ m above sea level	0.5	0.2	0
- for sites at altitude $H > 1000$ m above sea level	0.7	0.5	0.2
Wind loads on buildings	0.6	0.2	0

Source: Table NA A1.1 of UK National Annex to EC0.

If a structural element is to be designed for one permanent load G_k and two variable loads $Q_{k,1}$ and $Q_{k,2}$, the ULS design should be checked for three combinations of loading:

$$1.35G_k + 1.50\psi_0 Q_{k,1} + 1.50\psi_0 Q_{k,2} \quad \text{(EC0 equation 6.10a)}$$
$$1.15G_k + 1.50\psi_0 Q_{k,1} + 1.50 Q_{k,2} \quad \text{(EC0 equation 6.10b with } \zeta = 0.925\text{)}$$
$$\text{and } 1.15G_k + 1.50 Q_{k,1} + 1.50\psi_0 Q_{k,2} \quad \text{(EC0 equation 6.10b with } \zeta = 0.925\text{)}$$

As an example, consider a roof purlin which carries the following unfactored loads:

Dead load:	$G_k = 12\,\text{kN}$
Imposed load for access to roof:	$Q_{k,1} = 15\,\text{kN}$
Snow load:	$Q_{k,2} = 8\,\text{kN}$

Assuming that the building is not more than 1000 m above sea level, a reasonable assumption for buildings in the UK, then from Table 1.10 the ψ_0 values are 0.7 for the access load and 0.5 for the snow load.

The ULS design load is the greatest of

$$1.35G_k + 1.50\psi_0 Q_{k,1} + 1.50\psi_0 Q_{k,2} = 1.35 \times 12 + 1.50 \times 0.7 \times 15 + 1.50 \times 0.5 \times 8 = 38.0\,\text{kN}$$
$$1.15G_k + 1.50\psi_0 Q_{k,1} + 1.50 Q_{k,2} = 1.15 \times 12 + 1.50 \times 0.7 \times 15 + 1.50 \times 8 = 41.6\,\text{kN}$$
$$1.15G_k + 1.50 Q_{k,1} + 1.50\psi_0 Q_{k,2} = 1.15 \times 12 + 1.50 \times 15 + 1.50 \times 0.5 \times 8 = 42.3\,\text{kN}$$

The purlin should be designed for an ultimate load of 42.3 kN.

Combined loads at SLS

As noted for the ULS, it is very unlikely that all the variable loads will have their maximum value at the same time. In addition, it is assumed that the imposed loads (e.g. those given in Table 1.5) are unlikely to occur very often in the life of a building and that most of the time the actual loads will be less. EC0 allows variable loads to be reduced by ψ factors as shown in Table 1.11, which is based on three classifications of load.

Table 1.11: Loads to be used for assessing structures at SLS

Risk	Loads to be used	Dead loads	First variable load	Other variable loads
Function of structure, damage to structural and non-structural elements	Characteristic	Full	Full	Multiply by ψ_0
User comfort, operation of machinery, ponding of water	Frequent	Full	Multiply by ψ_1	Multiply by ψ_2
Appearance of structure	Semi-permanent	Full	Multiply by ψ_2	Multiply by ψ_2

Source: Clause NA.2.2.6 of UK National Annex to EC0.
Values of ψ_0, ψ_1 and ψ_2 are given in Table 1.10.

- Characteristic loads: these are the highest expected loads and occur only rarely.
- Frequent loads: these are lower than the characteristic loads and occur quite often.
- Semi-permanent loads: these are lower than frequent loads and are likely to be present most of the time.

Thus, if a structural element is to be designed for one permanent load G_k and two variable loads $Q_{k,1}$ and $Q_{k,2}$, the SLS design should be checked for the load combinations shown in Table 1.12.

Table 1.12: Loads to be used for assessing at SLS a structure subject to G_k, $Q_{k,1}$ and $Q_{k,2}$

Risk	Loads to be used	Load combinations
Function of structure, damage to structural and non-structural elements	Characteristic	$G_k + Q_{k,1} + \psi_0 Q_{k,2}$ and $G_k + \psi_0 Q_{k,1} + Q_{k,2}$
User comfort, operation of machinery, ponding of water	Frequent	$G_k + \psi_1 Q_{k,1} + \psi_2 Q_{k,2}$ and $G_k + \psi_2 Q_{k,1} + \psi_1 Q_{k,2}$
Appearance of structure	Semi-permanent	$G_k + \psi_2 Q_{k,1} + \psi_2 Q_{k,2}$

Source: Clause NA.2.2.6 of UK National Annex to EC0.

Consider the roof purlin from the previous section which carries the following unfactored loads:

Dead load:	G_k	$= 12\,\text{kN}$
Imposed load for access to roof:	$Q_{k,1}$	$= 15\,\text{kN}$
Snow load:	$Q_{k,2}$	$= 8\,\text{kN}$

To check against damage to finishes, the deflection under characteristic loads is required. From Table 1.10, for the imposed load $\psi_0 = 0.7$ and for the snow load $\psi_0 = 0.5$.

$$G_k + Q_{k,1} + \psi_0 Q_{k,2} = 12 + 15 + 0.5 \times 8 = 31\,\text{kN}$$
$$G_k + \psi_0 Q_{k,1} + Q_{k,2} = 12 + 0.7 \times 15 + 8 = 30.5\,\text{kN}$$

The deflection under a load of 31 kN should be found.

To check for water ponding, the deflection under frequent loads is required. From Table 1.10, for the imposed load $\psi_1 = 0$ and $\psi_2 = 0$, and for the snow load $\psi_1 = 0.2$ and $\psi_2 = 0$.

$$G_k + \psi_1 Q_{k,1} + \psi_2 Q_{k,2} = 12 + 0 \times 15 + 0 \times 8 = 12\,\text{kN}$$
$$G_k + \psi_2 Q_{k,1} + \psi_1 Q_{k,2} = 12 + 0 \times 15 + 0.2 \times 8 = 13.6\,\text{kN}$$

The deflection under a load of 13.6 kN should be found.

To check the appearance of the structure, the deflection under semi-permanent load is required. From Table 1.10, for the imposed load $\psi_2 = 0$ and for the snow load $\psi_2 = 0$.

$$G_k + \psi_2 Q_{k,1} + \psi_2 Q_{k,2} = 12 + 0 \times 15 + 0 \times 8 = 12\,\text{kN}$$

The deflection under a load of 12 kN should be found.

1.5 Determining Loads on Individual Structural Elements

A structural element is a portion of a structure that can be usefully considered as a separate entity. At an appropriate time in the design process a slab, a beam, a wall or a column can be visualised as isolated from the rest of the structure and calculations can be performed on that element to determine a suitable choice of size or section.

The following examples illustrate the calculation of design loads for beams and columns.

Example 1.1 Load on timber floor beam

Timber beams spanning 4.0 m and spaced at 3.0 m centres, as shown in Figure 1.2(a), support a timber floor comprising joists and boards with a plaster ceiling. Other design data:

- Self-weight of boards and floor joists 0.23 kN/m^2
- Self-weight of ceiling 0.22 kN/m^2

- Imposed load on floor 1.50 kN/m^2
- Self-weight of timber beam 0.6 kN (assumed)

A weight for the beam is assumed because at this stage in the design process the size of the beam is not known.

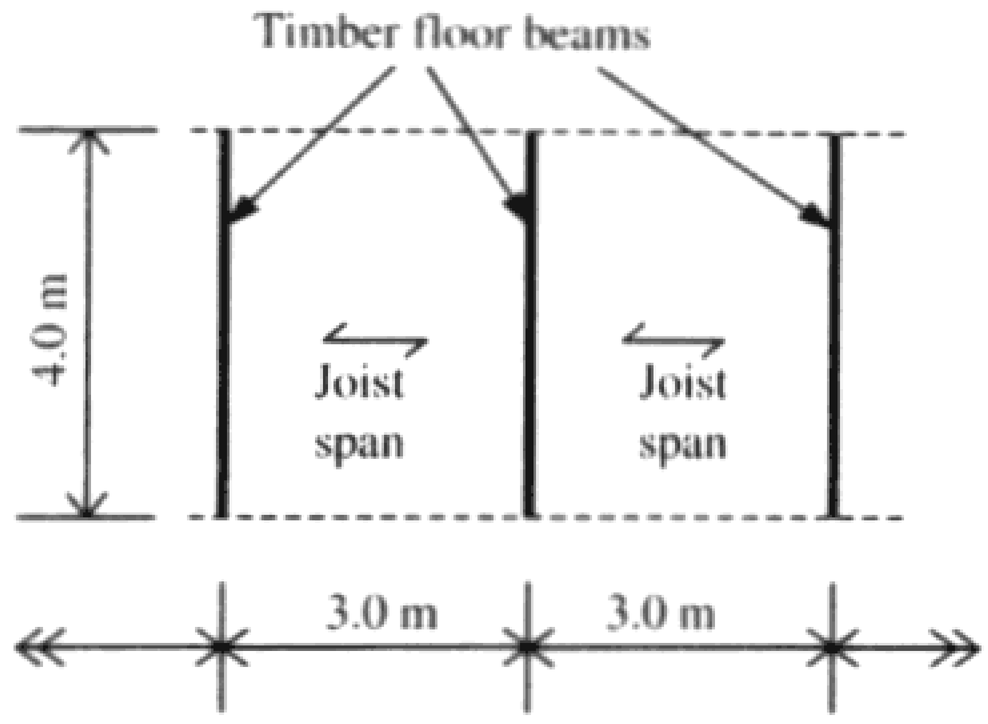


Figure 1.2(a): Floor plan

Calculations for Example 1.1

To determine the uniformly distributed load (UDL), visualise a single beam removed from the structure as in Figure 1.2(b)

The load on one square metre of floor is multiplied by the area supported, 4.0 m × 3.0 m, to give the total UD load on the beam as follows

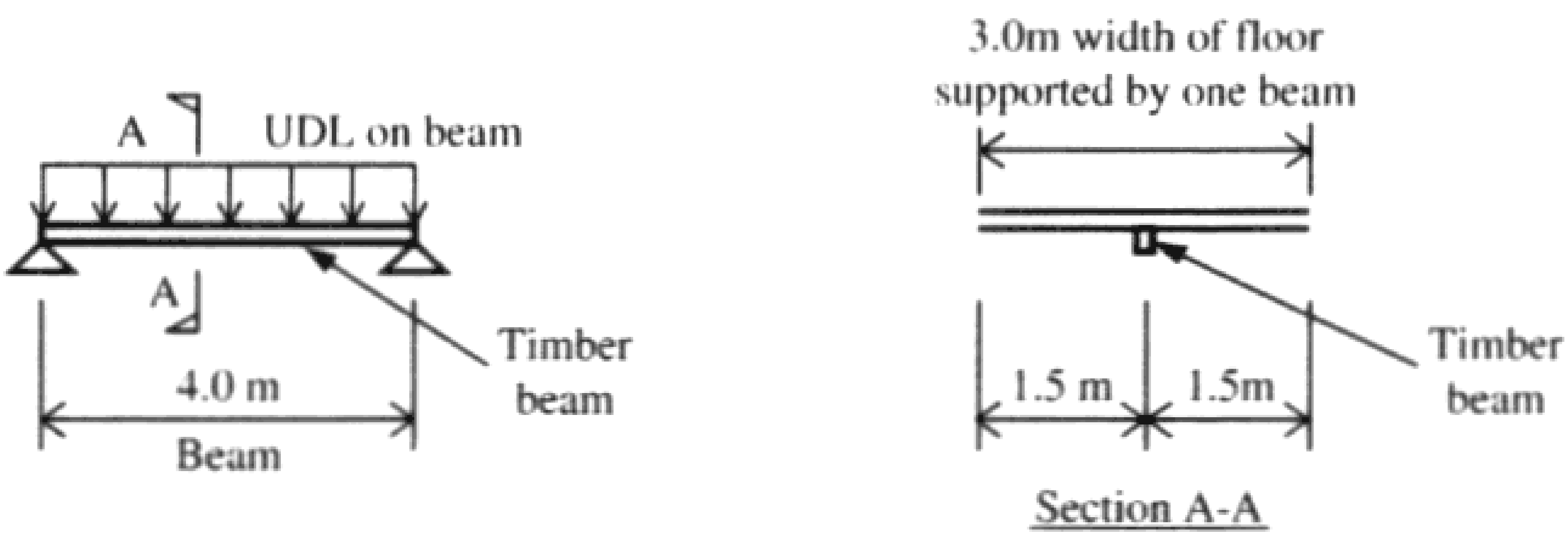


Figure 1.2(b): Isolated timber floor beam

Loads on timber beam		Dead loads (self-weight)	Imposed loads
Joists and boards	0.23 × 4.0 × 3.0 =	2.76 kN	-
Ceiling	0.22 × 4.0 × 3.0 =	2.64 kN	-
Timber beam		0.60 kN	-
Imposed load	1.5 × 4.0 × 3.0 =	-	18.0 kN
Totals		6.0 kN	18.0 kN

Continued on next page

Calculations for Example 1.1 (Continued from previous page)	
Total UD load for serviceability limit states $= 6.0 \times 1.00 + 18.0 \times 1.00$	$= 24.0\,\text{kN}$
Total UD load for ultimate limit states $= 6.0 \times 1.35 + 18.0 \times 1.50$	$= 35.1\,\text{kN}$
After the size of the beam has been determined, the assumed self-weight can be checked. If the chosen beam is 250 mm deep × 100 mm wide in C16 timber, which has a density of $3.7\,\text{kN/m}^3$, then its self-weight will be $0.25 \times 0.10 \times 4.0 \times 3.7 = 0.37\,\text{kN}$. Thus the assumed weight of 0.6 kN was satisfactory	

Example 1.2 Load on steel floor beams

Steel floor beams arranged as shown in Figure 1.3 support a reinforced concrete slab which carries a screed. Calculate the total UD load on each beam using this design data:

- slab thickness 150 mm
- screed weight $1.2\,\text{kN/m}^2$
- imposed load on slabs $5.0\,\text{kN/m}^2$
- mass of steel beams 60 kg/m.

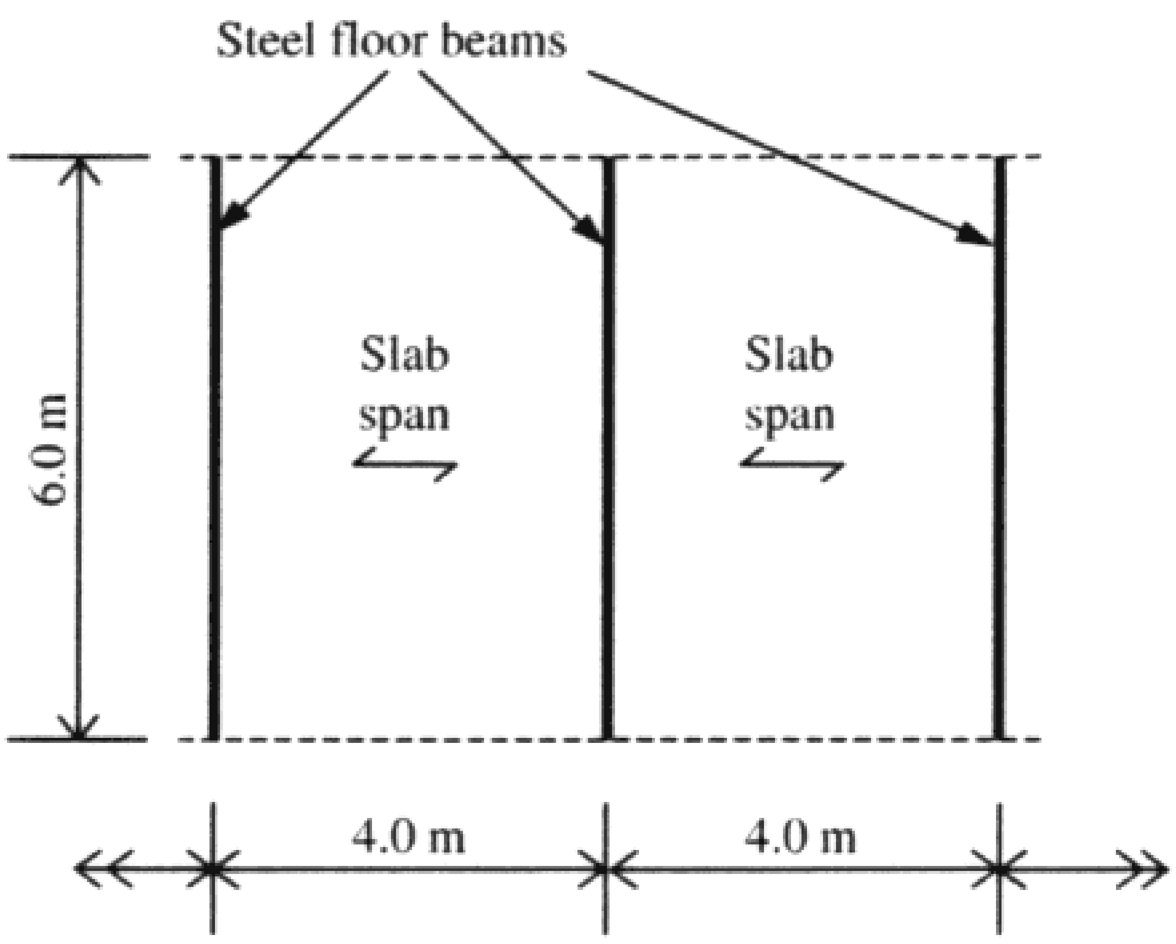
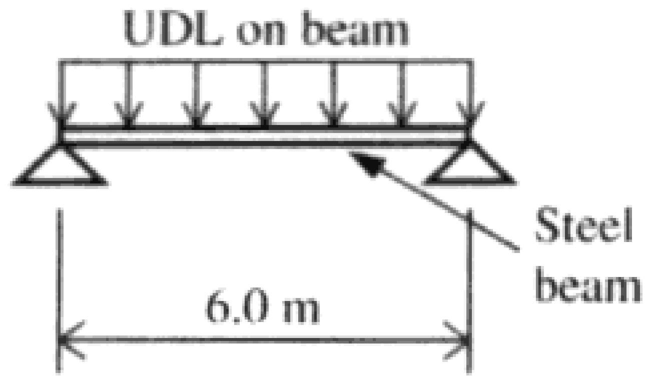


Figure 1.3

Calculations for Example 1.2	
Unit weight of reinforced concrete	$= 25\,\text{kN/m}^2$
Concrete slab self-weight $= 0.15 \times 25$	$= 3.75\,\text{kN/m}^2$
Steel beam self-weight $= 60 \times 0.0098$	$= 0.59\,\text{kN/m}$



Loads on steel beam		Dead loads (self-weight)	Imposed loads
Screed	$1.2 \times 6.0 \times 4.0 =$	28.8 kN	-
Concrete slab	$3.75 \times 6.0 \times 4.0 =$	90.0 kN	-
Steel beam	$0.59 \times 6.0 =$	3.5 kN	-
Imposed load	$5.0 \times 6.0 \times 4.0 =$	-	120.0 kN
Totals		122.3 kN	120.0 kN
Total UD load for serviceability limit states		$= 122.3 \times 1.00 + 120.0 \times 1.00$	$= 242.3$ kN
Total UD load for ultimate limit states		$= 122.3 \times 1.35 + 120.0 \times 1.50$	$= 345.1$ kN

Example 1.3

Find the beam loads and the reactions transmitted to the walls for the steelwork arrangement shown in Figure 1.4 using this design data:

- reinforced concrete (RC) slab thickness 100 mm
- screed weight 1.0 kN/m^2
- imposed load on slabs 3.0 kN/m^2
- mass of steel beams 80 kg/m.

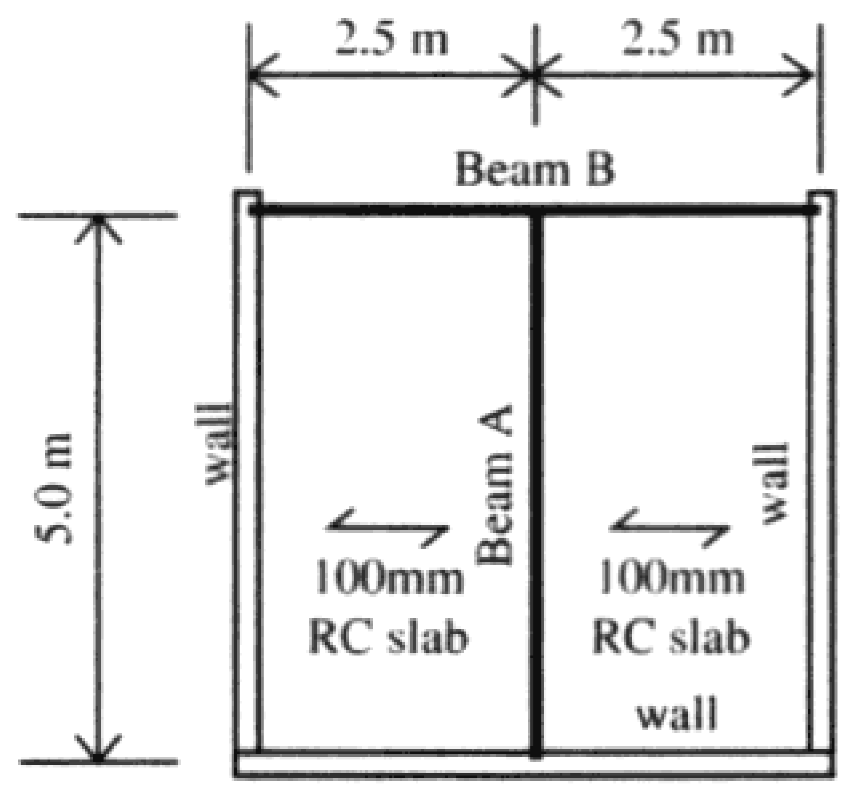
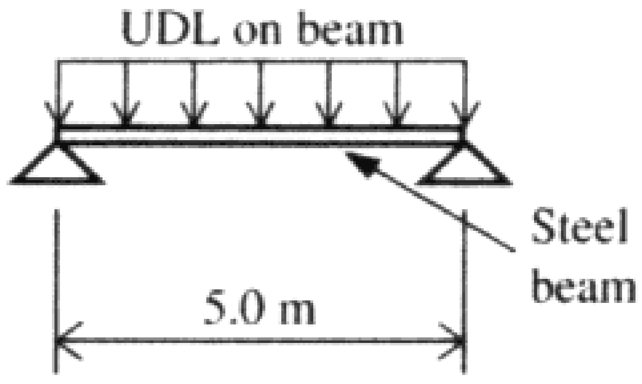


Figure 1.4: Floor plan

Calculations for Example 1.3

Concrete slab self weight	$= 0.10 \times 25$	$= 2.50\,kN/m^2$
Steel beam self weight	$= 80 \times 9.81/1000$	$= 0.78\,kN/m$

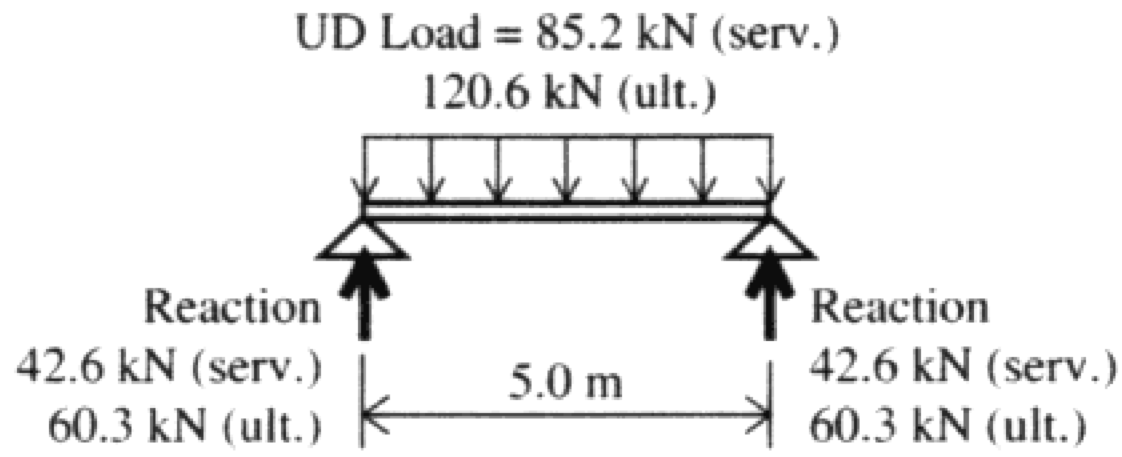
Beam A



UD loads on beam A		Dead loads (self-weight)	Imposed loads
Screed	$1.0 \times 5.0 \times 2.5 =$	12.5 kN	-
Concrete slab	$2.5 \times 5.0 \times 2.5 =$	31.3 kN	-
Steel beam	$0.78 \times 5.0 =$	3.9 kN	
Imposed load	$3.0 \times 5.0 \times 2.5 =$	-	37.5 kN
Totals		47.7 kN	37.5 kN

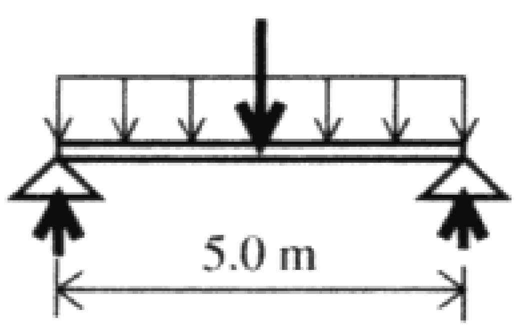
Total UD load for serviceability limit states	$= 47.7 \times 1.00 + 37.5 \times 1.00$	$= 85.2\,kN$
Total UD load for ultimate limit states	$= 47.7 \times 1.35 + 37.5 \times 1.50$	$= 120.6\,kN$

As beam A is loaded symmetrically each end reaction is half the total load Loads and reactions are shown in diagram below



Beam A isolated

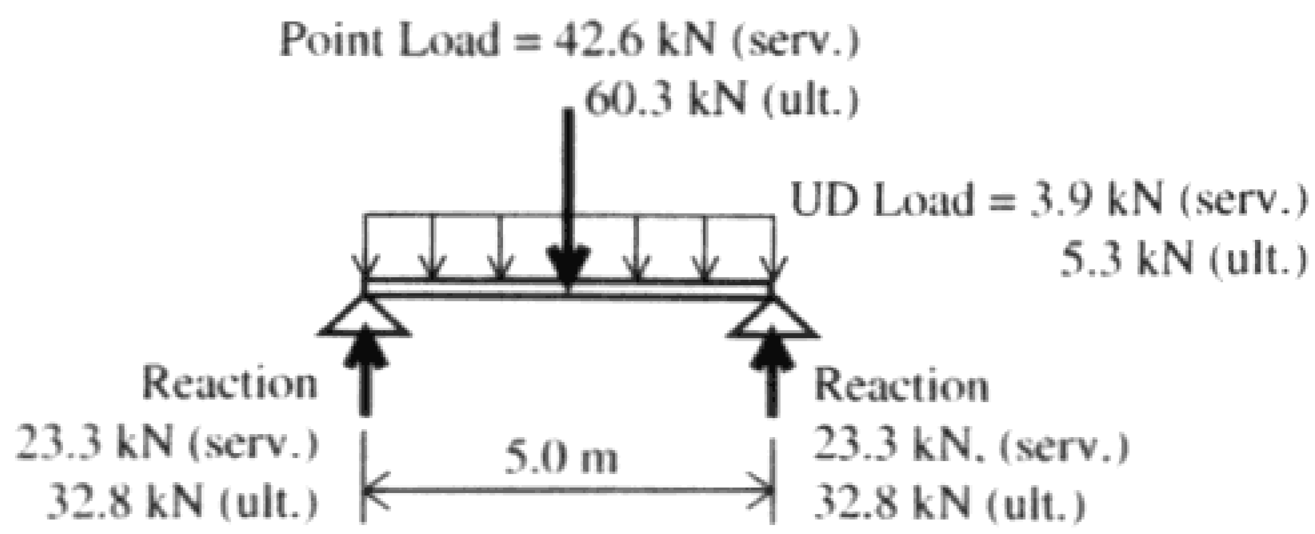
Beam B



UD loads on beam B		Dead loads (self-weight)	Imposed loads
Steel beam	$0.78 \times 5.0 =$	3.9 kN	-
Total		3.9 kN	nil

Total UD load for serviceability limit states	= 3.9 × 1.00	= 3.9 kN
Total UD load for ultimate limit states	= 3.9 × 1.35	= 5.3 kN

The diagram below shows the loads on beam B and the reactions transmitted to the walls



Note that each reaction is half the total load on the beam

Beam B isolated

Example 1.4

A reinforced concrete mezzanine floor slab is simply supported on steel beams and columns as shown in Figure 1.5(a). Calculate the beam and column loads using this design data:

- slab thickness 200 mm
- weight of screed 1.5 kN/m²

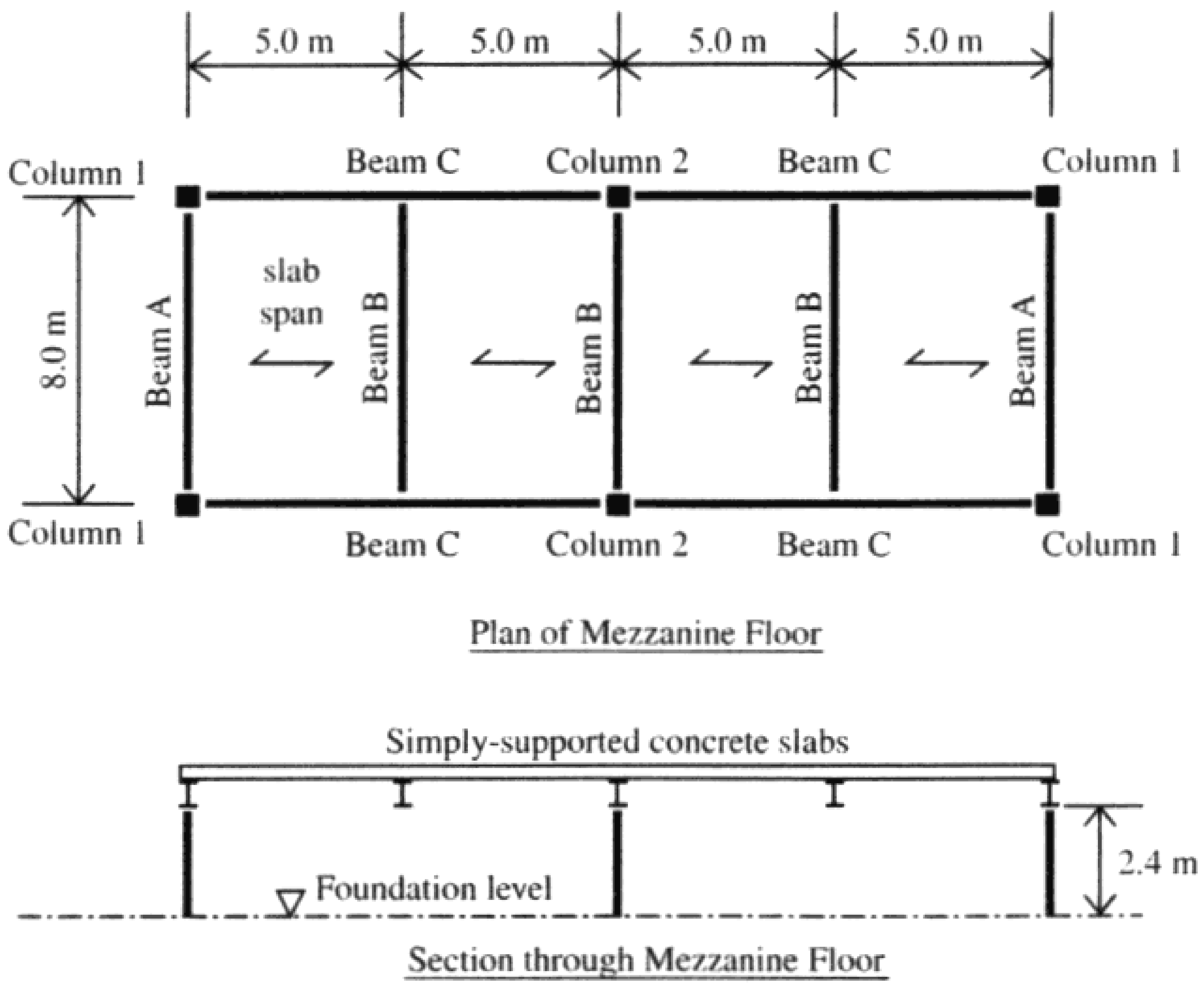


Figure 1.5(a): Arrangement of mezzanine floor in Example 1.4

- mass of steel beams 120 kg/m
- mass of steel columns 100 kg/m
- imposed load on slab 4.5 kN/m^2

Calculations for Example 1.4

Concrete slab self-weight	$= 0.20 \times 25$	$= 5.0$ kN/m^2
Total self-weight of slab + screed	$= 5.0 + 1.5$	$= 6.5$ kN/m^2
Steel beam self-weight	$= 120 \times 9.81/1000$	$= 1.18$ kN/m
Steel column self-weight	$= 100 \times 9.81/1000$	$= 0.98$ kN/m

Beam A

UD loads on beam A		Dead loads (self-weight)	Imposed loads
Concrete slab + screed	$6.5 \times 8.0 \times 2.5 =$	130.0 kN	-
Steel beam	$1.18 \times 8.0 =$	9.4 kN	-
Imposed load	$4.5 \times 8.0 \times 2.5 =$	-	90.0 kN
Totals		139.4 kN	90.0 kN

Total UD load for serviceability limit states	$= 139.4 \times 1.00 + 90.0 \times 1.00$	$= 229.4$ kN
Total UD load for ultimate limit states	$= 139.4 \times 1.35 + 90.0 \times 1.50$	$= 323.2$ kN

Figure 1.5(b) shows the loads and reactions for beam A

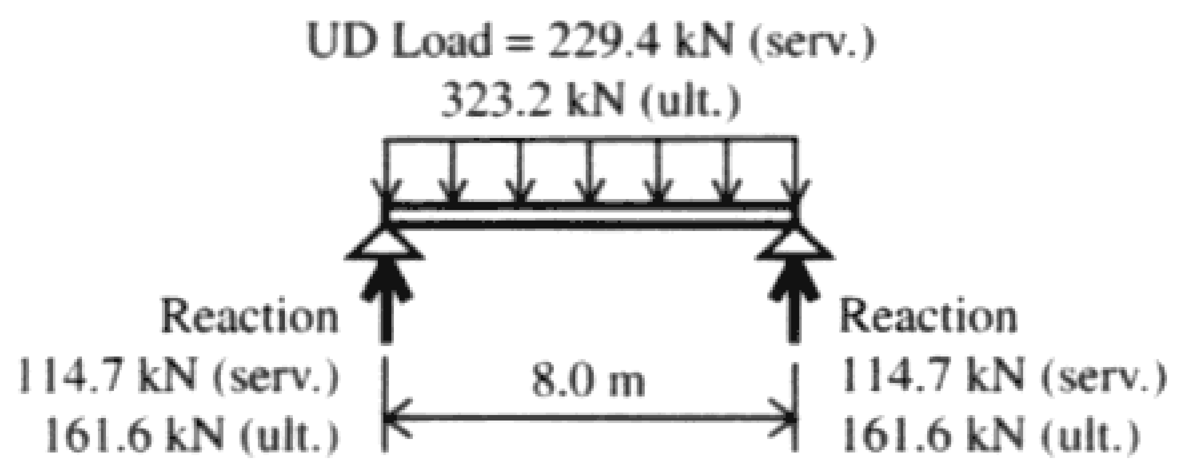


Figure 1.5(b): Beam A isolated

Beam B

UD loads on beam B		Dead loads (self-weight)	Imposed loads
Concrete slab + screed	$6.5 \times 8.0 \times 5.0 =$	260.0 kN	-
Steel beam	$1.18 \times 8.0 =$	9.4 kN	-
Imposed load	$4.5 \times 8.0 \times 5.0 =$	-	180.0 kN
Totals		269.4 kN	180.0 kN

Total UD load for serviceability limit states	$= 269.4 \times 1.00 + 180.0 \times 1.00$	$= 449.4$ kN
Total UD load for ultimate limit states	$= 269.4 \times 1.35 + 180.0 \times 1.50$	$= 633.7$ kN

Figure 1.5(c) shows the loads and reactions for beam B

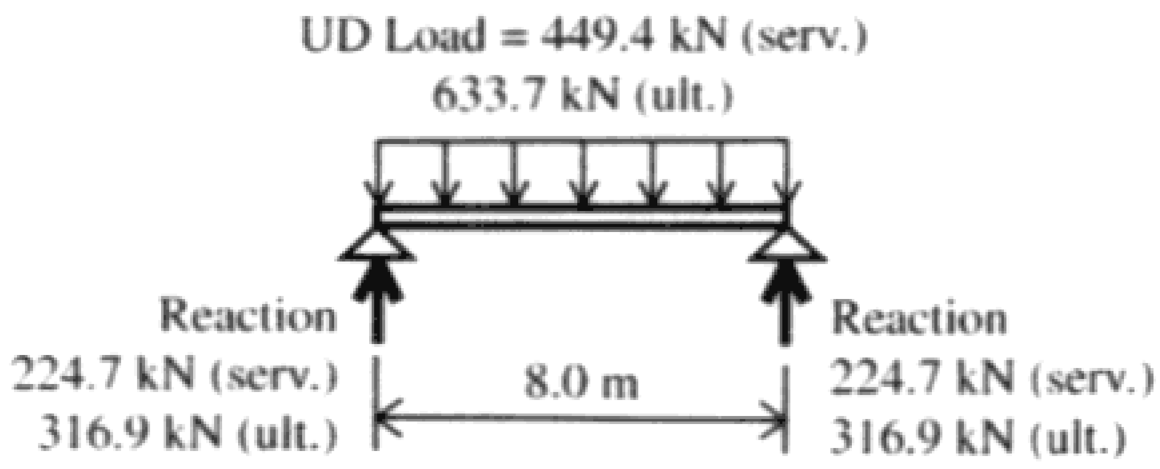


Figure 1.5(c): Beam B isolated

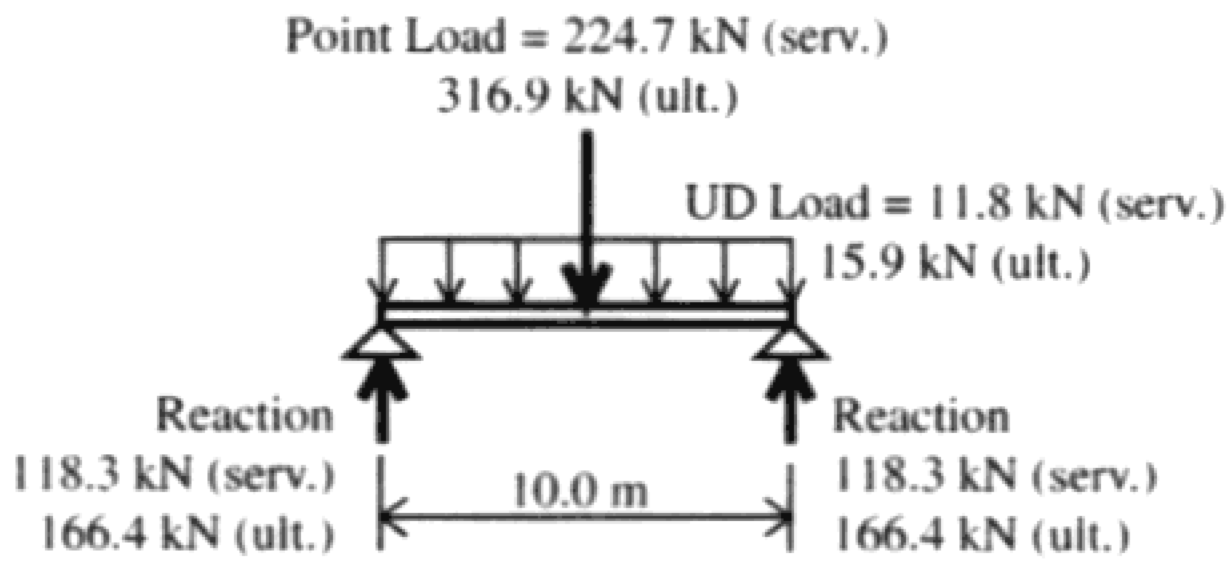
Beam C

UD loads on beam C		Dead loads (self-weight)	Imposed loads
Steel beam	$1.18 \times 10.0 =$	11.8 kN	-
Total		11.8 kN	nil

Total UD load for serviceability limit states $= 11.8 \times 1.00$ $= 11.8$ kN

Total UD load for ultimate limit states $= 11.8 \times 1.35$ $= 15.9$ kN

Figure 1.5(d) shows the loads and reactions for beam C



Note that each reaction is half the total load on the beam

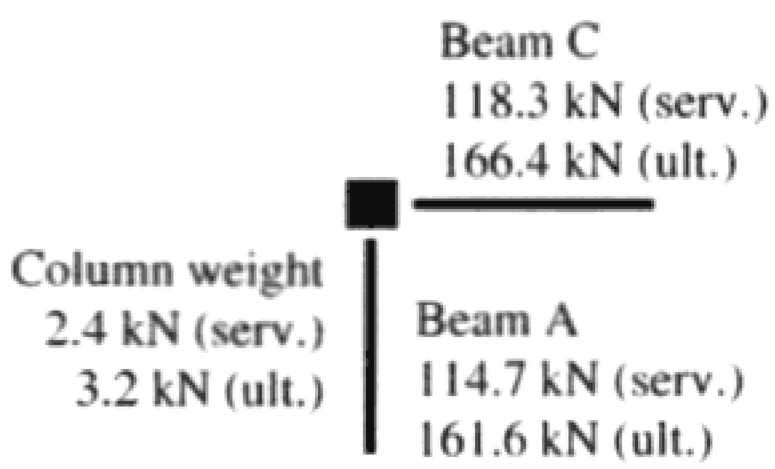
Figure 1.5(d): Beam C isolated

Columns

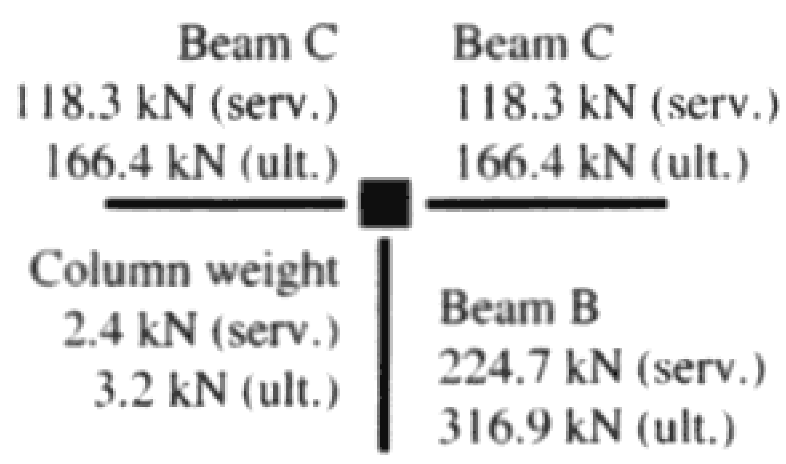
Self-weight of one column SLS $= 0.98 \times 2.4$ $= 2.4$ kN

Self-weight of one column ULS $= 1.35 \times 2.4$ $= 3.2$ kN

Column One



Column Two



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Calculations for Example 1.4 (Continued from previous page)

Total column load for SLS = 118.3 + 114.7 + 2.4 = 235.4 kN Total column load for ULS = 166.4 + 161.6 + 3.2 = 331.2 kN	Total column load for SLS = 2 × 118.3 + 224.7 + 2.4 = 463.7 kN Total column load for ULS = 2 × 166.4 + 316.9 + 3.2 = 652.9 kN

Example 1.5

A series of reinforced concrete beams at 5.0-m centres span 7.5 m onto reinforced concrete columns 3.5 m high, as shown in Figure 1.6. The beams, which are 575 mm deep by 250 mm wide, carry a 175-mm-thick reinforced concrete slab, which can be considered as simply supported. The columns are 250 mm by 250 mm in cross-section. The slab carries a screed weighing 1.4 kN/m^2 and an imposed load of 3.0 kN/m^2.

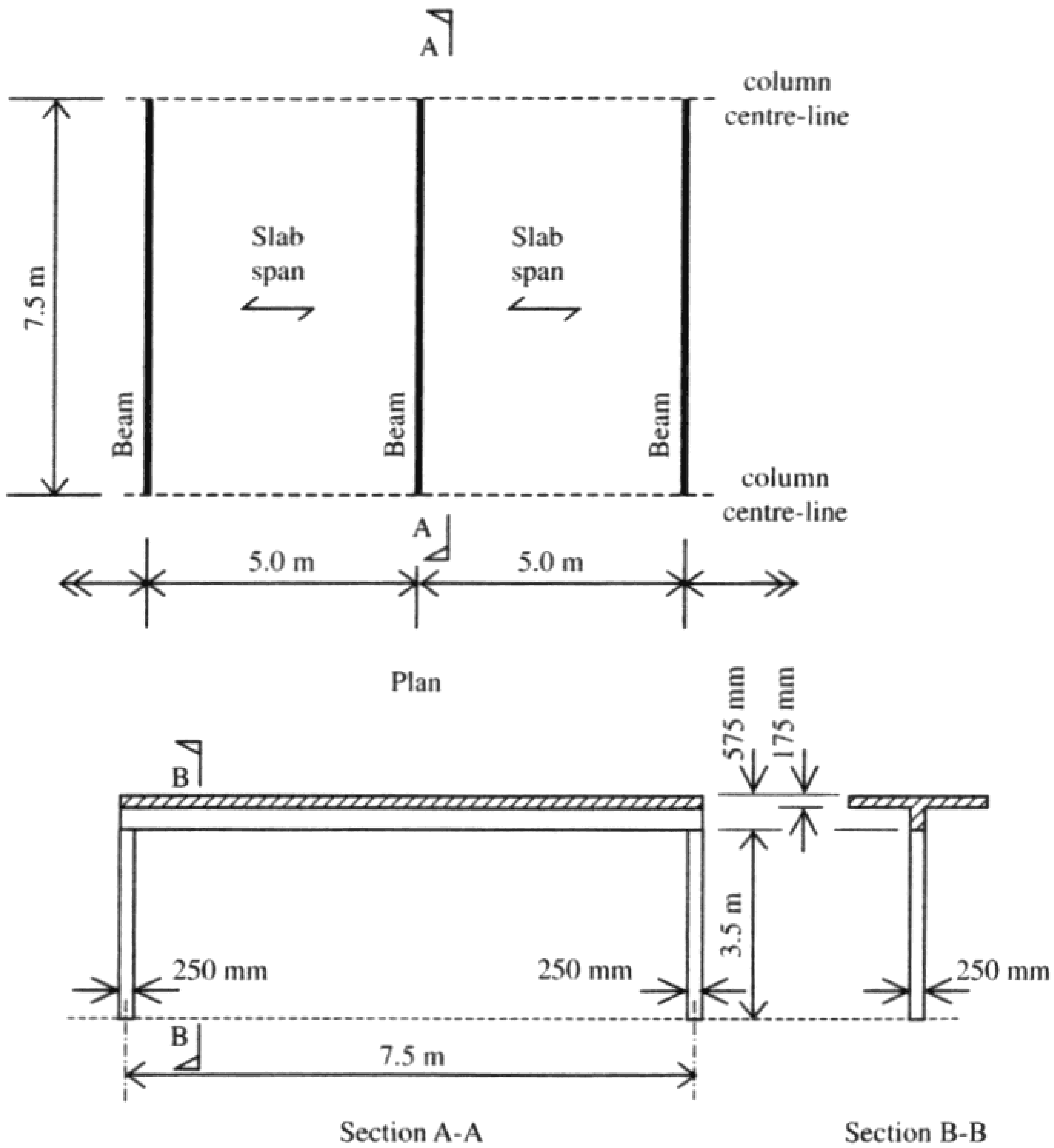


Figure 1.6: Arrangement of beams and columns for Example 1.5

Find the total UD load on one beam, the reaction to one column and the loads from one column to its foundation.

Calculations for Example 1.5			
Concrete slab self-weight	$= 0.175 \times 25$		$= 4.4\,kN/m^2$
Total self-weight of slab + screed	$= 4.4 + 1.4$		$= 5.8\,kN/m^2$
Concrete beam self-weight	$= 0.40 \times 0.25 \times 25$		$= 2.5\,kN/m$
Concrete column self-weight	$= 0.25 \times 0.25 \times 25$		$= 1.6\,kN/m$
UD loads on beam		Dead loads (self-weight)	Imposed loads
Concrete slab + screed	$5.8 \times 7.5 \times 5.0 =$	217.5 kN	-
Concrete beam	$2.5 \times 7.5 =$	18.8 kN	-
Imposed load	$3.0 \times 7.5 \times 5.0 =$	-	112.5 kN
Totals		236.3 kN	112.5 kN
Total UD load for SLS	$= 236.3 \times 1.00 + 112.5 \times 1.00$		$= 348.8\,kN$
Total UD load for ULS	$= 236.3 \times 1.35 + 112.5 \times 1.50$		$= 487.8\,kN$
Beam reaction transmitted to column at SLS	$= 348.8/2$		$= 174.4\,kN$
Loads on columns			
Beam reaction transmitted to column at ULS		$= 487.8/2$	$= 243.9\,kN$
Column self-weight		$= 1.6 \times 3.5$	$= 5.6\,kN$
Column foundation load at SLS = beam reaction + column self-weight		$= 174.4 + 5.6$	$= 180.0\,kN$
Column foundation load at ULS		$= 243.9 + 5.6 \times 1.35$	$= 251.5\,kN$

1.6 Structural Mechanics

Before the size of a structural element can be determined it is first necessary to know the forces, shears and bending moments acting on that element. It is also necessary to know how these will be resisted by the element. The principles of structural mechanics are needed for these calculations.

This manual is not a textbook on structural mechanics and readers unfamiliar with this subject should seek guidance elsewhere. However, two particular aspects of structural mechanics are sufficiently important to be repeated here. They are:

- The theory of bending
- The behaviour of compression members.

Shears and bending moments in beams

Starting from the loads carried by a beam, the following steps are suggested.

- Calculate the reactions
- Draw a load diagram for the beam
- Draw the shear force (SF) diagram for the beam
- Draw the bending moment (BM) diagram for the beam, noting that the BM will have a maximum value at a point where the SF is zero.

Figure 1.7(a) shows these diagrams, plus the formula for maximum deflection, for a simply supported beam carrying a uniformly distributed load. In the formulas for deflection, E is the elastic modulus of the material and I is the second moment of area of the beam cross-section. Figure 1.7(b) is for a similar beam with a central point load. Other standard cases

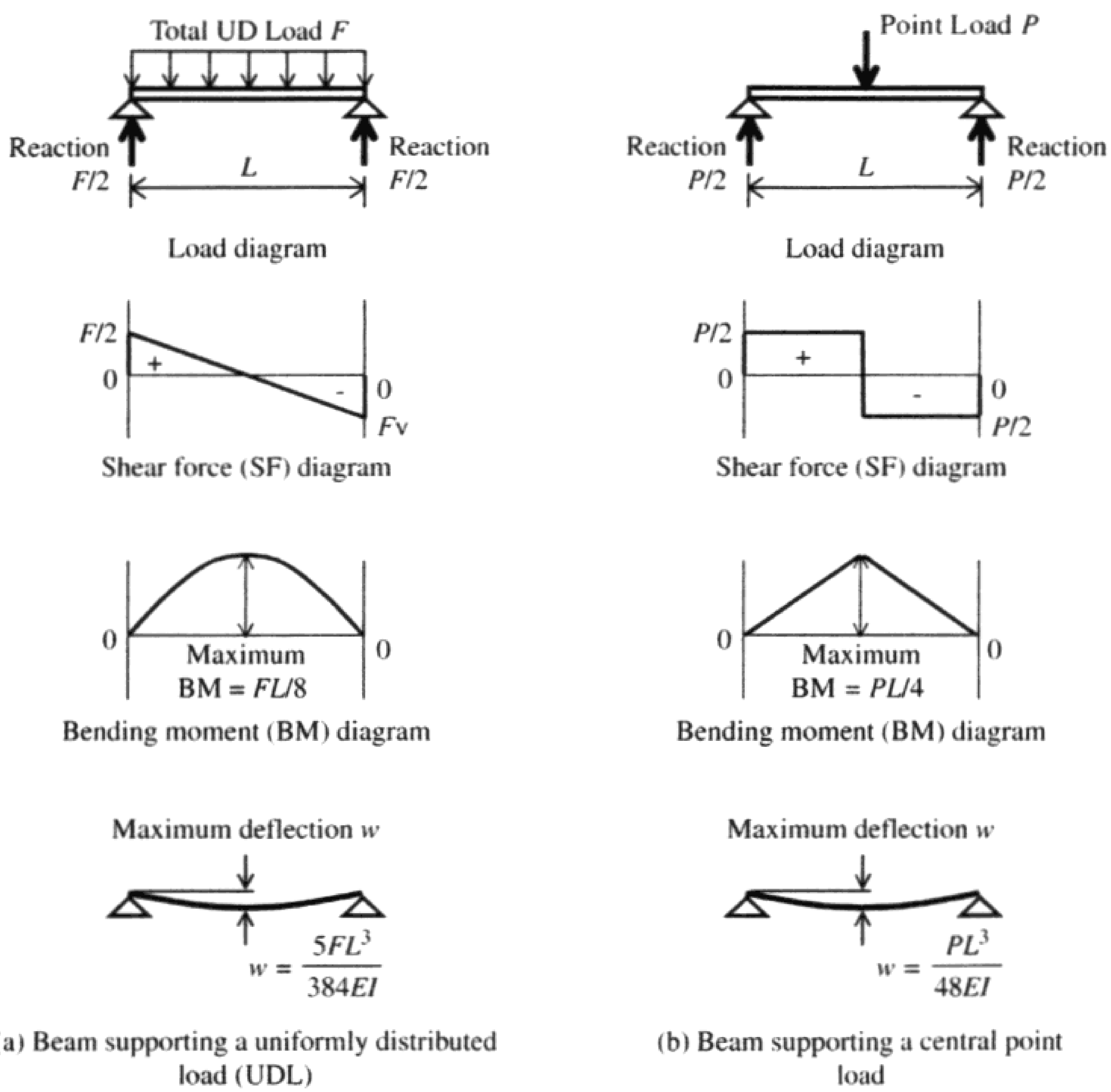


Figure 1.7: Reactions, shear forces, bending moments and deflections for standard loading conditions on simply supported beams

can be found in various design manuals, and non-standard cases can be calculated from first principles.

1.7 Design of Beams for Bending Moment

If a beam section is not to fail under load, an internal moment of resistance (MR) must be available within the beam at least equal to the maximum BM produced by the loads. That is:

Internal MR is at least as great as external

The comparison may be carried out under one or both of the following conditions.

(i) Under serviceability loads, when the behaviour of the material in the beam is elastic.

(ii) Under ultimate loads, when the behaviour of the material in the beam may be plastic.

Elastic and plastic material behaviour of ductile materials

Under serviceability loads the material behaviour will generally be elastic and will follow Hooke's law:

$$\text{Stress} = E \times \text{strain}$$

where E = Young's modulus for the material.

If the stress reaches the ultimate strength of the material and the loads are increased, it may be possible for the material to deform without breaking. The idealised stress/strain relationship for a ductile material is shown in Figure 1.8.

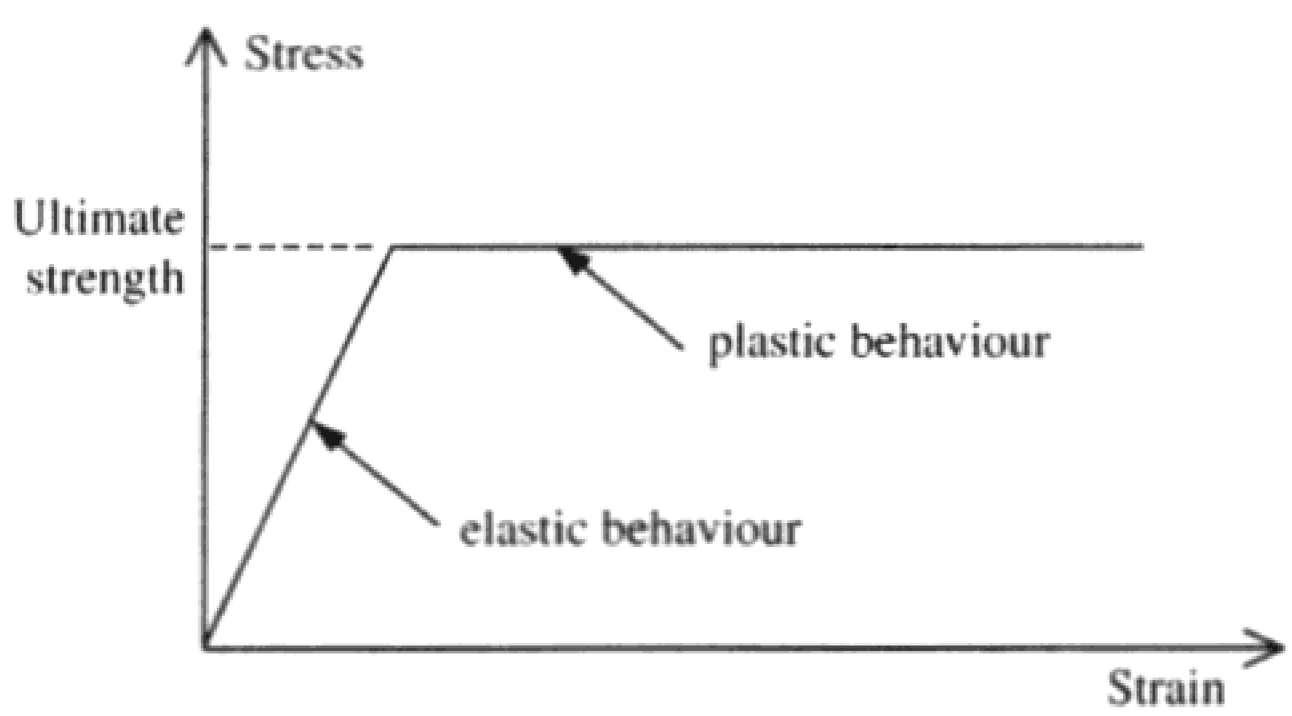


Figure 1.8: Idealised stress/strain relationship for a ductile material

Bending stresses in beams: elastic behaviour

Stresses will vary from zero on the neutral axis (NA) to maximum at top and bottom.

The governing equation for elastic bending is

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

where

M = bending moment
I = second moment of area of the beam, which is a geometric property of the beam shape.
f = stress in the material
y = distance from the NA
E = Young's modulus of elasticity for the material.
R = radius of curvature after bending.

The term E/R relates to the deformation of the beam and is used in calculating deflections. It is not required for strength calculations, so the equation reduces to

$$\frac{M}{I} = \frac{f}{y}$$

which can be rearranged as

$$M = f\frac{I}{y} \text{ or as } f = M\frac{y}{I}$$

We are interested in maximum stresses which occur when y is greatest, that is at the top or bottom of the beam. Both I and y are properties of the shape of the section, and we can usefully define a new property W_{el}.

$$W_{el} = \text{elastic section modulus} = I/y_{max}$$

The equations above can be rewritten as

$$\text{(i)} \quad M = fW_{el} \qquad \text{(ii)} \quad f = \frac{M}{W_{el}} \qquad \text{(iii)} \quad W_{el} = \frac{M}{f}$$

These equations can be used in design as follows:

- Equation (i) may be used to calculate the greatest moment M that can be carried by a beam of known size (W_{el} known) and material (maximum permissible f known).
- Equation (ii) may be used to calculate the maximum stress f occurring within a beam of known size (W_{el} known) when subject to an applied bending moment (M known).

- Equation (iii) may be used to find the beam property W_{el} needed if a beam of known material (maximum permissible f known) is to carry a certain applied bending moment (M known).

Bending stresses in beams: plastic behaviour

A beam made of a ductile material will have a moment of resistance (MR) greater than M in equation (i) above. This is because the material in the top and bottom of the beam can deform plastically, so allowing the stresses in material near the neutral axis to increase. When the BM on the beam reaches the MR, all the material in the beam will be at the same stress.

By analogy with elastic behaviour we can define

$$W_{pl} = \text{plastic section modulus}$$

and re-write two of the elastic equations:

$$\text{(i)} \quad M = fW_{pl} \qquad \text{(iii)} \quad W_{pl} = \frac{M}{f}$$

These equations can be used in design as follows:

- Equation (i) may be used to calculate the greatest moment M that can be carried by a beam of known size (W_{pl} known) and material (maximum permissible f known).
- Note that equation (ii) for elastic bending has no equivalent in the context of plastic behaviour.
- Equation (iii) may be used to find the beam property W_{pl} needed if a beam of known material (maximum permissible f known) is to carry a certain applied bending moment (M known).

Figure 1.9 illustrates the stresses and equivalent forces in a rectangular beam under elastic and plastic conditions. For a rectangular beam W_{pl} is 50% greater than W_{el}, so the internal MR under plastic conditions is 50% greater than the internal MR under elastic conditions.

Figure 1.10 illustrates stresses and forces in a rectangular beam, an I beam and a reinforced concrete (RC) beam.

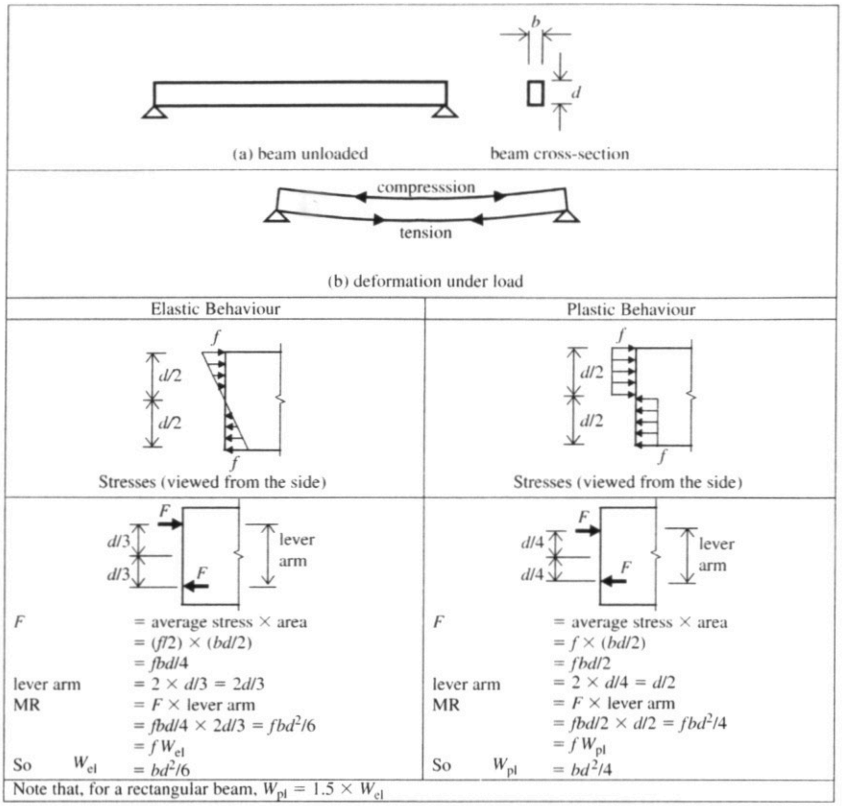


Figure 1.9: Elastic and plastic bending stresses in a rectangular beam

Examples 1.6 to 1.9 show how beams can be sized on the basis of elastic behaviour under serviceability loads. Later chapters of this manual that relate to specific materials explain sizing based on behaviour under ultimate loads.

Example 1.6

A rectangular timber beam spanning 4.5 m supports a UDL F of 3.75 kN including its self-weight, as shown in Figure 1.11. Taking the breadth b of the beam as 47 mm and the allowable stress f (assuming elastic behaviour) to be 7 N/mm^2, what depth d of beam is required?

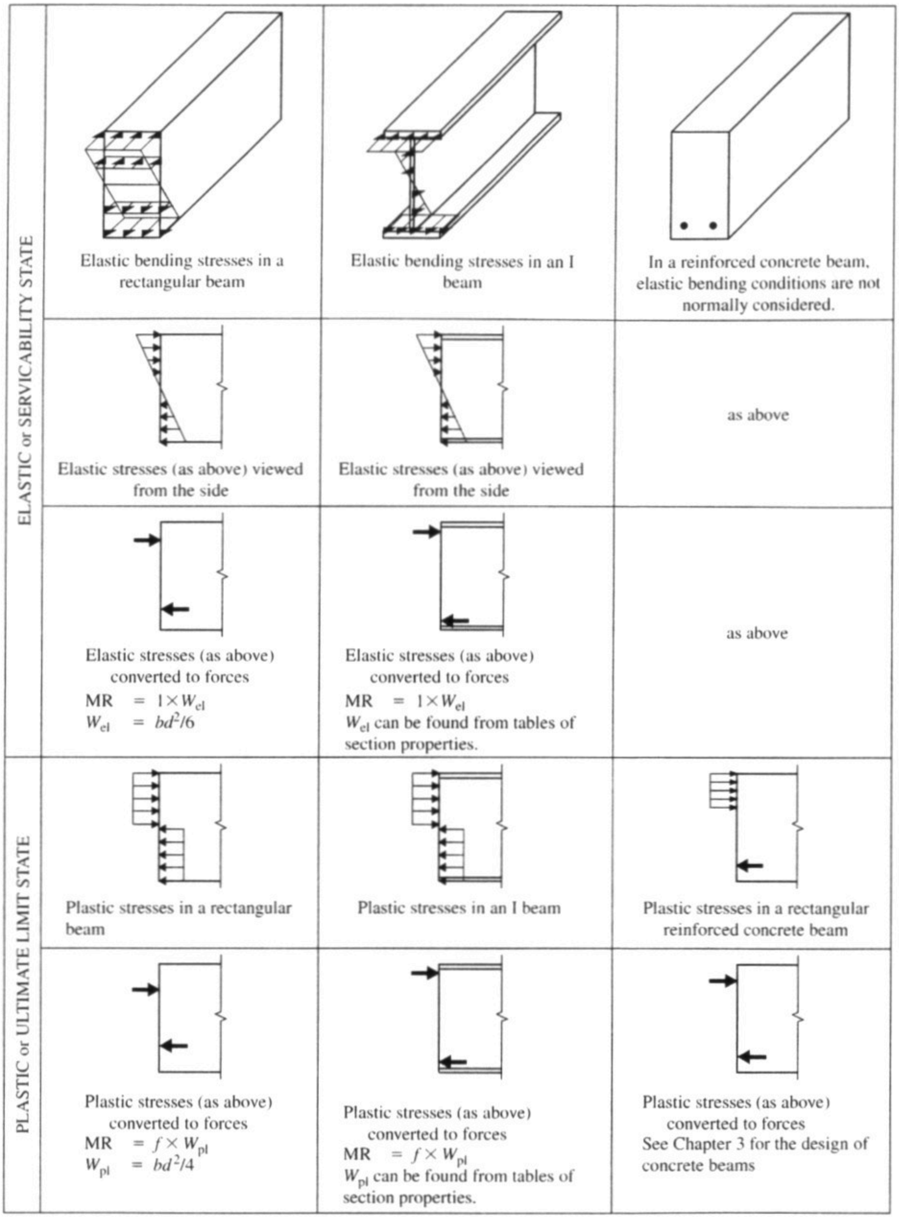

ELASTIC or SERVICABILITY STATE
Elastic bending stresses in a rectangular beam
Elastic bending stresses in an I beam
In a reinforced concrete beam, elastic bending conditions are not normally considered.
Elastic stresses (as above) viewed from the side
Elastic stresses (as above) viewed from the side
as above
Elastic stresses (as above) converted to forces
MR = $1 \times W_{el}$
W_{el} = $bd^2/6$
Elastic stresses (as above) converted to forces
MR = $1 \times W_{el}$
W_{el} can be found from tables of section properties.
as above
PLASTIC or ULTIMATE LIMIT STATE
Plastic stresses in a rectangular beam
Plastic stresses in an I beam
Plastic stresses in a rectangular reinforced concrete beam
Plastic stresses (as above) converted to forces
MR = $f \times W_{pl}$
W_{pl} = $bd^2/4$
Plastic stresses (as above) converted to forces
MR = $f \times W_{pl}$
W_{pl} can be found from tables of section properties.
Plastic stresses (as above) converted to forces
See Chapter 3 for the design of concrete beams

Figure 1.10

Calculations for Example 1.6

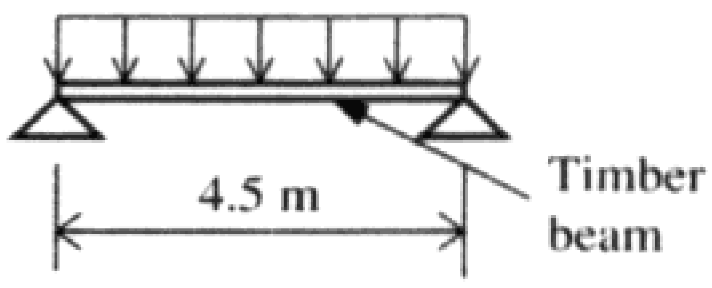


Figure 1.11: Load diagram for Example 1.6

The elastic section modulus of the rectangular beam $W_{el} = bd^2/6$ $= 47d^2/6\,\text{mm}^3$

The internal MR of the beam is $fW_{el} = 7 \times 47d^2/6$ $= 54.8d^2\,\text{Nmm}$

The external BM $= FL/8 = 3.75 \times 4.5/8$ $= 2.11\,\text{kNm}$

It is necessary to use compatibly units for both quantities, so convert the external BM into newton and millimetre units. Multiply the force units by 10^3 to convert from kilonewtons to newtons, and multiply the length units by 10^3 to convert from metres to millimetres

External BM $= 3.75 \times 10^3 \times 4.5 \times 103/8$ $= 2.11 \times 10^6\,\text{Nmm}$

The internal MR should be at least as big as the external BM $54.8d^2 \geq 2.11 \times 10^6$

$$d^2 \geq 2.11 \times 10^6/54.8 = 38\,503\,\text{mm}^2$$

$$d \geq \sqrt{38\,503} = 196\,\text{mm}$$

Use a 47 mm × 200 mm timber beam.

Example 1.7

Calculate the depth required for the timber beam shown in Figure 1.12(a) if the breadth of the beam b is 75 mm and the permissible bending stress (assuming elastic behaviour) is 8.5 N/mm^2. An allowance for the self-weight of the beam has been included with the point loads.

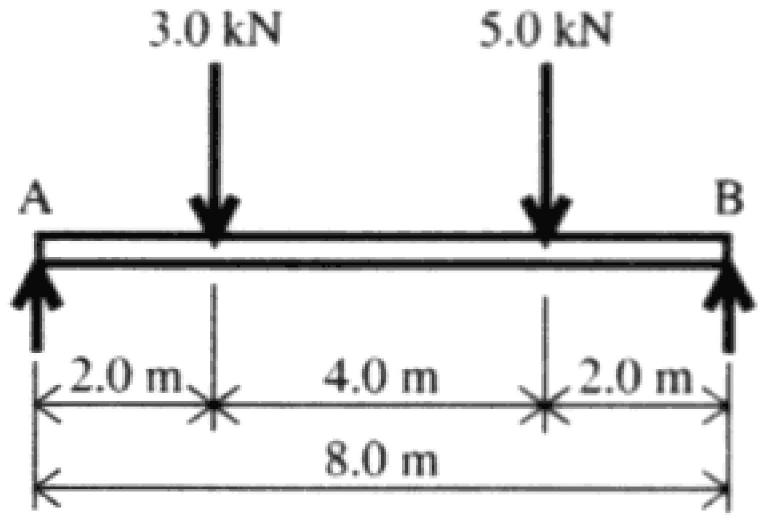


Figure 1.12(a): Load diagram

Calculations for Example 1.7

To complete the load diagram it is first necessary to calculate the reactions (Figure 1.12b)

Take moments about end B, with clockwise moments positive and anti-clockwise moments negative:

$8R_A - (3.0 \times 6) - (5.0 \times 2) = 0$

$8R_A - 18 - 10 = 0$

$8R_A = 28$

$R_A = 28/8$ $\qquad R_A = 3.5\,\text{kN}$

So $R_B = 3.0 + 5.0 - 3.5$ $\qquad R_B = 4.5\,\text{kN}$

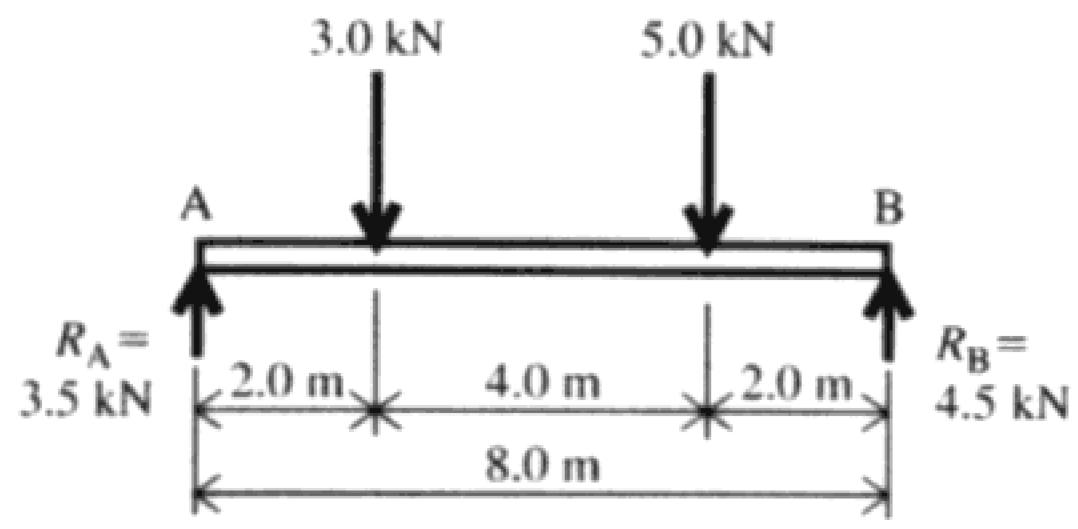


Figure 1.12(b): Load diagram and reactions

Knowing the reactions, the SF diagram Figure 1.12(c) can be constructed

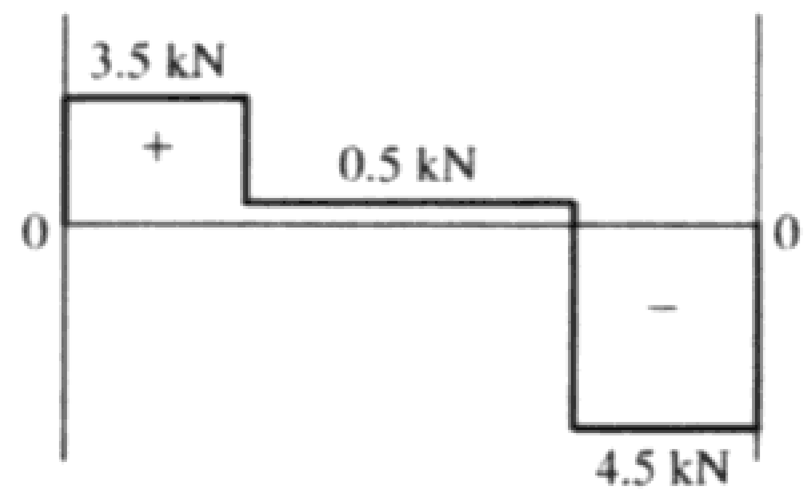


Figure 1.12(c): SF diagram

This shows that a point of zero SF, and therefore a point of maximum BM, occurs under the 5.0-kN point load

BM under the 3.0-kN point load $= 3.5 \times 2$ $\qquad = 7.0\,\text{kNm}$

BM under the 5.0-kN point load $= 4.5 \times 2$ $\qquad = 9.0\,\text{kNm}$

The BM diagram for the beam is in Figure 1.12(d)

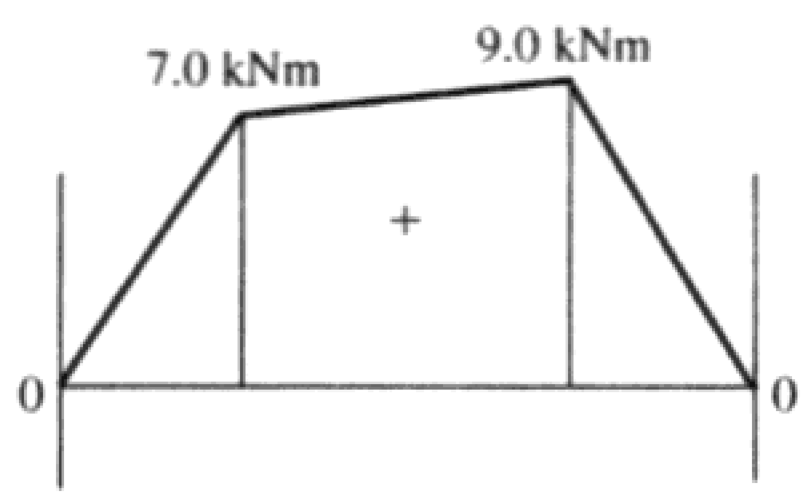


Figure 1.12(d): BM diagram

Continued on next page

Calculations for Example 1.7 (Continued from previous page)	
The elastic section modulus of the rectangular beam $W_{el} = bd^2/6$	$W_{el} = 75d^2/6\,mm^3$
The internal MR of the beam is $fW_{el} = 8.5 \times 75d^2/6$	$fW_{el} = 106.3d^2\,Nmm$
The maximum external BM = 9.0 kNm	$= 9.0 \times 10^6\,Nmm$
The internal MR should be at least as big as the external BM	
$106.3d^2 \geq 9.0 \times 10^6$	
$d^2 \geq 9.0 \times 10^6/106.3 = 84666\,mm^2$	
$d \geq \sqrt{84666}$	$d \geq 291\,mm$
	Use a 75 mm × 300 mm timber beam

Example 1.8

A steel beam supports a total UDL F, including its self-weight, of 70 kN over a span L of 5.0 m. If the permissible bending stress f (assuming elastic behaviour) is 180 N/mm², find the elastic modulus W_{el} required for the beam and choose a suitable Universal Beam section.

Calculations for Example 1.8	
Internal MR should be at least the maximum external BM	
$fW_{el} \geq FL/8$	
$180W_{el} \geq (70 \times 10^3 \times 5.0 \times 10^3)/8$	
$W_{el} \geq (70 \times 10^3 \times 5.0 \times 10^3)/(8 \times 180)$	$W_{el} \geq 243056\,mm^3$
In tables of steel section properties (see Chapter 5), the elastic modulus W_{el} is given in cm³	
$W_{el} \geq 243056 \times 10^{-3}\,cm^3$	$W_{el} \geq 243.1\,cm^3$
From the section tables (see Chapter 5) a 254 × 102 × 25 kg/m Universal Beam section which has $W_{el,y} = 266\,cm^3$ would be suitable	

Example 1.9

A timber beam spanning $L = 5.0$ m supports a UDL F of 4.0 kN which includes its self-weight. The beam is 100 mm wide and 200 mm deep, and the E value of the timber is 6600 N/mm². Find the maximum bending stress f in the timber and the deflection w produced by the load.

Calculations for Example 1.9	
We know $b = 100$ mm and $d = 200$ mm; f is to be found	
$f \times bd^2/6 = FL/8$ $\quad$ $(f \times 100 \times 200^2)/6 = (4.0 \times 10^3 \times 5.0 \times 10^3)/8$	
So $f = (4.0 \times 5.0 \times 10^6 \times 6)/(8 \times 100 \times 200^2)$	$F = 3.75\,N/mm^2$
From standard deflections results for a uniformly distributed load	$W = 5FL^3/384EI$

The second moment of area I for a rectangular section $b \times d$ is given by

$$I = bd^3/12 = (100 \times 200^3)/12 \qquad I = 66.7 \times 10^6\,\text{mm}^4$$

So $w = (5 \times 4.0 \times 10^3 \times 5000^3)/(384 \times 6600 \times 66.7 \times 10^6)$ $\qquad W = 14.8\,\text{mm}$

1.8 Compression Members

Compression members are those elements within a structure that primarily resist compressive forces. The most obvious examples in a building are the main vertical support members to the roof and floors, and other examples are certain components of trusses or bracing systems. Different names are given to the vertical support elements in different materials.

Structural material	**Name given to vertical support element**
Timber	Post or stud
Reinforced concrete	Column
Steelwork	Stanchion
Masonry	Wall or pier

The vertical loads they support can be concentric or eccentric. If the load is concentric its line of application coincides with the neutral axis (NA) of the member (see Figure 1.13a). Such compression members are said to be axially loaded and the stress induced is a direct compressive stress.

$$\text{Direct compressive stress} = (\text{Axial force})/(\text{Cross-section area})$$

In practice the vertical load is often applied eccentrically so that its line of action is eccentric to the NA of the member (see Figure 1.13b). This induces both bending stresses and compressive stresses in the member.

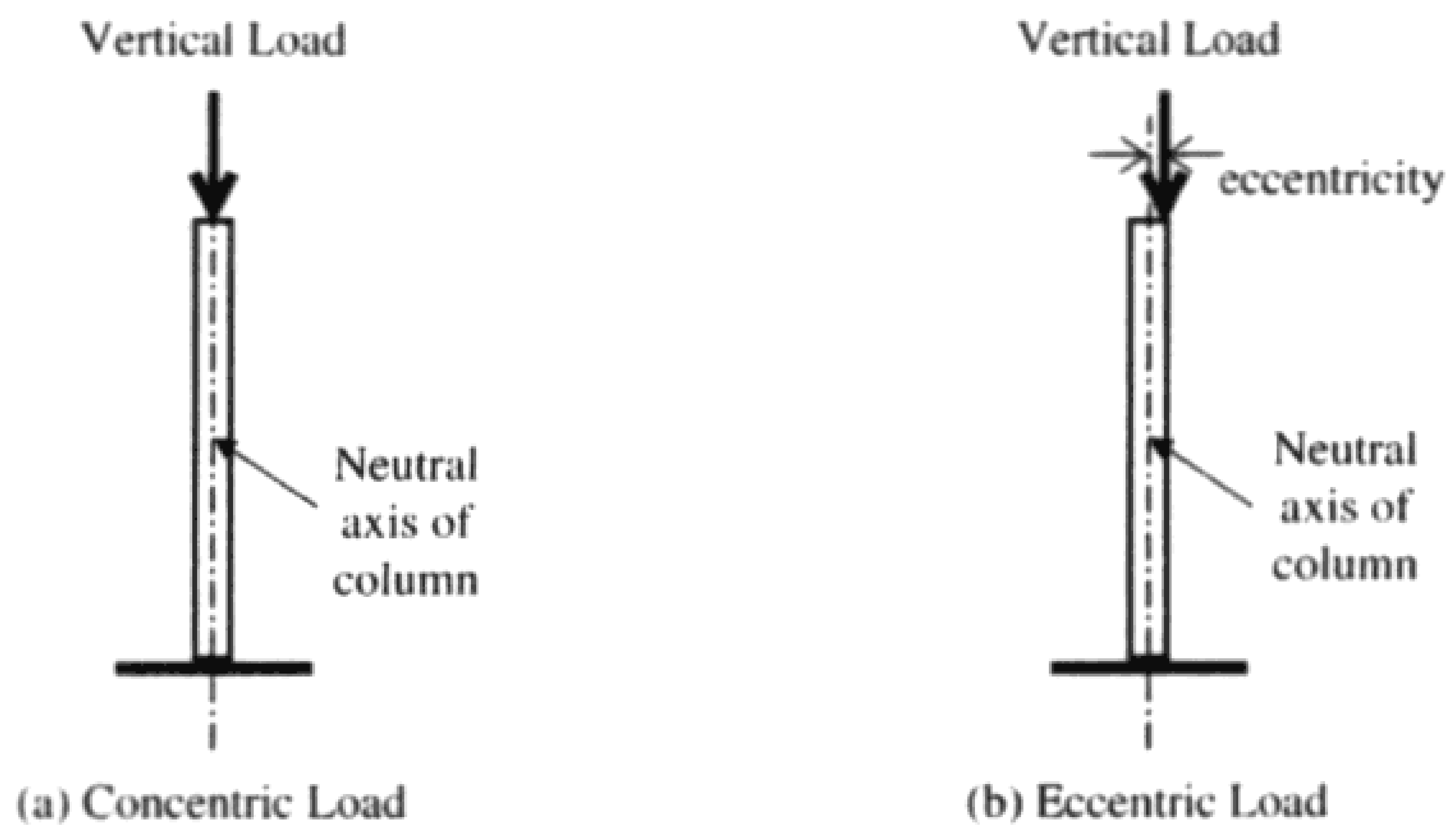


Figure 1.13: Concentric and eccentric loads on columns

In relation to their modes of failure, compression members may fail

- either by material crushing due to direct compression
- or by buckling due to combination of direct compression and compressive bending stresses.

Different words are used to describe compression members in relation to their mode of failure (see Figure 1.14).

Predominant mode of failure	**Classification**
Material crushing	Short or stocky
Buckling	Long or slender

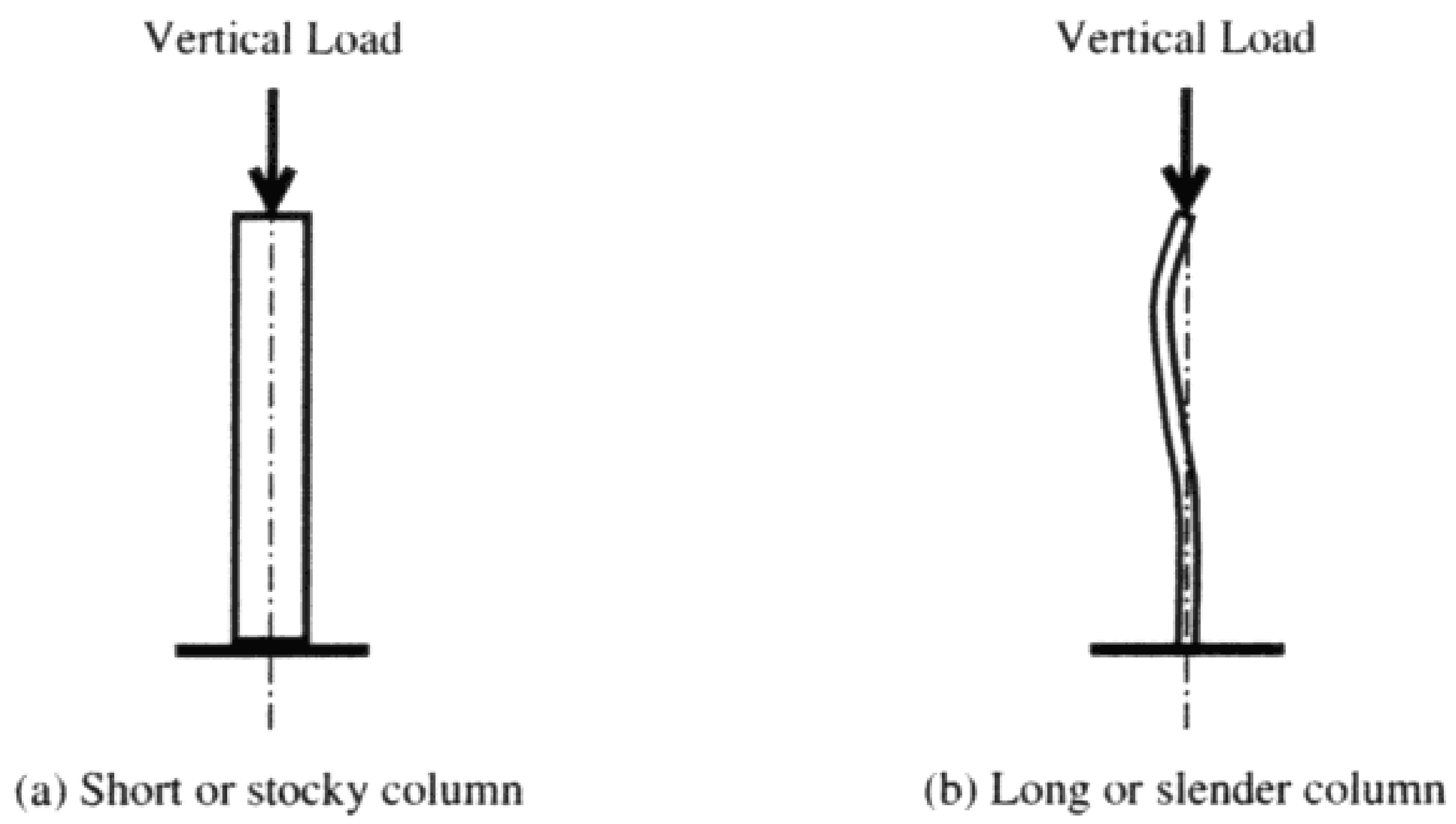


Figure 1.14: Failure modes of columns

Since long columns fail due to a combination of crushing and lateral buckling, the permissible stress is related to their slenderness. This depends on the column height, its cross-sectional geometry, and how it is held at the top and bottom.

The factor which governs the permissible stress of a long column is its slenderness ratio λ. This is the ratio of the effective length to the radius of gyration of the column.

$$\text{Slenderness ratio}\lambda = \frac{\text{effective length}}{\text{radius of gyration}}$$

Designers should be aware that for masonry design, dealt with in Chapter 4 of this manual, the term 'slenderness ratio' is normally taken to mean

$$\text{Slenderness ratio}\lambda = \frac{\text{effective length}}{\text{effective thickness}}$$

which is inconsistent with the more correct definition employing the radius of gyration. Thus the term 'slenderness ratio' has two meanings, and care is needed to avoid confusion.

The effective length of a column is controlled by the way that it is held at each end: its end fixity. By effective length we mean that length of the column which is subject to lateral buckling.

A column located in position at both ends but not held rigidly will buckle over a length equal to its actual height. A column held rigidly at both ends will buckle over a distance less than the full height. These and other situations are illustrated in Figure 1.15, and calculations of effective length for design purposes are given in the chapters of this manual that relate to specific materials.

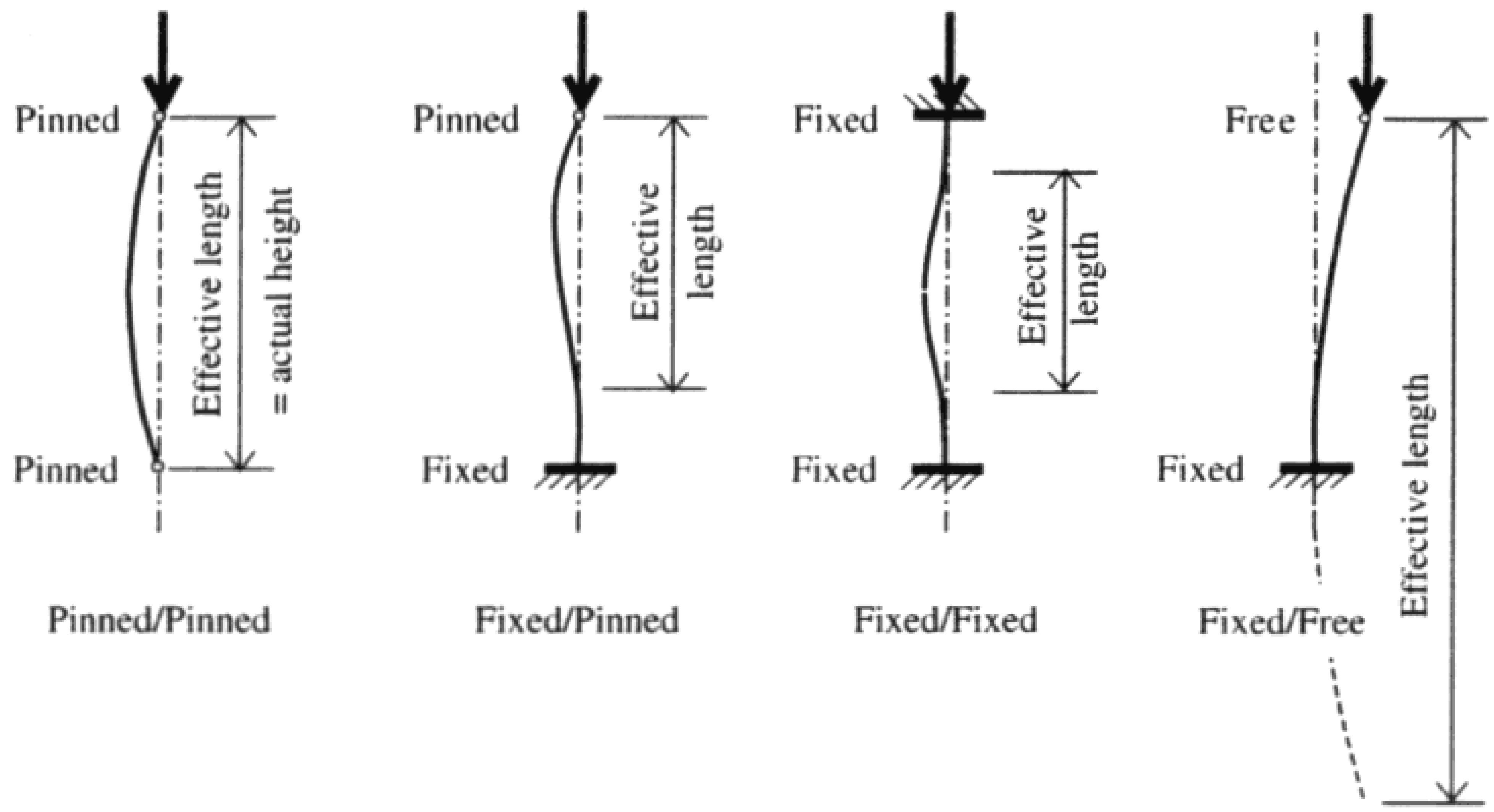


Figure 1.15: Effective lengths of columns

1.9 Material Properties

Structural design is carried out using characteristic values of material strength and stiffness. As mentioned in Section 1.4, the design of individual structural elements is carried out using partial safety factors for material strength, denoted by γ_m. The value of γ_m to be used varies depending on the material in use. These aspects are explained further in the chapters of this manual which relate to specific materials.

1.10 Summary

Use of a methodical procedure for structural design will minimise the chance of error. In relation to the general matters dealt with in this chapter, such a procedure may be summarised by the following steps:

(a) Evaluate the loads acting on the structure.

(b) Determine the loads acting on the individual structural elements.

(c) Calculate the forces, shears and bending moments induced in each member by the loads.

The remainder of this manual gives guidance for the final step:

(d) Design the members.

CHAPTER 2

Timber Elements

Contents

This chapter covers the design of solid timber elements, and some information on other timber products is also included for comparison.

2.1 Structural Design of Timber

Guidance on the design of all timber structures is given in BS EN 1995 (EC5) which is published in several sections as shown in Table 2.1.

Table 2.1: Codes relating to the design of timber

	BSI reference	Title
EC5 Part 1-1	**BS EN 1995 Part 1-1**	**General – Common rules and rules for buildings**
EC5 Part 1-2	BS EN 1995 Part 1-2	General rules – Structural fire design
EC6 Part 2	BS EN 1995 Part 2	Bridges

Each code should be read with the appropriate National Annex. The part shown in **bold** is relevant to this manual.

Additional useful information, including advice on UK practice as it relates to the use of EC5, is given in the *Manual for the design of timber building structures to Eurocode 5*, IStructE/TRADA, December 2007.

EC5 uses limit state design methods as set out in Chapter 1 of this manual. Generally the following load combinations are used:

For checking ultimate limit states (ULS)	$\gamma_G G_k + \gamma_Q Q_k = 1.35G_k + 1.50Q_k$
For checking serviceability limit states (SLS)	$\gamma_G G_k + \gamma_Q Q_k = 1.00G_k + 1.00Q_k$

The partial safety factors used for materials are:

For checking ULS	γ_m from Table 2.2
For checking SLS	$\gamma_m = 1.00$

Table 2.2: Partial factors γ_m for material properties and resistances at ULS

Material	γ_m
Solid timber, treated or untreated	1.3
Glued laminated timber (Glulam)	1.25
Laminated veneer lumber (LVL), plywood, oriented strand board (OSB)	1.2
Fibreboard	1.3
Connections, except for punched metal fasteners	1.3
Punched metal plate fasteners, anchorage strength	1.3
Punched metal plate fasteners, plate (steel) strength	1.15

Source: Table 2.3 of EC5 Part 1-1

Member axes

Member properties are referred to the principal axes, which are shown in Figure 2.1. The *x-x* axis is along the length of the member in the direction of the grain, the *y-y* and *z-z* axes are the major and minor bending axes.

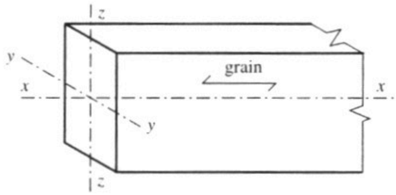


Figure 2.1: Member axes

Symbols

Table 2.3 shows some of the symbols used in the design of structural timber and gives some guidance on their meaning.

Table 2.3: Symbols used in the design of structural timber

Symbol	Normal units	Meaning	Comment
A	mm^2	Area of cross section	
b	mm	Breadth of section	
d	mm	Diameter of fastener (e.g. nail or bolt)	
E	kN/mm^2	Elastic modulus of timber	Values are given in Tables 2.5 and 2.6
f	N/mm^2	Strength of timber	See Table 2.4 for suffixes used with the symbol f
G	kN/mm^2	Shear modulus of timber	Normally for timber $G = E/16$
h	mm	Depth of section	
i	mm	Radius of gyration of section	For a rectangular section $i_y = 0.289h$, $i_z = 0.289b$
I	mm^4	Second moment of area of section	For a rectangular section, $I_y = bh^3/12$, $I_z = b^3\,h/12$
k_c		Factor for the effect of buckling on axial compressive strength	See Table 2.24
$k_{c,90}$		Factor giving enhanced strength at bearings	See Figure 2.2 $k_{c,90}$ takes values between 1 and 4
k_{def}		Factor for the effect of service class on long-term deflection	See Table 2.21
k_h		Depth factor, giving enhanced bending strength for members smaller than a stated size	See Table 2.16
k_m		Factor for combined bending	See Table 2.15 For rectangular sections of solid timber, $k_m = 0.7$
k_{mod}		Factor for the effect of load duration and service class on strength	See Table 2.14
k_{sys}		System strength factor, used when several members can share a load	If four or more similar parallel members can share the load $k_{sys} = 1.1$
k_v		Factor reducing shear strength at notched ends	See Section 2.8.3
w	mm	Vertical deflection of a beam	
W	mm^3	Elastic section modulus	For a rectangular section, $W_{yy} = bh^2/6$, $W_{zz} = b^2\,h/6$
γ_m		Partial safety factor for materials	For solid timber at ULS $\gamma_m = 1.3$ See Table 2.2 for other products
λ		Slenderness ratio	λ = effective length/radius of gyration
ρ	kg/m^3	Density of timber	
σ	N/mm^2	Stress in timber	See Table 2.4 for suffixes used with the symbol σ
ψ		Combination factors from EC0	See Chapter 1, Section 1.4

EC5 makes extensive use of suffixes in symbols. Where a symbol f or σ has two or three suffixes separated by commas, Table 2.4 gives guidance on the meaning of the suffixes.

Table 2.4: Suffixes for symbols f and σ

First suffix	Middle suffix if present	Last suffix
m – bending t – tension c – compression v – shear	0 – direction parallel to grain 90 – direction perpendicular to grain y – bending about y-y axis z – bending about z-z axis	d – design value mean – mean value k – characteristic (95%) value 0.05 – 5% value

2.2 Timber Strength Classes

Of all the materials used in construction, timber is unique by virtue of being entirely natural. Whilst this gives it a deserved aesthetic appeal, it also creates an initial problem for the structural designer.

In order to design any structural component efficiently it is necessary to know in advance the properties of the material to be used. Timber presents a problem in this respect since we have no apparent control over its quality. To overcome this difficulty, pieces of natural timber are tested and allocated a strength class, which indicates their properties.

Pieces of timber are first stress graded, either visually or by machine. Visual grading, which is carried out by trained graders, assesses the size and frequency of specific physical defects such as knots, slope of grain, rate of growth, wane, resin pockets and distortion. Machine grading measures the bending stiffness of each piece of timber by passing it through a set of rollers and measuring the force required to flex the piece by a set amount. Methods of stress grading are described in BS EN 14081, and timber suppliers are required to establish test regimes which reliably correlate stress grade and timber species information to strength classes. Each piece of timber is marked to show its strength class. Although it is not necessary for designers to have a detailed knowledge of stress grading methods, they do need to appreciate the necessity for individual pieces of timber to be graded in order to be allocated a strength class. If a previously graded piece of timber is significantly reduced in cross-section, particularly if it is sawn lengthways into two pieces, then the original grading no longer applies and consequently it would be necessary to re-grade the timber.

Unless there are particular requirements for appearance or durability it is not necessary for the designer to specify either the tree species or the stress grade. The simple specification of a strength class will give the supplier a wide range of possible timber sources and so allow an economical choice to be made.

Table 2.5 gives the strength properties for softwood and poplar in strength classes C14 to C50, and Table 2.6 gives this information for hardwoods in strength classes D30 to D70. Unless there are particular requirements for appearance, durability or very high strength, softwood products are likely to be more economical than hardwoods. Designers should normally specify either C16 or C24 timber as these are the most readily available.

Table 2.5: Strength classes: characteristic values for softwoods and poplar

		C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50
Strength properties (in N/mm^2)													
Bending	$f_{m,k}$	14	16	18	20	22	24	27	30	35	40	45	50
Tension parallel	$f_{t,0,k}$	8	10	11	12	13	14	16	18	21	24	27	30
Tension perpendicular	$f_{t,90,k}$	0.4	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6
Compression parallel	$f_{c,0,k}$	16	17	18	19	20	21	22	23	25	26	27	29
Compression perpendicular	$f_{c,90,k}$	2.0	2.2	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.2
Shear	$f_{v,k}$	1.7	1.8	2.0	2.2	2.4	2.5	2.8	3.0	3.4	3.8	3.8	3.8
Stiffness properties (in kN/mm^2)													
Mean E parallel	$E_{0,mean}$	7	8	9	9.5	10	11	11.5	12	13	14	15	16
5% E parallel	$E_{0,0.05}$	4.7	5.4	6.0	6.4	6.7	7.4	7.7	8.0	8.7	9.4	10.0	10.7
Mean E perpendicular	$E_{90,mean}$	0.23	0.27	0.30	0.32	0.33	0.37	0.38	0.40	0.43	0.47	0.50	0.53
Mean G	G_{mean}	0.44	0.50	0.56	0.59	0.63	0.69	0.72	0.75	0.81	0.88	0.91	1.00
Density (in kg/m^3)													
Density	ρ_k	290	310	320	330	340	350	370	380	400	420	440	460
Mean density	ρ_{mean}	350	370	380	390	410	420	450	460	480	500	520	550

These properties relate to timber at a temperature of 20°C and a relative humidity of 65%.
It may be difficult to obtain timber of class C45 or C50.
Source: Table 1 of BS EN 338.

Table 2.6: Strength classes: characteristic values for hardwoods

		D30	D35	D40	D50	D60	D70
Strength properties (in N/mm^2)							
Bending	$f_{m,k}$	30	35	40	50	60	70
Tension parallel	$f_{t,0,k}$	18	21	24	30	36	42
Tension perpendicular	$f_{t,90,k}$	0.6	0.6	0.6	0.6	0.6	0.6
Compression parallel	$f_{c,0,k}$	23	25	26	29	32	34
Compression perpendicular	$f_{c,90,k}$	8.0	8.4	8.8	9.7	10.5	13.5
Shear	$f_{v,k}$	3.0	3.4	3.8	4.6	5.3	6.0

(*Continued*)

Table 2.6: (Continued)

Stiffness properties (in kN/mm²)							
Mean E parallel	$E_{0,mean}$	10	10	11	14	17	20
5% E parallel	$E_{0,0.05}$	8.0	8.7	9.4	11.8	14.3	16.8
Mean E perpendicular	$E_{90,mean}$	0.64	0.69	0.75	0.93	1.13	1.33
Mean G	G_{mean}	0.60	0.65	0.70	0.88	1.06	1.25
Density (in kg/m³)							
Density	ρ_k	530	560	590	650	700	900
Mean density	ρ_{mean}	640	670	700	780	840	1080

These properties relate to timber at a temperature of 20°C and a relative humidity of 65%.
Source: Table 1 of BS EN 338.

2.3 Variation of Timber Stiffness and Strength with Load Duration and Service Class

When any material is stressed by an amount less than its ultimate strength it will undergo an immediate strain, and if the stress is maintained at a constant level for some time the strain will increase. This phenomenon is known as creep. All structural materials including concrete, steel and masonry are subject to creep, but timber is particularly susceptible. If the timber is kept dry then the strain after 10 years may be 60% more than the immediate strain, or 200% more if the timber becomes wet. In a similar way the strength of timber for long-term loads may be 50% less than its strength for short-term loads, and dry timber is stronger than wet timber. Thus the design stiffness and design strength vary with the duration of the load and with the moisture content of the timber.

The limit state design method takes account of the duration of the load by assigning a load duration class, and these are defined in Table 2.7.

Table 2.7: Load duration classes

	Approximate duration	**Examples of loading**
Permanent	More than 10 years	Self-weight
Long-term	6 months to 10 years	Storage loading (including in lofts), water tanks
Medium-term	1 week to 6 months	Imposed floor load
Short-term	Less than 1 week	Snow, maintenance or man loading on roofs, residual structure after accidental event
Instantaneous		Wind, impact loading, explosion

Source: Table NA2 of UK National Annex to EC5 Part 1-1.

Timber will rapidly adjust its moisture content according to the environmental conditions. The limit state design process takes account of moisture content by assigning a service class, and these are defined in Table 2.8.

Table 2.8: Service classes

Service Class	Definition	Examples
1	Temperature of 20 °C, relative humidity of surrounding air only exceeding 65% for a few weeks per year. Moisture content of timber generally not more than 16%	Warm roofs, intermediate floors, internal and party walls
2	Temperature of 20 °C, relative humidity of surrounding air only exceeding 85% for a few weeks per year. Moisture content of timber generally not more than 20%	Cold roofs, ground floors, external walls and external uses where timber is protected from direct wetting
3	Conditions leading to higher moisture contents than in service class 2	External uses, fully exposed

Source: Clause 2.3.1.3 of EC5 Part 1-1 and Table NA2 of UK National Annex to EC5 Part 1-1.

In the limit state design process the load duration class and service class are used as follows.

- The ULS variation of timber strength with load duration and service class is taken into account by use of the k_{mod} factor (Table 2.14) as explained in Section 2.6.
- The SLS variation of timber stiffness with load duration and service class is taken into account by use of the k_{def} factor (Table 2.21) as explained in Section 2.8.4.

2.4 Solid Timber

Timber suppliers normally stock a number of standard sizes. Table 2.9 shows the commonly available lengths of timber, and it may not always be easy to obtain longer lengths. Designers should consider the use of proprietary timber beams, referred to in Section 2.9, when members longer than 5.4 m are required.

Table 2.9: Commonly available lengths of softwood structural timber

Length (m)			
2.4	3.00 3.30 3.60 3.90	4.20 4.50 4.80	5.10 5.40

Source: Table NA1 of BS EN 336.

Timber may be supplied sawn or planed. Sawn timber is generally more economical because it requires less machining and there is less wastage of timber in its production, but it has a rough surface and planed timber is normally preferred when the finished work will be exposed to view. Tolerances on cross-section dimensions are shown in Table 2.10. Sawn timber should comply with tolerance class T1, planed timber with tolerance class T2.

Table 2.10: Tolerances on cross-section dimensions

	Dimension up to 100 mm	Dimension over 100 mm
Tolerance class T1 – sawn timber	−1 mm, +3 mm	−2 mm, +4 mm
Tolerance class T2 – planed timber	−1 mm, +1 mm	−1.5 mm, +1.5 mm

Source: Clause 4.3 of BS EN 336.

Tables 2.11 and 2.12 give the target cross-section sizes commonly available for sawn and planed timber respectively. It can be seen that the stock sizes of planed timber are produced from the stock sizes of sawn timber by removing between 1.5 mm and 2.5 mm from the faces. The tables also show the geometric properties of the sections, which are calculated from section dimensions $b \times h$ as follows:

Area:	$A = bh$	
Elastic section modulus:	$W_{yy} = bh^2/6$	$W_{zz} = b^2h/6$
Second moment of area:	$I_{yy} = bh^3/12$	$I_{zz} = b^3h/12$
Radius of gyration:	$i_y = 0.289h$	$i_z = 0.289b$

The sizes are standardized at 20% moisture content. Cross-section dimensions change approximately +0.25% for every +1% change in moisture content up to 30%. Thus at 18% moisture content the cross-section sizes will be about 0.5% smaller than the tabulated values, and at 16% moisture content they will be about 1% smaller. However, EC5 states that geometric data for cross-sections may be taken as target values for solid timber or as nominal sizes for proprietary products, so it is not necessary to take account of these dimension changes in the structural design process.

2.5 Durability

In most situations a designer will specify a strength class of timber and the choice of timber species will be made by the supplier. If a particular species is not acceptable (e.g. for reasons of durability), the designer should make this clear in the project specification.

Where timber is used inside an occupied and heated building then adequate natural durability can be assumed. In situations where the moisture content of the timber is likely to remain above 18% then BS EN 350-2 gives guidance on species which have good natural durability, and advice on preservative treatments is given in BS EN 351-1 and BS EN 460. It should be noted that preservative treatments may affect the strength and stiffness properties of the timber and may also promote corrosion of metal fastenings.

Table 2.11: Commonly available sawn softwood structural timber: target sizes and section properties

Target size (mm)	Area A (10^3mm^2)	Elastic section modulus		Second moment of area		Radius of gyration	
		W_{yy} (10^3mm^3)	W_{zz} (10^3mm^3)	I_{yy} (10^6mm^4)	I_{zz} (10^6mm^4)	i_y (mm)	i_z (mm)
22 × 100	2.20	36.7	8.1	1.83	0.09	28.9	6.4
38 × 100	3.80	63.3	24.1	3.17	0.46	28.9	11.0
38 × 150	5.70	142.5	36.1	10.69	0.69	43.3	11.0
38 × 175	6.65	194.0	42.1	16.97	0.80	50.5	11.0
38 × 200	7.60	253.3	48.1	25.33	0.91	57.7	11.0
38 × 225	8.55	320.6	54.2	36.07	1.03	65.0	11.0
47 × 75	3.53	44.1	27.6	1.65	0.65	21.7	13.6
47 × 100	4.70	78.3	36.8	3.92	0.87	28.9	13.6
47 × 125	5.88	122.4	46.0	7.65	1.08	36.1	13.6
47 × 150	7.05	176.3	55.2	13.22	1.30	43.3	13.6
47 × 175	8.23	239.9	64.4	20.99	1.51	50.5	13.6
47 × 200	9.40	313.3	73.6	31.33	1.73	57.7	13.6
47 × 225	10.58	396.6	82.8	44.61	1.95	65.0	13.6
47 × 250	11.75	489.6	92.0	61.20	2.16	72.2	13.6
63 × 150	9.45	236.3	99.2	17.72	3.13	43.3	18.2
63 × 175	11.03	321.6	115.8	28.14	3.65	50.5	18.2
63 × 200	12.60	420.0	132.3	42.00	4.17	57.7	18.2
63 × 225	14.18	531.6	148.8	59.80	4.69	65.0	18.2
75 × 100	7.50	125.0	93.8	6.25	3.52	28.9	21.7
75 × 150	11.25	281.3	140.6	21.09	5.27	43.3	21.7
75 × 175	13.13	382.8	164.1	33.50	6.15	50.5	21.7
75 × 200	15.00	500.0	187.5	50.00	7.03	57.7	21.7
75 × 250	18.75	781.3	234.4	97.66	8.79	72.2	21.7
75 × 300	22.50	1125.0	281.3	168.75	10.55	86.6	21.7
100 × 100	10.00	166.7	166.7	8.33	8.33	28.9	28.9
100 × 150	15.00	375.0	250.0	28.13	12.50	43.3	28.9
100 × 200	20.00	666.7	333.3	66.67	16.67	57.7	28.9
100 × 225	22.50	843.8	375.0	94.92	18.75	65.0	28.9
100 × 250	25.00	1041.7	416.7	130.21	20.83	72.2	28.9
100 × 300	30.00	1500.0	500.0	225.00	25.00	86.6	28.9
150 × 150	22.50	562.5	562.5	42.19	42.19	43.3	43.3
150 × 300	45.00	2250.0	1125.0	337.50	84.38	86.6	43.3
300 × 300	90.00	4500.0	4500.0	675.00	675.00	86.6	86.6

Timber in other sizes is available to order.
In the UK these sizes are commonly available in strength classes C16 and C24.
Source: sizes from Table NA2 of BS EN 336.

Table 2.12: Commonly available softwood structural timber planed on four sides: target sizes and section properties

Target size (mm)	Area A (10^3 mm^2)	Elastic section modulus		Second moment of area		Radius of gyration	
		W_{yy} (10^3 mm^3)	W_{zz} (10^3 mm^3)	I_{yy} (10^6 mm^4)	I_{zz} (10^6 mm^4)	i_y (mm)	i_z (mm)
35 × 97	3.40	54.9	19.8	2.66	0.35	28.0	10.1
35 × 145	5.08	122.6	29.6	8.89	0.52	41.9	10.1
35 × 170	5.95	168.6	34.7	14.33	0.61	49.1	10.1
35 × 195	6.83	221.8	39.8	21.63	0.70	56.3	10.1
35 × 220	7.70	282.3	44.9	31.06	0.79	63.5	10.1
44 × 72	3.17	38.0	23.2	1.37	0.51	20.8	12.7
44 × 97	4.27	69.0	31.3	3.35	0.69	28.0	12.7
44 × 120	5.28	105.6	38.7	6.34	0.85	34.6	12.7
44 × 145	6.38	154.2	46.8	11.18	1.03	41.9	12.7
44 × 170	7.48	211.9	54.9	18.01	1.21	49.1	12.7
44 × 195	8.58	278.9	62.9	27.19	1.38	56.3	12.7
44 × 220	9.68	354.9	71.0	39.04	1.56	63.5	12.7
44 × 245	10.78	440.2	79.1	53.92	1.74	70.7	12.7
60 × 145	8.70	210.3	87.0	15.24	2.61	41.9	17.3
60 × 170	10.20	289.0	102.0	24.57	3.06	49.1	17.3
60 × 195	11.70	380.3	117.0	37.07	3.51	56.3	17.3
60 × 220	13.20	484.0	132.0	53.24	3.96	63.5	17.3
72 × 97	6.98	112.9	83.8	5.48	3.02	28.0	20.8
72 × 145	10.44	252.3	125.3	18.29	4.51	41.9	20.8
72 × 170	12.24	346.8	146.9	29.48	5.29	49.1	20.8
72 × 195	14.04	456.3	168.5	44.49	6.07	56.3	20.8
72 × 220	15.84	580.8	190.1	63.89	6.84	63.5	20.8
72 × 245	17.64	720.3	211.7	88.24	7.62	70.7	20.8
97 × 97	9.41	152.1	152.1	7.38	7.38	28.0	28.0
97 × 145	14.07	339.9	227.4	24.64	11.03	41.9	28.0
97 × 195	18.92	614.7	305.8	59.94	14.83	56.3	28.0
97 × 220	21.34	782.5	345.0	86.07	16.73	63.5	28.0
97 × 245	23.77	970.4	384.2	118.87	18.63	70.7	28.0
97 × 295	28.62	1406.9	462.6	207.52	22.44	85.2	28.0
145 × 145	21.03	508.1	508.1	36.84	36.84	41.9	41.9
145 × 295	42.78	2103.1	1033.7	310.21	74.95	85.2	41.9

Timber in other sizes is available to order.
In the UK these sizes are commonly available in strength classes C16 and C24.
Source: sizes from Table NA2 of BS EN 336.

Fasteners may require protection against corrosion, and Table 2.13 is a summary of the minimum requirements of EC5 Part 1-1, Table 4.1. For further details on the requirements see EC5.

Table 2.13: Corrosion protection for fasteners

Fastener	Service class		
	1	2	3
Nails and screws with d up to 4 mm	None	Galvanized	Heavy galvanized
Bolts, dowels, nails and screws with $d > 4$ mm	None	None	Heavy galvanized
Staples	Galvanized	Galvanized	Stainless steel
Punched metal plate fasteners, steel plates up to 3 mm thick	Galvanized	Galvanized	Stainless steel
Steel plates 3–5 mm thick	None	Galvanized	Heavy galvanized
Steel plates over 5 mm thick	None	None	Heavy galvanized

For engineered timber products the durability of the glue should also be checked.

2.6 Load Duration and Service Class: The k_{mod} Factor for Design at ULS

2.6.1 Timber Strength at ULS

As noted in Section 2.3 the strength of timber varies with load duration and moisture content. Characteristic strength values from Tables 2.5 and 2.6 should be multiplied by the appropriate k_{mod} factor from Table 2.14 which is based on Table 3.1 of EC5 Part 1-1. Missing entries in the table indicate where materials are unsuitable for use. For instance, oriented strand board (OSB) is not suitable for use under service class 3 conditions.

It is worth noting that the strength and stiffness timber properties shown in Tables 2.5 and 2.6 are based on short-term (i.e. lasting a few minutes) tests on dry timber. Thus dry timber strengths for instantaneous loads are generally 10% higher ($k_{mod} = 1.1$) and dry timber strengths for short-term loads which may last several days are generally 10% lower ($k_{mod} = 0.9$). Timber in service class 3 has a higher moisture content and so a lower strength ($k_{mod} < 1$ for all load duration classes).

2.6.2 A Rule of Thumb for ULS Strength Checks

In general it may be necessary to check a timber element at ULS for several combinations of loading with a different k_{mod} factor for each combination. For instance, the checks for a solid timber element in service class 1 or 2 carrying dead load and imposed load would be:

Check under dead load only with $k_{mod} = 0.60$ for permanent actions

Check under dead load plus imposed load with $k_{mod} = 0.80$ for medium-term actions.

Table 2.14: Values of k_{mod} for timber and wood-based products at ULS

Material	Standard	Service class	Load duration class, see Table 2.7				
			Permanent action	Long-term action	Medium-term action	Short-term action	Instantaneous
Solid timber	EN 14081-1	1	0.60	0.70	0.80	0.90	1.10
		2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
Glulam	EN 14080	1	0.60	0.70	0.80	0.90	1.10
		2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
LVL	EN 14374,	1	0.60	0.70	0.80	0.90	1.10
	EN 14279	2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
OSB	EN 300						
	– OSB/2	1	0.30	0.45	0.65	0.85	1.10
	– OSB/3 or OSB/4	1	0.40	0.50	0.70	0.90	1.10
	– OSB/3 or OSB/4	2	0.30	0.40	0.55	0.70	0.90

Source: Table 3.1 of EC5 Part 1-1.
If a load combination includes loads belonging to different load duration classes, a value of k_{mod} corresponding to the load with the shortest duration should be used.

Example 2.1 Load cases for ULS strength check

Consider two timber beams each carrying dead load and imposed load over a span of 4.5 m. The dead load is the same on both beams but the imposed load on Beam Two is much smaller than the imposed load on Beam One.

Beam One: Dead load $G_k = 10.0$ kN, Imposed load $Q_k = 12.0$ kN
Beam Two: Dead load $G_k = 10.0$ kN, Imposed load $Q_k = 2.5$ kN.

Calculations for Example 2.1
Beam One:
Load Case 1: Dead load only
Ultimate load $= 1.35G_k = 1.35 \times 10.0 = 13.5$ kN
Moment $= 13.5 \times 4.5^2/8 = 34.2$ kNm
$k_{mod} = 0.60$ for permanent actions
Design strength required $= 34.2/0.60 = 57.0$ kNm

Load Case 2: Dead load plus Imposed load

Ultimate load $= 1.35G_k + 1.50Q_k = 1.35 \times 10.0 + 1.50 \times 12.0 = 31.5\,\text{kN}$

Moment $= 31.5 \times 4.5^2/8 = 79.7\,\text{kNm}$

$k_{mod} = 0.80$ for medium term actions

Design strength required $= 79.7/0.80 = 99.6\,\text{kNm}$

Beam size will be determined by Load Case 2

Beam Two:

Load Case 1: Dead load only

Ultimate load $= 1.35G_k = 1.35 \times 10.0 = 13.5\,\text{kN}$

Moment $= 13.5 \times 4.5^2/8 = 34.2\,\text{kNm}$

$k_{mod} = 0.60$ for permanent actions

Design strength required $= 34.2/0.60 = 57.0\,\text{kNm}$

Load Case 2: Dead load plus Imposed load

Ultimate load $= 1.35G_k + 1.50Q_k = 1.35 \times 10.0 + 1.50 \times 2.5 = 17.3\,\text{kN}$

Moment $= 17.3 \times 4.5^2/8 = 43.8\,\text{kNm}$

$k_{mod} = 0.80$ for medium-term actions

Design strength required $= 43.8/0.80 = 54.8\,\text{kNm}$

Beam size will be determined by Load Case 1

Generally it can be shown that Load Case 1 will only be critical if the imposed load is less 0.3 times the dead load, and this is unlikely because timber floors are generally light in relation to the load that they carry. Superfluous calculation may be avoided by noting the following:

Authors' rule of thumb for timber beams and joists:

For solid timber elements carrying dead load and imposed load, the dead-load-only ULS check is not required if the imposed load is more than 0.3 times the dead load.

2.7 System Strength: The k_{sys} Factor for Design at ULS

When several equally spaced similar members are connected by a continuous load distribution system, the member strength properties may be multiplied by a system strength factor $k_{sys} = 1.1$. The load distribution system may be floor- or roof-boarding, battens, purlins or binders, and must be capable of transferring loads from one member to neighbouring members. In most floor and roof structures with four or more similar parallel members, the members can be designed with $k_{sys} = 1.1$.

2.8 Timber Beams and Joists

Horizontal timber members that resist vertical load primarily in bending are generally referred to as either beams or joists. A single isolated member is usually called a beam while several similar parallel timbers, supporting a floor or roof, are usually called joists.

2.8.1 Bending ULS

When a member is bent about both axes the stresses should satisfy these equations from EC5 Part 1-1, Clause 6:

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \text{ and } k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1$$

The factor k_m makes allowance for re-distribution of stresses and the inhomogeneities of the material in a cross-section. Values of k_m are given in Table 2.15.

Table 2.15: Values of k_m

Material	Section shape	Value of k_m
Solid timber, Glulam, LVL	Rectangular sections	$k_m = 0.7$
	Other cross sections	$k_m = 1.0$
All other wood products	Any section	$k_m = 1.0$

In most situations members are bent about one axis only, so the expressions simplify to

$$\sigma_{m,d} \leq f_{m,d}$$

The tension and bending strengths in Tables 2.5 and 2.6 apply to solid timber members with cross-section dimensions of 150 mm or more. Smaller sections have higher tension and bending strengths calculated using the k_h factor from Table 2.16.

Table 2.16: Depth factor k_h for solid timber in bending

Depth factor	Timber depth h (mm)										
	40 or less	50	60	70	80	90	100	110	120	130	140	150 or more
k_h	1.30	1.25	1.20	1.16	1.13	1.11	1.08	1.06	1.05	1.03	1.01	1.00

Formula: k_h is the minimum of $(150/h)^{0.2}$ or 1.3, but not less than 1.0.
Source: Equation 3.1 of EC5 Part 1-1.

The design bending strength $f_{m,d}$ is found from the characteristic timber strength $f_{m,k}$ by applying the k_{mod}, k_h and γ_m factors:

$$f_{m,d} = k_{mod} \times k_h \times f_{m,k} / \gamma_m$$

Example 2.2 Check of a timber beam at the bending ULS

A planed softwood beam of C16 timber is 72 mm wide × 220 mm deep, has an effective span of $L = 4.25$ m and carries the following unfactored loads:

Dead load	2.0 kN
Imposed load	3.5 kN.

The beam is part of the ground floor of an office building and is laterally restrained by the flooring boards. Check whether its bending strength at ULS is adequate.

Calculations for Example 2.2	
A ULS check under dead load only is not required because the imposed load is more than 0.3 times the imposed load	
Service class, from Table 2.8, for a ground floor	2
Load duration class, from Table 2.7, for imposed loading	Medium
Elastic section modulus W_{yy}, from Table 2.12, for a joist 72 mm × 220 mm	$W_{yy} = 580.8 \times 10^3\,mm^3$
Bending strength $f_{m,k}$ from Table 2.5, for C16 timber	$f_{m,k} = 16\,N/mm^2$
Depth factor from Table 2.16, for $d = 220$ mm	$k_h = 1.0$
Partial safety factor γ_m from Table 2.2, for solid timber	$\gamma_m = 1.3$
Modification factor k_{mod} from Table 2.14, for solid timber in Service Class 2 and medium-term loading	$k_{mod} = 0.8$
Design bending strength $f_{m,d} = k_{mod} \times k_h \times f_{m,k}/\gamma_m = 0.8 \times 1.0 \times 16/1.3$	$f_{m,d} = 9.85\,N/mm^2$
Ultimate moment of resistance $M_{ult} = f_{m,d} \times W_{yy}$ $= 9.85 \times 580.8 \times 10^3$	$M_{ult} = 5.72 \times 10^6\,Nmm = 5.72\,kNm$
From the data given	
Total dead load = 2.0 kN	$G_k = 2.0\,kN$
Total imposed load = 3.5 kN	$Q_k = 3.5\,kN$
Ultimate load $= \gamma_G G_k + \gamma_Q Q_k = 1.35 \times 2.0 + 1.50 \times 3.5$	$F = 7.95\,kN$
$M = FL/8 = 7.95 \times 4.25/8$	$M = 4.22\,kNm$
M is less than M_{ult}	Beam is satisfactory at bending ULS

Lateral torsional buckling

In many cases the compression edge of a joist or beam will be fully restrained against lateral displacement by decking or by battens and bracing. If the compression edge is not restrained in this way, then it may be necessary to reduce the design compressive strength of the timber to take account of lateral torsional bucking (LTB). However, if the height-to-breadth ratio of the beam is within the limits given in Table 2.17 then the beam is not subject to LTB.

EC2 gives methods for determining the resistance of beams which are subject to LTB but these are not covered by this manual. Most solid timber beams will be within the limits in Table 2.17.

Table 2.17: Maximum depth-to-breadth ratios of solid timber beams to avoid LTB

Lateral support	Maximum ratio
None	2:1
Ends restrained against rotation	3:1
Ends restrained against rotation and member held in line as by purlins or tie rods at centres not more than 30 times the breadth of the member	4:1
Ends restrained against rotation and compression edge held in line, as by direct connection of sheathing, deck or joists	5:1
Ends restrained against rotation, and compression edge held in line as by direct connection of sheathing, deck or joists, together with adequate bridging or blocking at intervals not exceeding six times the depth of the member	6:1
Ends restrained against rotation and both edges held firmly in line	7:1

Source: Table 4.3 of the IStructE *Manual for the design of timber building structures to Eurocode 5.*

Example 2.3 Bending strength of a softwood joist

A simply supported, planed softwood joist 44 mm wide and 120 mm deep in C24 timber has a span of 3.5 m. The joist supports nailed chipboard decking which provides lateral support to the top of the joist. Assuming Service Class 1 and short-term loads, find the design bending strength of the joist at the ULS.

Calculations for Example 2.3	
The compression flange has continuous restraint from the chipboard decking	
From Table 2.17 the limiting height/breadth ratio is 5:1	
Actual height/breadth ratio = 120:44 = 2.7:1	
So the beam is not subject to lateral torsional buckling	Beam is not subject to LTB
Elastic section modulus W_{yy}, from Table 2.12, for a joist 44 mm × 120 mm	$W_{yy} = 105.6 \times 10^3\,\text{mm}^3$
Bending strength $f_{m,k}$ from Table 2.5, for C24 timber	$f_{m,k} = 24\,\text{N/mm}^2$

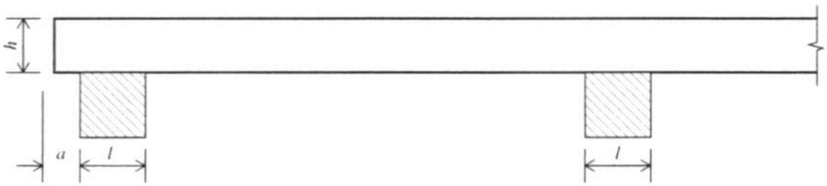
Depth factor from Table 2.16, for d = 120 mm	$k_h = 1.05$
Partial safety factor γ_m from Table 2.2, for solid timber	$\gamma_m = 1.3$
Modification factor k_{mod} from Table 2.14, for solid timber in service class 1 and short-term loading	$k_{mod} = 0.9$
Design bending strength $f_{m,d} = k_{mod} \times k_h \times f_{m,k}/\gamma_m = 0.9 \times 1.05 \times 24/1.3$	$f_{m,d} = 17.4\,\text{N/mm}^2$
Ultimate bending moment $M_u = f_{m,d} \times W_{yy} = 17.4 \times 105.6 \times 10^3$ $= 1.84 \times 10^6\,\text{Nmm}$	$M_u = 1.84\,\text{kNm}$

2.8.2 Bearing ULS: Compression Perpendicular to the Grain at Support Bearings

The general requirement is that the compressive stress perpendicular to the grain $\sigma_{c,90,d}$ should not be more than the design bearing strength $f_{c,90,d}$. However, a greater stress is allowed when, as is usually the case, the bearing area is small. EC5 requires that

$$\sigma_{c,90,d} \leq k_{c,90} f_{c,90,d}$$

where $k_{c,90}$ is a factor taking into account the load configuration, the possibility of splitting and the degree of compressive deformation. The value of $k_{c,90}$ is between 1 and 4, and different expressions are given for end bearings and intermediate bearings as shown in Figure 2.2.



End support bearing. $a = < h/3$

$k_{c,90}$ is $1 \leq \left(2.38 - \dfrac{l}{250}\right)\left(1 + \dfrac{h}{12l}\right) \leq 4$

Internal support bearing

$k_{c,90}$ is $1 \leq \left(2.38 - \dfrac{l}{250}\right)\left(1 + \dfrac{h}{6l}\right) \leq 4$

Figure 2.2: Values of $k_{c,90}$ at support bearings. From EC5 Part 1-1, Equations 6.4 and 6.5

Example 2.4 Bearing capacity

A sawn softwood beam in C16 timber, 63 mm wide × 200 mm deep, is supported on 100 mm wide bearings. The load is medium term and Service Class 2 applies. Find the ultimate capacity of the bearing:

(i) If the bearing is at the end of the joist, so that $a = 0$

(ii) If the bearing is internal, so that $a \geq 200/3 = 67$ mm.

Calculations for Example 2.4	
Bearing strength $f_{c,90,k}$ from Table 2.5, for C16 timber	$f_{c,90,k} = 2.2\,N/mm^2$
Modification factor k_{mod} from Table 2.14, for solid timber in service class 2 and medium-term loading	$k_{mod} = 0.8$
Partial safety factor γ_m from Table 2.2, for solid timber	$\gamma_m = 1.3$
Design bearing strength $f_{c,90,d} = k_{mod} \times f_{c,90,k}/\gamma_m = 0.8 \times 2.2/1.3$	$f_{c,90,d} = 1.35\,N/mm^2$
At end bearing	
$k_{c,90}$ is $1 \leq \left(2.38 - \frac{l}{250}\right)\left(1 + \frac{h}{12l}\right) \leq 4$	
$\left(2.38 - \frac{100}{250}\right)\left(1 + \frac{200}{12 \times 100}\right) = 2.31$	At end bearing, $k_{c,90} = 2.31$
$\sigma_{c,90,d} \leq k_{c,90}f_{c,90,d} = 2.31 \times 1.35$	$\sigma_{c,90,d} \leq 3.12\,N/mm^2$
Ultimate capacity of bearing $= \sigma_{c,90,d} \times$ bearing area $= 3.12 \times 63 \times 100 = 19\,660\,N$	End bearing capacity at ULS = 19.7 kN
At internal bearing	
$k_{c,90}$ is $1 \leq \left(2.38 - \frac{l}{250}\right)\left(1 + \frac{h}{6l}\right) \leq 4$	
$\left(2.38 - \frac{100}{250}\right)\left(1 + \frac{200}{6 \times 100}\right) = 2.64$	At internal bearing, $k_{c,90} = 2.64$
$\sigma_{c,90,d} \leq k_{c,90}f_{c,90,d} = 2.64 \times 1.35$	$\sigma_{c,90,d} \leq 3.56\,N/mm^2$
Ultimate capacity of bearing $= \sigma_{c,90,d} \times$ bearing area $= 3.56 \times 63 \times 100 = 22\,430\,N$	Internal bearing capacity at ULS = 22.4 kN

Larger values of $k_{c,90}$ may be used when the load on the beam or joist includes significant point loads near the support, and guidance on this is given in EC5.

2.8.3 Shear ULS

The critical position for shear on a normally loaded flexural member is at the support where the maximum shear force occurs. The maximum shear stress τ_d in a rectangular section occurs on the neutral axis and can be calculated from the shear force V using the following expression

$$\tau_d = 1.5V / bh_{ef}$$

where bh_{ef} is the cross-section area of the beam.

The characteristic shear strength of the timber $f_{v,k}$ can be found from Table 2.5 or 2.6, and the design shear strength $f_{v,d}$ is calculated from

$$f_{v,d} = \frac{k_{mod} f_{v,k}}{\gamma_m}$$

If the beam or joist is notched at the support as shown in Figure 2.3 then the design shear strength is modified by the factor k_v. For a top-edge notch k_v is 1.0, and for a bottom-edge notch k_v can be calculated from the formula from EC5 Part 1-1, Equation 6.62 given in Figure 2.4. The design will be satisfactory if

For supports without a notch	$\tau_d \leq f_{v,d}$
For notched supports	$\tau_d \leq k_v f_{v,d}$

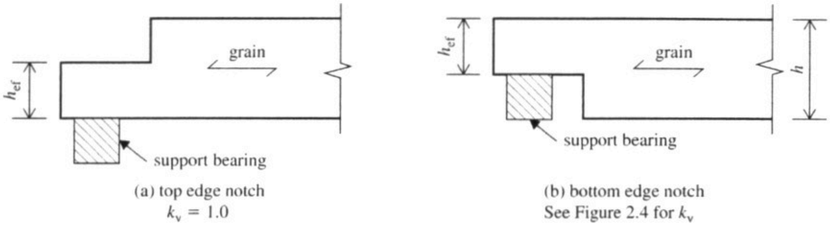
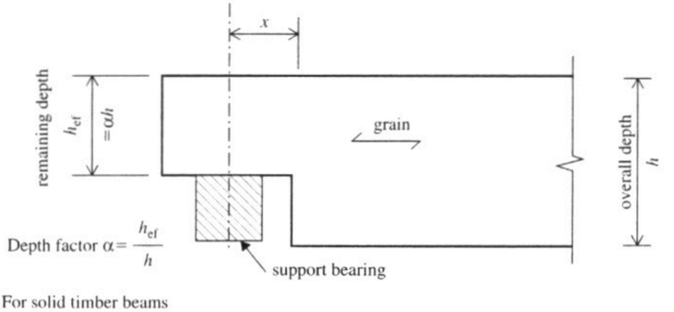


Figure 2.3: Notched supports



For solid timber beams

$$k_v = \frac{5}{\sqrt{h}\left[\sqrt{\alpha(1-\alpha)} + 8.0\frac{x}{h}\sqrt{\frac{1}{\alpha} - \alpha^2}\right]} \text{ but not more than } 1.$$

Figure 2.4: k_v factor for bottom-edge-notched supports

Example 2.5 Shear capacity at a support

The end of a sawn softwood beam in C16 timber, 63 mm wide × 200 mm deep, is supported on a 100 mm wide bearing. The load is medium term and Service Class 2 applies. Find the ultimate shear capacity at the support for the three configurations shown in Figure 2.5.

(a) No notch

(b) Top notch 125 mm long × 80 mm deep

(c) Bottom notch 125 mm long × 80 mm deep.

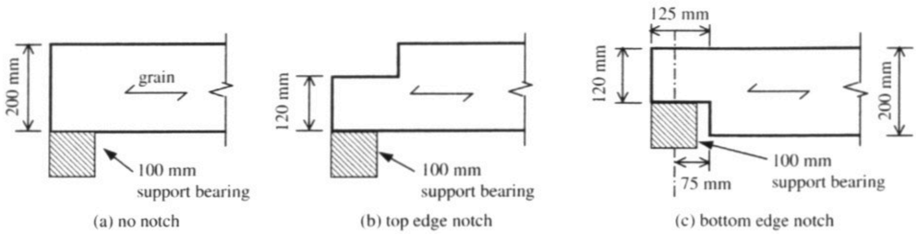


Figure 2.5: Support configurations for Example 2.4

Calculations for Example 2.5	
Shear strength $f_{v,k}$ from Table 2.5, for C16 timber	$f_{v,k} = 1.8\,\text{N/mm}^2$
Modification factor k_{mod} from Table 2.14, for solid timber in service class 2 and medium-term loading	$k_{mod} = 0.8$
Partial safety factor γ_m from Table 2.2, for solid timber	$\gamma_m = 1.3$
Design shear strength $f_{v,d} = \dfrac{k_{mod} f_{v,k}}{\gamma_m} = \dfrac{0.8 \times 1.8}{1.3}$	$f_{v,d} = 1.11\,\text{N/mm}^2$
Support (a): Not notched	
τ_d must not exceed $f_{v,d}$ N/mm^2	
$\tau_d = 1.5\,V/bh_{ef}$, so $V = \tau_d bh_{ef}/1.5$	
Hence V must not exceed $f_{v,d}\,bh_{ef}/1.5 = 1.11 \times 63 \times 200/1.5 = 9324\,\text{N}$	Shear capacity = 9.3 kN
Support (b): Notched at the top, $k_v = 1.0$, $h_{ef} = 120\,\text{mm}$	
The calculation of $f_{v,d}$ is the same as for support (a).	
V must not exceed $k_v f_{v,d}\,bh_{ef}/1.5 = 1.0 \times 1.11 \times 63 \times 120/1.5 = 5544\,\text{N}$	Shear capacity = 5.5 kN

Support (c): Notched at the bottom

From Figures 2.4 and 2.5(c)

$h_{ef} = 120\,\text{mm}$

$x = 75\,\text{mm}$

$\alpha = h_{ef}/h = 120/200 = 0.6$

$$k_v = \frac{5}{\sqrt{h}\left[\sqrt{\alpha(1-\alpha)} + 0.8\frac{x}{h}\sqrt{\frac{1}{\alpha} - \alpha^2}\right]} = \frac{5}{\sqrt{200}\left[\sqrt{0.6(1-0.6)} + 0.8 \times \frac{75}{200}\sqrt{\frac{1}{0.6} - 0.6^2}\right]}$$

$k_v = 0.425 < 1.0$

V must not exceed $k_v f_{v,d}\, b h_{ef}/1.5 = 0.425 \times 1.11 \times 63 \times 120/1.5 = 2380\,\text{N}$ Shear capacity = 2.4 kN

2.8.4 *Deflection SLS*

Deflections of flexural members must be limited to avoid damage to finishes, ceilings and partitions. Limits on vibration of floors may also be required.

The formulas for calculating deflections in simply supported beams due to bending were given in Chapter 1:

With uniformly distributed load F, the maximum deflection due to bending $w_m = \dfrac{5FL^3}{384EI}$

With a central point load P, the maximum deflection due to bending $w_m = \dfrac{PL^3}{48EI}$

The shear stiffness of timber is low when compared to its stiffness under direct stress, so timber beams and joists have significant shear deflections in addition to the bending deflections. It can be shown that for rectangular sections the shear deflections are as follows.

With uniformly distributed load F, the maximum deflection due to shear $w_v = \dfrac{1.2}{GA} \times \dfrac{FL}{8}$

With a central point load P, the maximum deflection due to shear $w_v = \dfrac{1.2}{GA} \times \dfrac{PL}{4}$

Where G is the shear modulus of the timber and A is the section area.

These expressions can be simplified by noting that:

With uniformly distributed load F, the maximum moment $M = \dfrac{FL}{8}$

With a central point load P, the maximum moment $M = \dfrac{PL}{4}$

So that, for both types of load, the maximum deflection due to shear $w_v = \dfrac{1.2}{GA} \times M$

Inspection of Tables 2.5 and 2.6 shows that, for all the timber grades listed, $G = E/16$, so

$$w_v = \frac{1.2}{(E/16)A} \times M = \frac{19.2M}{EA}$$

Adding the shear deflection to the bending deflection gives

With uniformly distributed load F, the maximum total deflection $w = \dfrac{5FL^3}{384EI} + \dfrac{19.2M}{EA}$

With a central point load P, the maximum total deflection $w = \dfrac{PL^3}{48EI} + \dfrac{19.2M}{EA}$

Instantaneous and final deflections

EC5 does not limit the instantaneous deflection under SLS loads. A structure which meets the limit on final deflection will be satisfactory for instantaneous deflection. However, floors in residential buildings should satisfy a vibration criterion as explained in the next section.

Vibration of residential floors

Loads from people walking over a floor may cause vibrations which are a nuisance to those living below the floor, and the following checks will limit those vibrations to a reasonable level. Designers should note that the Building Regulations impose additional requirements for floors with respect to sound insulation between dwellings and these should be checked separately.

First the fundamental frequency f_1 should be found, and this can be approximately calculated as

$$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI}{m}}$$

where EI is the stiffness of the beams in $\mathrm{Nm^2/m}$, l is the span in metres and m is the mass of the unloaded floor in $\mathrm{kg/m^2}$.

If f_1 is less than 8 Hz a special investigation is required. If f_1 is greater than 8 Hz then the floor will be satisfactory if the immediate deflection under a 1 kN point load placed anywhere on the span does not exceed the value from Table 2.18. The 1 kN point load represents a person walking on the floor.

Table 2.18: Limiting immediate deflections under a 1 kN point load

	Span of floor in metres										
	4.0 or less	4.2	4.4	4.6	4.8	5.0	3.2	5.4	5.6	5.8	6.0
Limit (mm)	1.80	1.71	1.62	1.54	1.47	1.41	1.35	1.29	1.24	1.20	1.15

Formula: limit = $16500/l^{1.1}$ where l is the span in mm, or 1.8 mm if $l \leq 4000$ mm.
Source: Table NA5 in UK National Annex to EC5 Part 1-1.

The floor decking is assumed to be able to distribute the point load onto several joists. Conservatively EC5 allows the floor to be checked as if each single joist carries only 0.38 of the point load, or 380 N. Taking k_{amp}, an allowance for shear deformation, as 1.05, the deflection w is given by:

$$w = 399l^3 / 48EI$$

(Source: Equation NA1 of UK National Annex to EC5 with $k_{dist} = 0.38$, $k_{amp} = 1.05$)

where l is the span in mm, E is $E_{0,mean}$ for the timber and I is the second moment of area of the joist.

Note that the k_{amp} factor is used here to account for shear deformation. The k_{amp} factor is used when checking the vibration of floors and should not be used elsewhere in the design process.

This expression is conservative for solid timber joists; for other joists such as LVL or other proprietary products reference should be made to EC5.

These expressions can be used to calculate the minimum I value for joists, and Table 2.19 gives this information for joists of strength grades C16 and C24.

Table 2.19: Minimum *I* values of timber joists for domestic floors to meet the limiting deflection under a 1 kN point load

	Span of floor (m)							
	2.5	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
Minimum I value (mm^4 × 10^6) of joist in C16 timber, $E_{0,mean} = 8\,kN/mm^2$	9.0	15.6	24.7	36.9	59.9	99.2	136	195
Minimum I value (mm^4 × 10^6) of joist in C24 timber, $E_{0,mean} = 11\,kN/mm^2$	6.6	11.3	18.0	26.9	43.6	67.1	99.2	142

Note that a more complex method of calculation which can take into account the stiffening due to the floor decking, strutting and ceiling plaster is given in the National Annex. Use of this method may demonstrate that the vibration criterion can be met with smaller joists.

Example 2.6 Check of a residential floor against the vibration criterion

A residential floor of span 4.4 m comprises 63 mm × 225 mm sawn softwood joists of C16 timber at 400 mm centres. The decking is chipboard with a mass of 15 kg/m^2 and the ceiling is skimmed plasterboard with a mass of 20 kg/m^2. Check whether the floor meets the vibration criterion.

Calculations for Example 2.6	
Mass of the floor	
From Table 2.5, mean density of C16 timber = 370 kg/m	
So mass of one joist = 0.063 × 0.225 × 370 = 5.2 kg/m	
Mass of joists in one square metre = 5.2/0.4 = 13 kg/m^2	
Total mass of floor m = 13 + 15 + 20	m = 48 kg/m^2
***EI* value of one metre of floor**	
I value of one joist = $bh^3/12$ = 63 × 225^3/12 =	For one joist, I = 59.8 × 10^6 mm^4
I value of metre width of floor = 59.8 × 10^6/0.4	For one metre of floor, I = 150 × 10^6 mm^4
From Table 2.5 for C16 timber	$E_{0,\text{mean}}$ = 8 kN/mm^2
For one metre of floor, EI = 150 × 10^6 × 8 = 1.2 × 10^9 kNmm^2/m (multiply by 10^3, then divide by 10^6)	EI = 1.2 × 10^6 Nm^2/m
First fundamental frequency f_1	
First fundamental frequency $f_1 = \dfrac{\pi}{2l^2}\sqrt{\dfrac{EI}{m}} = \dfrac{\pi}{2 \times 4.4^2}\sqrt{\dfrac{1.2 \times 10^6}{48}}$	f_1 = 12.8 Hz
f_1 is more than 8 Hz, so a special investigation is not required	
Deflection under 1-kN point load	
Deflection under 1-kN point load on one joist $w = 399\,l^3/48\,EI$ = 399 × 4400^3/(48 × 59.8 × 10^6 × 8×10^3)	w = 1.48 mm
This is less than the limiting value of 1.62 mm from Table 2.18	Accept
Note that instead of the calculation above we could have found by interpolation in Table 2.19 that the minimum I value for one C16 joist with a span of 4.4 m is 55.3 × 10^6 mm^4, and so the 63 mm × 225 mm joist which has I = 59.8 × 10^6 mm^4 is satisfactory	

Final deflections

To avoid damage to finishes, ceilings, partitions and so on, the final deflection of fully loaded timber flexural members should not be excessive. EC5 states that limiting deflections should be specified for each project and agreed with the client, and the UK National Annex gives the values in Table 2.20 for guidance.

Table 2.20: Limiting final deflections of individual beams: values for guidance

Type of member	Limiting value for w_{fin} final deflections of individual beams	
	A member of span l between two supports	**A cantilever member of span l**
Roof or floor member with plastered or plasterboard ceiling	$l/250$	$l/125$
Roof or floor member without plastered or plasterboard ceiling	$l/150$	$l/75$

Source: Table NA4 of UK National Annex to EC5 Part 1-1.

The calculation of final deflections must take into account the following two considerations:

1. As explained in Section 1.4 of Chapter 1 it is unlikely that variable loads (except storage loads) will be at their maximum value for the whole of the life of the structure. Similarly if the member is subject to more than one variable load it is unlikely that these loads will all have their maximum value at the same time. Because of this, final deflections are calculated using loads reduced by the ψ factors given in that section.

2. As explained in Section 2.3 of this chapter, creep of the timber means that a long-term load will cause more deflection than an equal short-term load.

EC5 Clause 2.2.3(5) gives a simplified method of computing final deflections which takes account of these two considerations. First the instantaneous deflections w_{inst} under permanent and variable loads are found separately using $E_{0,mean}$ as the elastic modulus of the timber. The separate deflections are then multiplied by different factors to give the final deflections.

For a member carrying a permanent load G and two variable loads Q_1 and Q_2, the final deflection w_{fin} is given by:

$$w_{fin} = w_{fin,G} + w_{fin,Q1} + w_{fin,Q2}$$

where $w_{fin,G}$ = deflection caused by the permanent load $G = w_{inst,G}(1 + k_{def})$
$w_{fin,Q1}$ = deflection caused one variable load $Q_1 = w_{inst,Q1}(1 + \psi_{2,1}k_{def})$
$w_{fin,Q2}$ = deflection caused by the other variable loads $Q_2 = w_{inst,Q2}(\psi_{0,2} + \psi_{2,2}k_{def})$
k_{def} factors are given in Table 2.21
ψ factors are given in Table 1.10 of Chapter 1.

If there is one permanent load G and one variable load Q, these expressions become

$$w_{\text{fin}} = w_{\text{fin,G}} + w_{\text{fin,Q}}$$

where $w_{\text{fin,G}}$ = deflection caused by the permanent load $G = w_{\text{inst,G}}(1 + k_{\text{def}})$
and $w_{\text{fin,Q}}$ = deflection caused by the variable load $Q = w_{\text{inst,Q}}(1 + \psi_2 k_{\text{def}})$
k_{def} factors are given in Table 2.21
ψ factors are given in Table 1.10 of Chapter 1.

Table 2.21: Values of k_{def} for timber and wood-based products

Material	Standard	Service class		
		1	2	3
Solid timber	EN 14081-1	0.60	0.80	2.00
Glulam	EN 14080	0.60	0.80	2.00
LVL	EN 14374, EN14279	0.60	0.80	2.00
OSB	EN 300, OSB/2	2.25	–	–
	EN 300, OSB/3 or OSB/4	1.50	2.25	–

Source: Table 3.2 of EC5 Part 1-1.

Example 2.7 Check of a beam for deflection SLS

A planed softwood beam of C16 timber is 72 mm wide × 220 mm deep, has an effective span of 4.25 m and carries the following unfactored loads:

Dead load	2.0 kN
Imposed load	3.5 kN.

The beam is part of the ground floor of an office building and has no ceiling. Find the instantaneous and final deflections at the centre of the joist and check whether the final deflection meets the limits given in Table 2.20.

Calculations for Example 2.7

First calculate the instantaneous deflections

From Table 2.5 for C16 timber $\quad E_{0,\text{mean}} = 8\,\text{kN/mm}^2 = 8 \times 10^3\,\text{N/mm}^2$

From Table 2.12, for a joist 72 mm × 220 mm $\quad A = 15.84 \times 10^3\,\text{mm}^2$

$I_{yy} = 63.89 \times 10^6\,\text{mm}^4$

For the dead load $F = 2.0\,\text{kN}$, $M = FL/8 = 2.0 \times 4.25/8$	$M = 1.06\,\text{kNm}$
$w_{\text{inst.G}} = \dfrac{5FL^3}{384EI} + \dfrac{19.2M}{EA} = \dfrac{5 \times 2.0 \times 10^3 \times 4250^3}{384 \times 8 \times 10^3 \times 63.89 \times 10^6} + \dfrac{19.2 \times 1.06 \times 10^6}{8 \times 10^3 \times 15.84 \times 10^3}$	
$= 3.91 + 0.16$	$w_{\text{inst.G}} = 4.07\,\text{mm}$
For the imposed load $F = 3.5\,\text{kN}$, $M = FL/8 = 3.5 \times 4.25/8$	$M = 1.86\,\text{kNm}$
$w_{\text{inst.Q}} = \dfrac{5FL^3}{384EI} + \dfrac{19.2M}{EA} = \dfrac{5 \times 3.5 \times 10^3 \times 4250^3}{384 \times 8 \times 10^3 \times 63.89 \times 10^6} + \dfrac{19.2 \times 1.86 \times 10^6}{8 \times 10^3 \times 15.84 \times 10^3}$	
$= 6.84 + 0.28$	$w_{\text{inst.Q}} = 7.12\,\text{mm}$
So the total instantaneous deflection $w_{\text{inst}} = 4.07 + 7.12$	$w_{\text{inst}} = 11.2\,\text{mm}$
Now find the final deflection $w_{\text{fin}} = w_{\text{fin.G}} + w_{\text{fin.Q}}$	
From Chapter 1, Table 1.10, office loading is category B and	$\psi_2 = 0.3$
From Table 2.8, for a ground floor	Service Class 2
From Table 2.21, for solid timber in service class 2	$k_{\text{def}} = 0.8$
$w_{\text{fin.G}}$ = final deflection caused by the permanent load $G = w_{\text{inst.G}}(1 + k_{\text{def}})$ $= 4.07 \times (1 + 0.8) = 4.07 \times 1.8$	$w_{\text{fin.G}} = 7.33\,\text{mm}$
$w_{\text{fin.Q}}$ = final deflection caused by the variable load $Q = w_{\text{inst.Q}}(1 + \psi_2 k_{\text{def}})$ $= 7.12 \times (1 + 0.3 \times 0.8) = 7.12 \times 1.24$	$w_{\text{fin.Q}} = 8.83\,\text{mm}$
The total final deflection $w_{\text{fin}} = w_{\text{fin.G}} + w_{\text{fin.Q}} = 7.33 + 8.83$	$w_{\text{fin}} = 16.2\,\text{mm}$
From Table 2.22 the final deflection limit is span/150 = 4250/150	limit = 28 mm
16.2 mm is less than 28 mm	Beam is satisfactory at deflection SLS

Example 2.8 Design of a timber joist at ULS and SLS

A flat roof spanning 4.25 m is to be designed using C24 timber joists at 600 mm centres as shown in Figure 2.6. Loads from the proposed roof construction are:

Asphalt 20 mm thick	$0.45\,\text{kN/m}^2$
Insulation	$0.10\,\text{kN/m}^2$
Roof decking boards	$0.30\,\text{kN/m}^2$
Timber firrings	$0.01\,\text{kN/m}^2$
Suspended tile ceiling	$0.15\,\text{kN/m}^2$

The building is at an elevation of 50 m above sea level and the roof is to be designed for a snow load of $0.6\,\text{kN/m}^2$. As the insulation is above the roof boards, the construction is of the 'warm roof' type. The height available from the supports to the underside of the decking is 150 mm, and the ends of the joists may have to be notched to fit this dimension. Assuming that the roof decking provides full lateral restraint to the joists, choose a suitable size for the joist timbers.

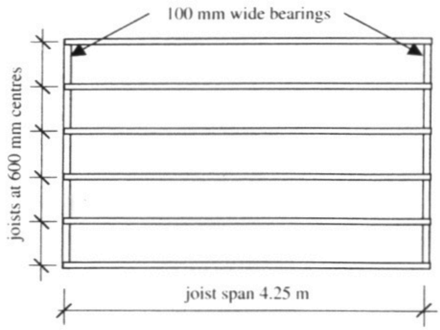


Figure 2.6: Arrangement of joists in Example 2.8

Calculations for Example 2.8

Loading

Assume that the self-weight of the joists is equivalent to 0.1 kN/m^2. This will be checked later

Load on one joist		Dead loads	Imposed loads
asphalt	$0.45 \times 0.6 \times 4.25 =$	1.15 kN	-
insulation	$0.10 \times 0.6 \times 4.25 =$	0.26 kN	-
roof decking boards	$0.30 \times 0.6 \times 4.25 =$	0.77 kN	-
timber firrings	$0.01 \times 0.6 \times 4.25 =$	0.03 kN	-
suspended tile ceiling	$0.15 \times 0.6 \times 4.25 =$	0.38 kN	-
joist self-weight	$0.10 \times 0.6 \times 4.25 =$	0.26 kN	-
snow load	$0.60 \times 0.6 \times 4.25 =$	-	1.53 kN
	Totals	$G_k = 2.85\,\text{kN}$	$Q_k = 1.53\,\text{kN}$

Load on one joist for SLS $= 1.00G_k + 1.00Q_k = 2.85 + 1.53$ $F = 4.38\,\text{kN}$

Load on one joist for ULS $= 1.35G_k + 1.50Q_k = 1.35 \times 2.85 + 1.50 \times 1.53$ $F = 6.14\,\text{kN}$

Timber properties and parameters

From Table 2.5, for C24 timber $f_{m,k} = 24\,\text{N/mm}^2$

$f_{c,90,k} = 2.5\,\text{N/mm}^2$

$f_{v,k} = 2.5\,\text{N/mm}^2$

$E_{o,mean} = 11\,\text{kN/mm}^2$

From Table 2.7, the snow load is short term (less than 1 week) Snow load duration class short term

From Table 2.2, for solid timber $\gamma_m = 1.3$

From Table 2.8, service class for timber in a warm roof Service Class 1

By the rule of thumb in Section 2.6.2

Dead load $G_k = 2.85\,\text{kN}$
Imposed load $Q_k = 1.53\,\text{kN}$ which is greater than $0.3 \times 2.85 = 0.85\,\text{kN}$
so a ULS check under dead load only is not required.

From Table 2.14 with load duration short term and service class 1 — $k_{mod} = 0.90$

Joists can share load, so — $k_{sys} = 1.1$

Table 2.16, depth is likely to be more than 150 mm so depth factor — $k_h = 1.0$

From Table 2.21, for solid timber and service class 1 — $k_{def} = 0.6$

Bending ULS

Design bending moment $M = FL/8 = 6.14 \times 4.25/8$ — $M = 3.26\,\text{kNm}$

Design bending strength $f_{m,d} = k_{sys} \times k_{mod} \times k_h \times f_{m,k}/\gamma_m$
$= 1.1 \times 0.9 \times 1.0 \times 24/1.3$ — $f_{m,d} = 18.3\,\text{N/mm}^2$

Ultimate moment of resistance $M_{ult} = f_{m,d} \times W_{yy}$
so $= W_{yy}$ must be at least $3.26 \times 10^6/18.3$ — W_{yy} must be at least $178 \times 10^3\,\text{mm}^3$

From Table 2.11, a sawn timber joist 47 mm × 175 mm has $W_{yy} = 239.9 \times 10^3\,\text{mm}^3$, which is sufficient

Use 47 mm × 175 mm sawn C24 joist

The compression flange has continuous restraint from the roof decking.

From Table 2.17 the limiting height/breadth ratio is 5:1

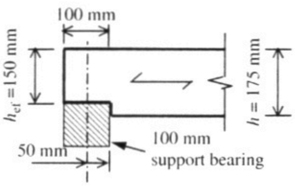
Actual height/breadth ratio = 175:47 = 3.7:1

So the beam is not subject to lateral torsional buckling — Beam is not subject to LTB

Shear ULS

Shear force V at support = 6.14/2 — $V = 3.07\,\text{kN}$

As the height available from the supports to the underside of the decking is 150 mm, the ends of the joists will have to be notched at the bottom by 25 mm as shown in the diagram opposite



From Figure 2.4

h_{ef} = 150 mm

x = 50 mm

$\alpha = 150/175 = 0.86$

From Figure 2.5(c)

$$k_v = \frac{5}{\sqrt{h}\left[\sqrt{\alpha(1-\alpha)} + 0.8\dfrac{x}{h}\sqrt{\dfrac{1}{\alpha} - \alpha^2}\right]} = \frac{5}{\sqrt{175}\left[\sqrt{0.86(1-0.86)} + 0.8 \times \dfrac{50}{175}\sqrt{\dfrac{1}{0.86} - 0.86^2}\right]}$$

$k_v = 0.76 < 1.0$

Shear stress $\tau_d = 1.5\,V/bh_{ef} = 1.5 \times 3.07 \times 10^3/(47 \times 150)$ — $\tau_d = 0.65\,\text{N/mm}^2$

Design shear strength $f_{v,d} = k_v \times k_{sys} \times k_{mod} \times f_{v,k}/\gamma_m$
$= 0.76 \times 1.1 \times 0.9 \times 2.5/1.3$ — $f_{v,d} = 1.45\,\text{N/mm}^2$

Continued on next page

Calculations for Example 2.8 (Continued from previous page)

$f_{y,d} > \tau_d$

Joist is satisfactory at shear ULS

Bearing ULS

Bearing stress $\sigma_{c,90,d} = V/\text{bearing area} = 3.07 \times 10^3/(100 \times 47)$ — $\sigma_{c,90,d} = 0.65\,\text{N/mm}^2$

The bearing is an end support. From Figure 2.4 with $a = 0$

$k_{c,90}$ is $1 \leq \left(2.38 - \dfrac{l}{250}\right)\left(1 + \dfrac{h}{12l}\right) \leq 4$, with $l = 100\,\text{mm}$ and $h = 175\,\text{mm}$

$$\left(2.38 - \frac{100}{250}\right)\left(1 + \frac{175}{12 \times 100}\right) = 2.27$$

$k_{c,90} = 2.27$

Design bearing strength $f_{c,90,d} = k_{c,90} \times k_{sys} \times k_{mod} \times f_{c,90,k}/\gamma_m$
$= 2.27 \times 1.1 \times 0.9 \times 2.5/1.3$ — $f_{c,90,d} = 4.3\,\text{N/mm}^2$

$\sigma_{c,90,d} < f_{c,90,d}$

Joist is satisfactory at bearing ULS

Deflection SLS

From Table 2.11, for a joist 47 mm × 175 mm — $A = 8.23 \times 10^3\,\text{mm}^2$

$I_{yy} = 20.99 \times 10^6\,\text{mm}^4$

For the dead load $F = 2.85\,\text{kN}$, $M = FL/8 = 2.85 \times 4.25/8$ — $M = 1.51\,\text{kNm}$

$$w_{inst,G} = \frac{5FL^3}{384EI} + \frac{19.2M}{EA} = \frac{5 \times 2.85 \times 10^3 \times 4250^3}{384 \times 11 \times 10^3 \times 20.99 \times 10^6} + \frac{19.2 \times 1.51 \times 10^6}{11 \times 10^3 \times 8.23 \times 10^3}$$

$= 12.3 + 0.32$ — $w_{inst,G} = 12.6\,\text{mm}$

For the imposed load $F = 1.53\,\text{kN}$, $M = FL/8 = 1.53 \times 4.25/8$ — $M = 0.81\,\text{kNm}$

$$w_{inst,Q} = \frac{5FL^3}{384EI} + \frac{19.2M}{EA} = \frac{5 \times 1.53 \times 10^3 \times 4250^3}{384 \times 11 \times 10^3 \times 20.99 \times 10^6} + \frac{19.2 \times 0.81 \times 10^6}{11 \times 10^3 \times 8.23 \times 10^3}$$

$= 6.6 + 0.17$ — $w_{inst,Q} = 6.8\,\text{mm}$

So the total instantaneous deflection $w_{inst} = 12.6 + 6.8$ — $w_{inst} = 19.4\,\text{mm}$

Now find the final deflection $w_{fin} = w_{fin,G} + w_{fin,Q}$

From Chapter 1 Table 1.10, for snow loads at altitude ≤ 1000 m above sea level — $\psi_2 = 0$

$w_{fin,G}$ = final deflection caused by the permanent load $G = w_{inst,G}(1 + k_{def})$
$= 12.6 \times (1 + 0.6) = 12.6 \times 1.6$ — $w_{fin,G} = 20.2\,\text{mm}$

$w_{fin,Q}$ = final deflection caused by the variable load $Q = w_{inst,Q}(1 + \psi_2 k_{def})$
$= 6.8 \times (1 + 0 \times 0.6) = 6.8 \times 1.0$ — $w_{fin,Q} = 6.8\,\text{mm}$

The total final deflection $w_{fin} = w_{fin,G} + w_{fin,Q} = 20.2 + 6.8$ — $w_{fin} = 27\,\text{mm}$

From Table 2.20, guidance deflection limit for a roof member without a plasterboard ceiling is span/150 = 4250/150 = 28 mm — Deflection limit = 28 mm

Joist is satisfactory at deflection SLS

2.9 Engineered Timber Products and Connections

While solid timber in rectangular sections is an economical structural material for many floors, roofs and wall panels, some structural applications require larger section sizes or greater lengths than are readily available. Engineered timber products are manufactured by fixing together small pierces of natural timber to give larger and longer sections. In many products the pieces are glued together, and a variety of glues are available to meet requirements of strength and durability. Three products extensively used in building structures are glued laminated beams (Glulam), laminated veneer lumber (LVL) and structurally engineered timber joists. These are illustrated in Figure 2.7 and described later in more detail.

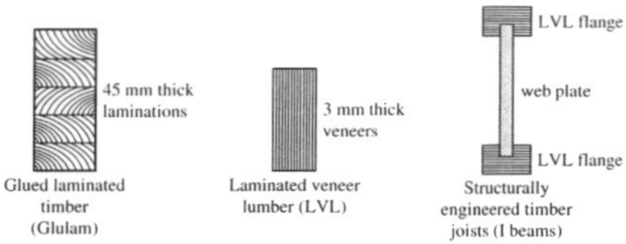


Figure 2.7: Engineered timber products

A great advantage of these products is that natural timber in short pieces or with defects can be used. When used for Glulam the defects can be cut out and the remaining near-perfect timber pieces jointed. When used for LVL some of the defects in the source timber can be eliminated by choice of veneers, and remaining defects will be spread thorough the product evenly and so will not have a great effect on the structural properties.

Where required, pieces of solid timber can be joined end to end with glued finger joints as shown in Figure 2.8. Made under factory conditions and with appropriate choice of glues the finger joints are as strong as the parent timber. Requirements for finger joints are given in BS EN 385: 2001.

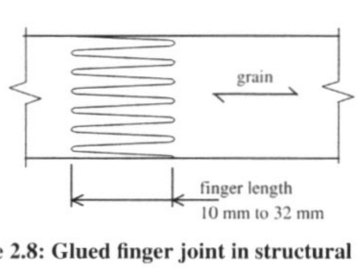


Figure 2.8: Glued finger joint in structural timber

As engineered products are made under factory conditions with good quality control, strength and size are less variable than with natural timber. However, the cost of the glues and of the production process means that natural timber will normally be a more economical material for small sections and short spans.

EC5 gives guidance on the design of these engineered timber products, but they are beyond the scope of this manual.

Glued laminated timber (Glulam)

These are built up from small finger-jointed laminations generally 45 mm thick, and the sizes normally manufactured are shown in Table 2.22. In the UK Glulam is generally made from timber of strength class C24. Most Glulam beams are straight, but cambered, curved and portal frame members can be made.

Table 2.22: Typical sizes of glued laminated timber beams

Beam depth (mm)	No. of laminations	Beam width (mm)						
		65	90	115	140	165	190	215
180	4	×						
225	5	×	×					
270	6	×	×					
315	7	×*	×*	×				
360	8		×	×				
405	9		×*	×*	×			
450	10		×	×	×			
495	11			×*	×	×		
540	12			×	×	×		
585	13				×	×		
630	14				×	×		
675	15				×	×	×	
720	16					×	×	
765	17					×	×	×
810	18					×	×	×
855	19						×	×
900	20						×	×
945	21						×	×
990	22							×
1035	23							×

Sizes marked * are available from stock.

Glued laminated beams are normally available up to 13.5 m long, with longer beams available to order.

Source: www.glulam.co.uk

Laminated veneer lumber (LVL)

Generally made from 3-mm-thick veneers, these sections are often used as beams or joists with the laminations vertical. They are particularly useful when the size required is just beyond what is available in natural timber, e.g. more than 300 mm deep.

Structurally engineered timber joists (I beams)

These have top and bottom flanges of LVL with a web plate of oriented strand board. They are used for long-span joists especially where the control of deflection is important, and are much lighter than a solid timber joist of the same strength and stiffness.

Sheet materials

Solid timber boards are not usually economic for decking and sheathing applications, though they may be used for aesthetic reasons. Many different timber-based sheet materials are available, all made by gluing together small fragments of timber. Some sheet materials are listed below.

Plywood, made from timber veneers

Oriented strand board (OSB), made from small slices of timber

Chipboard made from timber chips and sawdust

Fibreboards, made from sawdust.

Different grades can be made by using different adhesives and by varying the pressure applied to the board during manufacture. The lower grades are not suitable for use in damp conditions or for long-term loading.

Connections

Simple bearing connections between members can be made without structural design of fasteners. More complex joints in trusses, panels or other structural assemblies will require the use of metal fasteners such as screws, nails, bolts, dowels, plates or specialized shear connectors. In many structures the need to provide adequate connection between members will be more critical to the design than the strength of the members themselves. EC5 gives extensive guidance on the design of metal fasteners, but these are not within the scope of this manual.

2.10 Compression Members: Timber Posts, Columns and Struts

Compression members include posts, columns, vertical wall studs and the struts in trusses and girders. The design of a single isolated post or stud is considered in this manual.

The amount of bow permitted by most stress grading rules is not acceptable in columns, and timber for columns should be selected to limit the bow to 1/300 of the length.

Members carrying only axial load are designed for direct compression only. Members carrying eccentric loads or additional lateral loads (e.g. wind load) are subject to bending, and are designed for a combination of direct compression and bending. Design for lateral loads is not considered in this manual.

Compression parallel to the grain

The slenderness ratio of a member in compression is defined by

λ = effective length/radius of gyration

λ_y = slenderness ratio corresponding to bending about the y axis (deflection in the z direction)

= effective length/i_y, where for a rectangular section $i_y = 0.289h$ (see Table 2.3)

λ_z = slenderness ratio corresponding to bending about the z axis (deflection in the y direction)

= effective length/i_z, where for a rectangular section $i_z = 0.289b$ (see Table 2.3).

The capacity of timber members loaded in axial compression is limited by buckling if λ_{rel}, calculated as shown in Table 2.23, exceeds a critical value. If this is the case the timber strength is reduced by a factor k_c which is shown in Table 2.24.

Table 2.23: Lambda conversion factors for softwoods in direct compression

Timber grade	$f_{c,0,k}$ (N/mm^2)	$E_{0,0.05}$ (kN/mm^2)	lambda conversion factor
C14	16	4.7	0.0186
C16	17	5.4	0.0179
C18	18	6.0	0.0174
C20	19	6.4	0.0173
C22	20	6.7	0.0174
C24	21	7.4	0.0170
C27	22	7.7	0.0170
C30	23	8.0	0.0171
C35	25	8.7	0.0171
C40	26	9.4	0.0167
C45	27	10.0	0.0165
C50	29	10.7	0.0166

Based on EC5 Part 1-1, Equations 6.21 and 6.22.

$\lambda_{rel,y} = \lambda_y \times$ lambda conversion factor

$\lambda_{rel,z} = \lambda_z \times$ lambda conversion factor

The lambda conversion factor is calculated from $f_{c,0,k}$ the compression stress parallel to grain and $E_{0,0.05}$ the 5% elastic modulus parallel to grain using the formula $\frac{1}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,0.05}}}$. Values of $f_{c,0,k}$ and $E_{0,0.05}$ are taken from Table 2.5.

Table 2.24: Calculating the factors $k_{c,y}$ and $k_{c,z}$

First take $\beta_c = 0.2$ for solid timber or 0.1 for Glulam and LVL
Then calculate $k_y = 0.5(1 + \beta_c(\lambda_{rel,y} - 0.3) + \lambda^2_{rel,y})$ and $k_z = 0.5(1 + \beta_c(\lambda_{rel,z} - 0.3) + \lambda^2_{rel,z})$
Then calculate $k_{c,y} = \dfrac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}}$ and $k_{c,z} = \dfrac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}}$

Source: EC5 Part 1-1, Equations 6.25 to 6.28.

Stocky struts

Where both $\lambda_{rel,y}$ and $\lambda_{rel,z}$ are 0.3 or less the strut is stocky and its capacity is not limited by slenderness effects. The stresses should satisfy Equations 6.19 and 6.20 of EC5 Part 1-1, which are

$$\left[\frac{\sigma_{c,o,d}}{f_{c,0,d}}\right]^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \text{ and} \left[\frac{\sigma_{c,o,d}}{f_{c,0,d}}\right]^2 + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1$$

The k_m factor has been introduced in Section 2.8.1 and values are given in Table 2.15.

If the strut is carrying axial load only so that $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are both zero, then these two expressions become simply

$$\sigma_{c,o,d} \leq f_{c,0,d}$$

Slender struts

Where either $\lambda_{rel,y}$ or $\lambda_{rel,z}$ is more than 0.3, the stresses will be increased by slenderness effects and should satisfy Equations 6.23 and 6.24 of EC5 Part 1-1, which are

$$\frac{\sigma_{c,o,d}}{k_{c,y} f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \text{ and } \frac{\sigma_{c,o,d}}{k_{c,z} f_{c,0,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1$$

The factor k_m has been introduced in Section 2.8.1 and values are given in Table 2.15.

If the strut is carrying axial load only so that $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are both zero, then these two expressions become simply

$$\sigma_{c,o,d} \leq k_{c,y} f_{c,0,d} \text{ and } \sigma_{c,o,d} \leq k_{c,z} f_{c,0,d}$$

The factors $k_{c,y}$ and $k_{c,z}$ take account of the effect of slenderness in reducing the strength of the strut. The method of calculating the factors is given in the earlier Table 2.24.

Example 2.9 Axial load capacity of a timber post without bending moment
A planed softwood post of strength class C16 has a cross-section 97 mm × 145 mm (see Figure 2.9). The post is 3.5 m high, with the top and bottom restrained in position but not restrained against rotation. Assuming service class 2 and medium-term load, calculate the axial load that the post can carry.

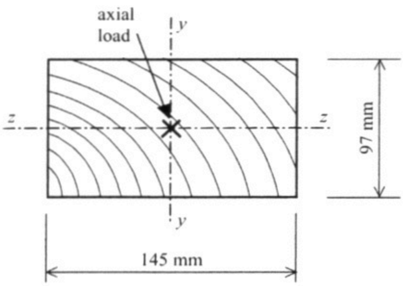


Figure 2.9: Timber post in Example 2.9

For completeness, the following calculations relate to buckling about both axes. In practice, if the effective length is the same about both axes it is only necessary to check the weak axis *z-z*, as the calculations for the strong axis *y-y* will always give a higher permitted load.

Calculations for Example 2.9	
Design compressive strength of timber	
From Table 2.5, for C16 timber	$f_{c,0,k} = 17\,\text{N/mm}^2$
From Table 2.14, for solid timber in service class 2 under medium-term loads	$k_{mod} = 0.80$
From Table 2.2, for solid timber	$\gamma_m = 1.3$
Design compressive strength of timber $f_{c,0,d} = k_{mod} f_{c,0,k}/\gamma_m = 0.80 \times 17/1.3$	$f_{c,0,d} = 10.5\,\text{N/mm}^2$

Calculations for buckling about the *z-z* axis	
Radius of gyration $i_z = 0.289 \times 97$	$i_z = 28\,\text{mm}$
Effective length = actual length	$l_{eff} = 3500\,\text{mm}$
Slenderness ratio $\lambda_z = 3500/28$	$\lambda_z = 125$
From Table 2.23, for C16 timber	lambda conversion factor = 0.0179
$\lambda_{rel,z} = \lambda_z \times$ lambda conversion factor $= 125 \times 0.0179$	$\lambda_{rel,z} = 2.24$
$\lambda_{rel,z}$ is more than 0.3, so use the equation $\sigma_{c,o,d} \leq k_{c,z} f_{c,0,d}$	
From Table 2.24, $k_z = 0.5(1 + \beta_c(\lambda_{rel,z} - 0.3) + \lambda^2_{rel,z})$, with $\beta_c = 0.2$ for solid timber	
$k_z = 0.5(1 + 0.2(2.24 - 0.3) + 2.24^2)$	$k_z = 3.20$
$k_{c,z} = \dfrac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \dfrac{1}{3.20 + \sqrt{3.20^2 - 2.24^2}}$	$k_{c,z} = 0.182$
$k_{c,z} f_{c,0,d} = 0.182 \times 10.5 = 1.91\,\text{N/mm}^2$	$\sigma_{c,0,d} \leq 1.91\,\text{N/mm}^2$
Ultimate axial load capacity $= 1.91 \times 97 \times 145 = 26\,860\,\text{N}$	Axial load capacity = 26.9 kN
Calculations for buckling about the *y-y* axis	
Radius of gyration $i_y = 0.289 \times 145$	$i_y = 41.9\,\text{mm}$
Effective length = actual length	$l_{eff} = 3500\,\text{mm}$
Slenderness ratio $\lambda_y = 3500/41.9$	$\lambda_y = 83.5$
From Table 2.23, for C16 timber	lambda conversion factor = 0.0179
$\lambda_{rel,y} = \lambda_y \times$ lambda conversion factor $= 83.5 \times 0.0179$	$\lambda_{rel,y} = 1.49$
$\lambda_{rel,y}$ is more than 0.3, so use the equation $\sigma_{c,o,d} \leq k_{c,y} f_{c,0,d}$	
From Table 2.24, $k_y = 0.5(1 + \beta_c(\lambda_{rel,y} - 0.3) + \lambda^2_{rel,y})$, with $\beta_c = 0.2$ for solid timber.	
$k_y = 0.5(1 + 0.2(1.49 - 0.3) + 1.49^2)$	$k_y = 1.73$
$k_{c,y} = \dfrac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = \dfrac{1}{1.73 + \sqrt{1.73^2 - 1.49^2}}$	$k_{c,y} = 0.38$
$k_{c,y} f_{c,0,d} = 0.38 \times 10.5 = 3.99\,\text{N/mm}^2$	$\sigma_{c,0,d} \leq 3.99\,\text{N/mm}^2$
Ultimate axial load capacity $= 3.99 \times 97 \times 145 = 56120\,\text{N}$	Axial load capacity = 56.1 kN
As expected, the *z-z* axis buckling is critical	Axial load capacity = 26.9 kN

Example 2.10 Timber post with axial load and bending moment

The timber post in Example 2.9 carries an ultimate vertical force of 20 kN which is 50 mm eccentric as shown in Figure 2.10. Determine whether the post is adequate at ULS.

As the load is eccentric it will generate a bending moment about the *y-y* axis, and it is necessary to check both *y-y* and *z-z* buckling. The post is 3.5 m high, with the top and bottom restrained in position but not restrained against rotation. Assume Service Class 2 and medium-term load.

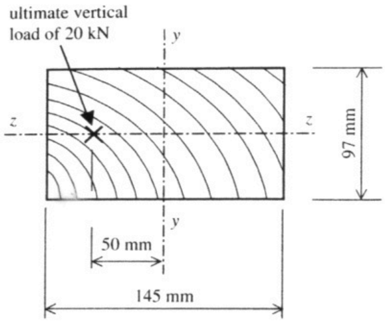


Figure 2.10: Timber post in Example 2.10

Calculations for Example 2.10	
Design bending and compressive strength of timber	
From Table 2.5, for C16 timber	$f_{m,k} = 18\,N/mm^2$
	$f_{c,0,k} = 17\,N/mm^2$
From Table 2.14, for solid timber in service class 2 under medium-term loads	$k_{mod} = 0.80$
From Table 2.2, for solid timber	$\gamma_m = 1.3$
Design bending strength of timber $f_{m,d} = k_{mod} f_{m,k}/\gamma_m = 0.80 \times 18/1.3$	$f_{m,d} = 11.1\,N/mm^2$
For y-y bending the depth of the section is 145 mm	
From Table 2.16 $k_h = 1.005$, so $f_{m,y,d} = 1.005 \times 11.1$	$f_{m,y,d} = 11.2\,N/mm^2$
For z-z bending the depth of the section is 97 mm	
From Table 2.16 $k_h = 1.09$, so $f_{m,z,d} = 1.09 \times 11.1$	$f_{m,z,d} = 12.1\,N/mm^2$
Design compressive strength of timber $f_{c,0,d} = k_{mod} f_{c,0,k}/\gamma_m = 0.80 \times 17/1.3$	$f_{c,0,d} = 10.5\,N/mm^2$
Design bending and compressive stresses	
Design compressive stress $\sigma_{c,0,d} = F/A = 20 \times 10^3/(97 \times 145)$	$\sigma_{c,0,d} = 1.42\,N/mm^2$
For y-y bending	
Bending moment M_y = Axial force × Eccentricity $= 20 \times 10^3 \times 50 = 1.0 \times 10^6\,Nmm$	
Elastic section modulus $W_{yy} = 97 \times 145^2/6 = 340 \times 10^3\,mm^3$	
Design bending stress in y-y bending $\sigma_{m,y,d} = M_y/W_{yy} = 1.0 \times 10^6/340 \times 10^3$	$\sigma_{m,y,d} = 2.94\,N/mm^2$
Design bending stress in z-z bending $\sigma_{m,z,d} = 0$	$\sigma_{m,z,d} = 0$
Calculations for buckling about the z-z axis	
Radius of gyration $i_z = 0.289 \times 97$	$i_z = 28\,mm$
Effective length = actual length	$l_{eff} = 3500\,mm$
Slenderness ratio $\lambda_z = 3500/28$	$\lambda_z = 125$
From Table 2.23, for C16 timber	lambda conversion factor = 0.0179

$\lambda_{rel,z} = \lambda_z \times$ lambda conversion factor $= 125 \times 0.0179$	$\lambda_{rel,z} = 2.24$
$\lambda_{rel,z}$ is more than 0.3, so use the equation $\dfrac{\sigma_{c,o,d}}{k_{c,z}f_{c,0,d}} + k_m\dfrac{\sigma_{m,y,d}}{f_{m,y,d}} + \dfrac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1$	
From Table 2.24, $k_z = 0.5(1 + \beta_c(\lambda_{rel,z} - 0.3) + \lambda^2_{rel,z})$, with $\beta_c = 0.2$ for solid timber.	
$k_z = 0.5(1 + 0.2(2.24 - 0.3) + 2.24^2)$	$k_z = 3.20$
$k_{c,z} = \dfrac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,y}^2}} = \dfrac{1}{3.20 + \sqrt{3.20^2 - 2.24^2}}$	$k_{c,z} = 0.182$
$\dfrac{\sigma_{c,o,d}}{k_{c,z}f_{c,0,d}} + k_m\dfrac{\sigma_{m,y,d}}{f_{m,y,d}} + \dfrac{\sigma_{m,z,d}}{f_{m,z,d}} = \dfrac{1.42}{0.182 \times 10.5} + 0.7 \times \dfrac{2.94}{11.2} + \dfrac{0}{12.1} = 0.93 \leq 1.0$	
The post is adequate at ULS for direct compression and buckling about the *z-z* axis	
Calculations for bending and buckling about the *y-y* axis	
Radius of gyration $i_y = 0.289 \times 145$	$i_y = 41.9\,\text{mm}$
Effective length = actual length	$l_{eff} = 3500\,\text{mm}$
Slenderness ratio $\lambda_y = 3500/41.9$	$\lambda_y = 83.5$
From Table 2.23, for C16 timber	lambda conversion factor = 0.0179
$\lambda_{rel,y} = \lambda_y \times$ lambda conversion factor $= 83.5 \times 0.0179$	$\lambda_{rel,y} = 1.49$
$\lambda_{rel,y}$ is more than 0.3, so use the equation $\dfrac{\sigma_{c,o,d}}{k_{c,y}f_{c,0,d}} + \dfrac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m\dfrac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1$	
From Table 2.24, $k_y = 0.5(1 + \beta_c(\lambda_{rel,y} - 0.3) + \lambda^2_{rel,y})$, with $\beta_c = 0.2$ for solid timber	
$k_y = 0.5(1 + 0.2(1.49 - 0.3) + 1.49^2)$	$k_y = 1.73$
$k_{c,y} = \dfrac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = \dfrac{1}{1.73 + \sqrt{1.73^2 - 1.49^2}}$	$k_{c,y} = 0.38$
$\dfrac{\sigma_{c,o,d}}{k_{c,y}f_{c,0,d}} + \dfrac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m\dfrac{\sigma_{m,z,d}}{f_{m,z,d}} = \dfrac{1.42}{0.38 \times 10.5} + \dfrac{2.94}{11.2} + 0.7 \times \dfrac{0}{12.1} = 0.62 \leq 1.0$	
The post is adequate at ULS for direct compression and for bending and buckling about the *y-y* axis	

2.11 References

BS EN 336: 2003 Structural timber: sizes, permitted deviations.

BS EN 338: 2003 Structural timber strength classes.

BS EN 385: 2001 Finger jointed structural timber: performance requirements and minimum production requirements.

BS EN 1995 Part 1-1: 2004 Eurocode 5: Design of timber structures – Part 1-1: General – Common rules and rules for buildings.

BS EN 14081-1: 2005 Timber structures: strength graded structural timber with rectangular section – Part 1: General requirements.

Manual for the design of timber building structures to Eurocode 5, IStructE/TRADA, December 2007.

CHAPTER 3

Concrete Elements

Contents

3.1 Structural Design of Reinforced Concrete

The limit state philosophy, set out in Chapter 1 of this manual, applies to reinforced concrete as follows.

Generally the following load combinations are used:

For checking ultimate limit states (ULS) $\gamma_G G_k + \gamma_Q Q_k = 1.35G_k + 1.50Q_k$
For checking serviceability limit states (SLS) $\gamma_G G_k + \gamma_Q Q_k = 1.00G_k + 1.00Q_k$

The partial safety factors used for materials are:

For checking ULS of bending, shear and compression $\gamma_m = 1.50$ for concrete, $\gamma_m = 1.15$ for steel.
For checking SLS of cracking and deflection $\gamma_m = 1.00$ for concrete, $\gamma_m = 1.0$ for steel.

In practice the SLS of cracking is checked by reference to rules about bar size and spacing, and the SLS of deflection is checked by reference to rules about the span/effective depth ratio. Thus explicit calculations using γ factors are not required for SLS checks.

Guidance on the structural use of reinforced concrete is given in BS EN 1992, Eurocode 2. This is published in several sections as shown in Table 3.1.

Table 3.1: Codes relating to the design of reinforced concrete

	BSI reference	Title
EC2 Part 1-1	**BS EN 1992 Part 1-1**	**General rules and rules for buildings**
EC2 Part 1-2	**BS EN 1992 Part 1-2**	**General rules – Structural fire design**
EC2 Part 2	BS EN 1992 Part 2	Reinforced and pre-stressed concrete bridges
EC2 Part 3	BS EN 1992 Part 3	Liquid retaining and containing structures

Each code should be read with the appropriate National Annex. Titles shown in **bold** are relevant to this manual.

Much useful additional information, including guidance on UK practice, is given in the following three documents:

(i) 'Manual for the design of concrete building structures to Eurocode 2', published by the Institution of Structural Engineers.

(ii) 'How to design concrete structures using Eurocode 2', a series of guides published by The Concrete Centre and available from www.eurocode2.info.

(iii) Concise Eurocode 2 for the design of in-situ concrete framed buildings to BS EN 1991-1-1:2004 and its UK National Annex: 2005, Narayanan R S and Goodchild CH, The Concrete Centre, 2006.

3.2 Symbols

Table 3.2 lists some of the symbols used in the design process. Where relevant the units commonly used for the quantity are shown.

Table 3.2: Symbols used in the design of reinforced concrete

Symbol	Normal units	Meaning	Comment
A_s	mm^2	Area of reinforcement	
A_{s1}	mm^2	Area of tension reinforcement	
A_{s2}	mm^2	Area of compression reinforcement	
A_{sw}	mm^2	Cross-section area of shear reinforcement	
b	mm	Breadth of section	
b_c	mm	Breadth of the compression face of a beam	
b_v	mm	Breadth of beam used to calculate the shear stress	

Table 3.2: (Continued)

Symbol	Normal units	Meaning	Comment
d	mm	Effective depth to reinforcement	
f_{ck}	N/mm^2	Characteristic cylinder strength of concrete = the lower figure in the grade designation	Normal concrete grade designations are C25/30, C28/35, C30/37, C32/40, C35/45, C40/50, C45/55 and C50/60
f_{yk}	N/mm^2	Characteristic yield strength of reinforcement	f_{yk} = 500 N/mm^2 for Grade H
g_k, G_k	kN, kN/m or kN/m^2	Characteristic permanent action, Dead Load	
h	mm	Overall depth of section	
K		Coefficient obtained from the design formula for rectangular beams	$K = M/bd^2f_{ck}$
K_{lim}		K_{lim} is the upper limit on K if compression reinforcement is not to be provided	$K_{lim} = 0.167$ for singly reinforced elements considered in this manual
q_k, Q_k	kN, kN/m or kN/m^2	Characteristic variable action, e.g. Imposed Load	
s	mm	Spacing of shear links	
SLS		Serviceability limit state	
ULS		Ultimate limit state	
V	kN	Shear force	
v	N/mm^2	Shear stress	
v_E	N/mm^2	Shear stress due to loads.	
v_R	N/mm^2	Shear resistance	
x	mm	Depth to neutral axis	
z	mm	Lever arm	z is the distance between the compressive force in the concrete and the tensile force in the reinforcement
γ_f		Partial factor of safety for load	At ULS these are normally 1.35 for permanent loads and 1.5 for variable loads
γ_m		Partial factor of safety for strength of material	At ULS, γ_m for concrete = 1.5 γ_m for reinforcement = 1.15
ρ_l		Reinforcement ratio for longitudinal reinforcement = A_s/bd	
ρ_w		Area of shear reinforcement = A_{sw}/s	
ϕ	mm	Diameter of reinforcing bar	Available sizes are 6, 8, 10, 12, 16, 20, 25, 32, and 40 mm, although 40-mm bars are not often used
	kN/m^3	Unit weight of unreinforced concrete	Normal value 24
	kN/m^3	Unit weight of reinforced concrete	Normal value 25

3.3 Material Properties

Properties are specified as characteristic values. It is expected that 95% of samples tested will equal or exceed the characteristic value and 5% are permitted to fall below it.

3.3.1 Reinforcing Bars

Tensile properties of steel reinforcing bars are shown in Table 3.3. Class H is normally specified, so the characteristic strength f_{yk} can always be taken as 500 N/mm^2.

Table 3.3: Tensile properties of steel reinforcing bars to BS4449

Class	A	B	C	H
Characteristic yield strength f_{yk}	500 N/mm^2	500 N/mm^2	500 N/mm^2	If class H is specified, class A, B or C may be supplied for bars up to and including 12 mm diameter, and class B or C for larger bars
Minimum value of f_t/f_y	1.05	1.08	1.15	
Strain at maximum force	at least 2.5%	at least 5.0%	at least 7.5%	

Note: Class H is normally specified.

Reinforcing bars are available designated H6, H8, H10, H12, H16, H20, H25, H32 and H40, although H40 bars are not often used. The bars are not smooth cylinders but have raised ribs on the surface to improve the bond between the steel and the concrete. The number in the designation is a nominal bar diameter which describes the cross-section area of the bar, for example an H12 bar has the same cross-section area as a circular bar of diameter 12 mm, that is $\pi \times 12^2/4 = 113\,\text{mm}^2$. Table 3.4 gives the cross-section area of one bar, its maximum lateral dimension over the ribs and its mass per metre.

Table 3.4: Cross-section areas, maximum lateral dimension and mass of reinforcing bars

Bar designation	H6	H8	H10	H12	H16	H20	H25	H32	H40
Cross-section area (mm^2)	28	50	79	113	201	314	491	804	1257
Maximum lateral dimension (mm)	8	11	13	14	19	23	29	37	46
Mass (kg/m)	0.222	0.395	0.616	0.888	1.579	2.466	3.854	6.313	9.864

At certain stages in the design process some bars are required to provide a certain total area, and Table 3.5 gives this information. At other times a set of bars at constant spacing is required to provide a known area per metre run, and Table 3.6(a) gives these.

Table 3.5: Sectional areas of groups of bars (mm^2)

Bar size	Number of bars								
	1	2	3	4	5	6	7	8	9
H6	28	57	85	113	141	170	198	226	254
H8	50	101	151	201	251	302	352	402	452
H10	79	157	236	314	393	471	550	628	707
H12	113	226	339	452	565	679	792	905	1018
H16	201	402	603	804	1005	1206	1407	1608	1810
H20	314	628	942	1257	1571	1885	2199	2513	2827
H25	491	982	1473	1963	2454	2945	3436	3927	4418
H32	804	1608	2413	3217	4021	4825	5630	6434	7238
H40	1257	2513	3770	5027	6283	7540	8796	10053	11310

Table 3.6(a): Sectional areas per metre width for various bar sizes and spacing (mm^2/m)

Bar size	Spacing of bars (mm)								
	75	100	125	150	175	200	250	300	350
H6	377	283	226	188	162	141	113	94	81
H8	670	503	402	335	287	251	201	168	144
H10	1047	785	628	524	449	393	314	262	224
H12	1508	1131	905	754	646	565	452	377	323
H16	2681	2011	1608	1340	1149	1005	804	670	574
H20	4189	3142	2513	2094	1795	1571	1257	1047	898
H25	6545	4909	3927	3272	2805	2454	1963	1636	1402
H32	10723	8042	6434	5362	4596	4021	3217	2681	2298
H40	16755	12566	10053	8378	7181	6283	5027	4189	3590

In reinforced concrete slabs and walls the time and labour of fixing loose bars can often be avoided by the use of welded steel fabric meshes. Standard fabrics are available in 4.8 m × 2.4 m sheets as shown in Table 3.6(b), and it is worth noting that the number in the fabric reference (e.g. the number 393 in the reference A393) refers to the area of longitudinal reinforcement in mm^2 per metre width of sheet. Non-standard fabrics can be made to order.

3.3.2 Concrete

Concrete is normally specified in terms of compressive strength, which may be tested by crushing cubes or cylinders. Cubes are normally used in the UK. If a cube and a cylinder are made of the same concrete then the cube will achieve a higher strength because the steel platens of the compression testing machine exert a greater lateral restraint on a stocky cube than they do on a more slender cylinder. A grade description of C25/30 means that the concrete is specified for compressive strength and that the characteristic strength 28 days after casting should be at least 25 N/mm^2 if cylinders are tested or 30 N/mm^2 if cubes are tested. Other concrete grades are shown in Table 3.7.

Table 3.6(b): Reinforcement fabrics to BS 4483: 2005

Fabric reference	Longitudinal wires			Cross wires			Mass (kg/m^2)
	Diameter (mm)	Pitch (mm)	Area (mm^2/m)	Diameter (mm)	Pitch (mm)	Area (mm^2/m)	
Square mesh							
A393	10	200	393	10	200	393	6.16
A252	8	200	252	8	200	252	3.95
A193	7	200	193	7	200	193	3.02
A142	6	200	142	6	200	142	2.22
A98	5	200	98	5	200	98	1.54
Structural mesh							
B1131	12	100	1131	8	200	252	10.9
B785	10	100	785	8	200	252	8.14
B503	8	100	503	8	200	252	5.93
B385	7	100	385	7	200	193	4.53
B283	6	100	283	7	200	193	3.73
B196	5	100	196	7	200	193	3.05
Long mesh							
C785	10	100	785	6	400	70.8	6.72
C636	9	100	636	6	400	70.8	5.55
C503	8	100	503	5	400	49	4.34
C385	7	100	385	5	400	49	3.41
C283	6	100	283	5	400	49	2.61
Wrapping mesh							
D98	5	200	98	5	200	98	1.54
D49	2.5	100	49	2.5	100	49	0.77
Stock sheet size	Longitudinal wires Length 4.8 m			Cross wires Width 2.4 m			Sheet area 11.52 m^2

Table 3.7: Concrete grades to BS 8500 and BS EN 206

Concrete grade	C25/30	C28/35	C30/37	C32/40	C35/45	C40/50	C45/55	C50/60
Cylinder strength f_{ck} (N/mm^2)	25	28	30	32	35	40	45	50
Cube strength (N/mm^2)	30	35	37	40	45	50	55	60

Design calculations are always based on f_{ck} which is the cylinder compressive strength and is the lower of the two values in the grade designation.

3.3.3 Partial Safety Factors

The use of partial safety factors and the reason why different factors are used in different situations are explained in Chapter 1, which also gives partial safety factors for loads. Partial safety factors for materials are given in Table 3.8. These factors are included in the design formulas and tables used in this manual.

Table 3.8: Partial safety factors γ_m for concrete and steel used in reinforced concrete design

Limit state	SLS	ULS
Concrete	1.0	1.50
Steel	1.0	1.15

3.4 Durability

Durable concrete should perform satisfactorily in its intended environment for the life of the structure. Durability considerations may influence the size of members, so it should be considered before the commencement of structural design. Several inter-related factors determine the durability of reinforced concrete.

(a) The shape and bulk of the concrete.

(b) The environmental conditions to which the concrete will be exposed.

(c) The amount of concrete cover to the reinforcement.

(d) The quality of the concrete, including cement type and content, water/cement ratio, and aggregate type.

(e) The workmanship necessary to deliver and place the concrete in the mould in good condition.

(f) The workmanship necessary to ensure proper compaction, protection and curing of the placed concrete.

Factors (a) to (d) must be considered at the design stage because they influence the form of the concrete elements, the specification of the concrete and the location of the reinforcement. Factors (e) and (f) require suitable clauses in the project specification and adequate site supervision.

3.4.1 Shape and Bulk of Concrete

If the concrete will be exposed when the building is finished, the shape and bulk of the members should be designed to encourage natural drainage and avoid standing water.

3.4.2 Concrete Cover to the Reinforcement

All reinforcement must have sufficient cover to prevent corrosion and to protect the reinforcement in the event of a fire. Cover to protect against fire is discussed in Section 3.5.

The amount of cover necessary to protect reinforcement against corrosion depends on the exposure conditions and the quality of the concrete used. EC2 defines 20 different exposure conditions, and Table 3.9 shows a selection of these covering most common situations.

Table 3.9: Exposure conditions: extract from Table 4.1 of EC2 Part 1-1

Exposure class	Conditions	Examples
Exposed to air and moisture		
XC1	Dry or permanently wet	Interior of buildings for normal habitation (e.g. homes, offices)
XC2	Wet, rarely dry	Completely buried in non-aggressive soil
XC3/4	Moderate humidity, or cyclic wet/dry	Normal outdoor condition, indoor subject to high humidity
In contact with water containing chlorides, including de-icing salt		
XD1	Moderate humidity	Highway structures away from direct spray
XD2	Wet, rarely dry	Totally immersed in water containing chlorides
XD3	Cyclic wet/dry	Highway structures within 10 m of carriageway, including car parks
In contact with sea water or air carrying salts from sea water		
XS1	Exposed to airborne salt but not in direct contact with seawater	Structures in coastal areas
XS2	Permanently submerged	Below mid-tide level
XS3	Tidal, splash and spray zones	Upper tidal, splash and spray zones

Durability depends on a combination of concrete grade and concrete cover. In some situations a thick cover of lower grade concrete can achieve the same durability as a thinner cover of higher grade concrete. Table 3.10 gives recommended covers and concrete grades for the exposure conditions in Table 3.9. Note that:

- Each entry in the table comprises three items:
  - The grade of concrete
  - The maximum water/cement ratio
  - The minimum cement content in kg of cement per m^3 of concrete.

All three of these must be satisfied to meet the durability requirement.

- The table gives several different ways of meeting the durability requirements for some exposure classes.

Table 3.10: Nominal concrete cover to reinforcement for concrete made with OPC for a 50-year design life

Exposure class	Nominal cover to all reinforcement (mm)							
	25	**30**	**35**	**40**	**45**	**50**	**55**	**60**
XC1	C20/25 0.7 240							
XC2			C25/30 0.65 260					
XC3/4		C40/50 0.45 340	C32/40 0.55 300	C28/35 0.6 280	C25/30 0.65 260			
XD1			C40/50 0.45 360	C32/40 0.55 320	C28/35 0.60 300			
XD2				C40/50 0.40 380	C32/40 0.50 340	C28/35 0.55 320		
XD3						C45/55 0.35 380	C40/50 0.40 380	C35/45 0.45 360
XS1				C50/60 0.35 380	C40/50 0.45 360	C35/45 0.50 340		
XS2				C40/50 0.40 380	C32/40 0.50 340	C28/35 0.55 320		
XS3							C45/55 0.35 380	C40/50 0.40 380
Key to entries:		C40/50 0.40 380	← Minimum concrete grade ← Maximum water/cement ratio ← Minimum cement content in kg/m^3					

Source: Table NA3 of UK National Annex to EC2 Part 1-1 with $\Delta_{c,dev}$ = 10 mm.

Notes:

(1) The specified cover should be provided to all reinforcement.

(2) When the work is carried out by a company with a recognised quality control system (specifically, a member of SpeCC, the Specialist Concrete Contractors Certification scheme), the nominal cover values may be reduced by 5 mm.

(3) If the element will be subject to freezing and thawing when it is wet, air-entrained concrete should be used.

- Table 3.10 is for mixes made with ordinary Portland cement (OPC). EC2 gives guidance on the use of other cement types which may achieve the required durability with lower cover or with mixes containing a smaller quantity of cement.

3.5 Resistance to Fire

Eurocode 2 Part 1-2 gives three methods for verifying that a structure has adequate fire resistance. These are:

1. Use of tabular data
2. Use of simple calculation methods
3. Use of advanced calculation methods, normally involving computer modelling.

Some information using Method 1 is given in this manual.

Buildings require fire resistance for several reasons, such as:

- To prevent spread of fire within the building or compartment
- To prevent spread of fire to adjacent properties
- To preserve the loadbearing capacity of the building
  - long enough to allow occupants to escape
  - long enough for firefighters to enter the building safely to fight the fire.

Some buildings may be fitted with active fire protection systems such as detectors or sprinklers, but in most cases it will be necessary for the structural elements to have a certain degree of passive fire resistance so that loadbearing capacity is maintained for a suitable period. The UK building regulations set down the method for determining the period required, which depends on the size of the building and on its usage. This is normally expressed in minutes, 30, 60 … up to 240. The structural designer should know this period at the start of the design process.

The required resistance may be stated in one of three ways:

R30 or R60,… fire resistance class for the loadbearing criterion for 30, or 60… minutes in standard fire exposure.

E30 or E60,… fire resistance class for the integrity criterion for 30, or 60… minutes in standard fire exposure.

I30 or I60,… fire resistance class for the insulation criterion for 30, or 60… minutes in standard fire exposure.

In most cases design for the R requirement will be sufficient.

Concrete is a good insulator and is inherently fire resistant, but steel soon loses its strength if it is heated. The most effective way to provide reinforced concrete elements with fire resistance is to ensure that the steel bars are covered by a sufficiently thick layer of concrete, and a minimum cross-section size is also necessary. Table 3.11 shows minimum sizes and minimum concrete cover, expressed as the minimum distance from the axis of a main reinforcing bar to the nearest concrete face, for columns and slabs.

Table 3.11: Resistance to fire: minimum sizes and minimum axis distances for columns and for simply supported slabs

Element		R30	R60	R90	R120	R180	R240
Column exposed to fire on all sides	Minimum cross-section dimension (mm)	200	250	350	350	450	–
	Minimum axis distance (mm)	32	46	53	57	70	–
Simply supported slab with plain soffit	Minimum thickness (mm)	60	80	100	120	150	175
	Minimum axis distance (mm)	10*	20	30	40	55	65

Column information from Table 5.2a of EC2 Part 1-2 with $\mu_{fi} = 0.7$.
Slab information from Table 5.8 of EC2 Part 1-2 for one-way spanning slabs. The minimum thicknesses shown will also satisfy the *E* and *I* resistance requirements.
*More cover will be required to ensure adequate durability.

Table 3.12 gives similar information for beams, and in this case the requirements for axis distance and breadth are related. Figure 3.1 shows two ways of ensuring that a beam satisfies the requirement for R60 fire resistance.

Table 3.12: Resistance to fire: minimum sizes and minimum axis distances for simply supported beams exposed to fire on three faces

Standard fire resistance	Parameter	Possible combinations (mm)			
R30	Minimum breadth	80	120	160	200
	Minimum axis distance from soffit	25	20	15*	15*
	Minimum axis distance from side	35	30	25	15*
R60	Minimum breadth	120	160	200	300
	Minimum axis distance from soffit	40	35	30	25
	Minimum axis distance from side	50	45	40	25
R90	Minimum breadth	150	200	300	400
	Minimum axis distance from soffit	55	45	40	35
	Minimum axis distance from side	65	55	50	35

(*Continued*)

Table 3.12: (Continued)

Standard fire resistance	Parameter	Possible combinations (mm)			
R120	Minimum breadth	200	240	300	500
	Minimum axis distance from soffit	65	60	55	50
	Minimum axis distance from side	75	70	65	50
R180	Minimum breadth	240	300	400	600
	Minimum axis distance from soffit	80	70	65	60
	Minimum axis distance from side	90	80	75	60
R240	Minimum breadth	280	350	500	700
	Minimum axis distance from soffit	90	80	75	70
	Minimum axis distance from side	100	90	85	70

Information from Table 5.5 of EC2 Part 1-2.
When the axis distance is 70 mm or more, surface reinforcement of 4 mm bars at 100 mm spacing should be provided to prevent the surface concrete falling off during the fire.
*More cover will be required to ensure adequate durability.

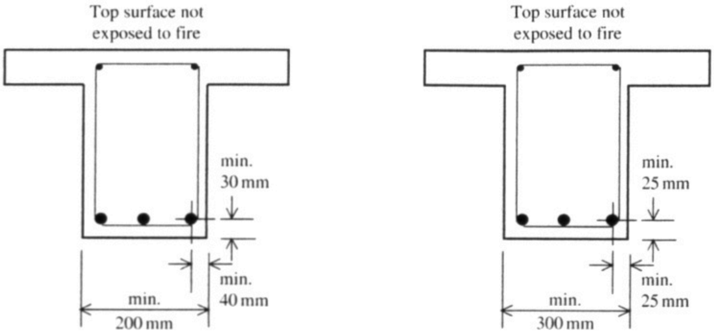


Figure 3.1: Two ways of ensuring that a beam has R60 fire resistance

3.6 Minimum Cover to Reinforcement

In addition to the requirements of durability and fire resistance, there must be

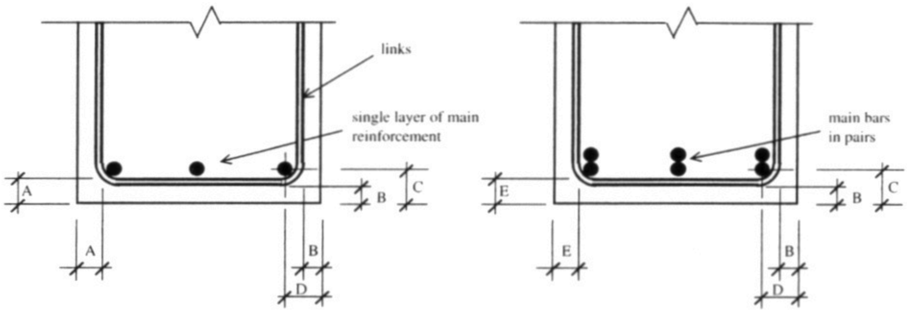
- sufficient space between the bar and the formwork to allow proper placing and compaction of the concrete, and
- sufficient concrete surrounding a reinforcing bar to develop good bond between the two materials.

Table 3.13 shows requirements for these two factors, and also summarises the other factors determining minimum cover. Figure 3.2 illustrates the different factors for the main and shear reinforcement in a beam.

Table 3.13: Factors determining minimum cover to reinforcing bars

Placing of concrete	Not less than the diameter of the bar ϕ + 10 mm
Bond between steel and concrete	Not less that the diameter of the bar ϕ + 10 mm If the bars are bundled, the minimum cover should be: • 1.4ϕ + 10 mm for a 2-bar bundle • 1.7ϕ + 10 mm for a 3-bar bundle • 2.0ϕ + 10 mm for a 4-bar bundle
Durability	See exposure classes in Table 3.9 and requirements in Table 3.10
Fire resistance	See Table 3.11 for slabs and columns, Table 3.12 for beams

Note that the first three of these factors determine the minimum cover to all bars while fire resistance requirements are given in terms of axis distance to the main reinforcement.



A = cover to a single main bar, not less than bar diameter + 10 mm. Determined by requirement for concrete compaction and for bond.
B = cover to all reinforcement, determined by durability requirements.
C,D = axis distance to main bar, determined by fire resistance requirement.
E = cover to group of 2 bars, not less than 1.4 bar diameters + 10 mm. Determined by requirement for concrete compaction and for bond.

Figure 3.2: Factors determining cover to reinforcement

3.7 Limits on Areas of Reinforcement and Bar Spacing

3.7.1 Minimum Reinforcement and Maximum Bar Spacing

As concrete dries and shrinks it develops microcracks, and these cracks enlarge when tensile stress is applied. If these cracks became too wide the effects would include

- loss of durability
- loss of fire resistance
- unattractive appearance.

A crack width of 0.3 mm is generally regarded as acceptable, and reinforcement is required to prevent larger cracks. It is important to realize that the function of the reinforcement is to control cracking not to prevent it, as a number of fine cracks are more acceptable than a single wide crack.

If watertight concrete is required for a reservoir or a basement then a lower crack width limit will be needed. EC2 Part 3 gives guidance on this.

For normal structural concrete, crack widths can be controlled by providing at least the required minimum amount of reinforcement, as in Table 3.14. The percentage should be based on the gross area of the concrete, not the effective area; for example, in a beam the percentage should be based on *bh*, not on *bd*. In slabs this should be provided in both directions.

Table 3.14: Minimum percentage of tensile reinforcement in beams and slabs

Conc. strength f_{ck} (N/mm^2)	25	28	30	32	35	40	45	50
Minimum % of reinforcement	0.14	0.15	0.15	0.16	0.17	0.19	0.20	0.22

This table uses $0.016 f_{ck}^{2/3}$ as recommended by the *IStructE manual for the design of concrete building structures to Eurocode 2* in place of $0.0156 f_{ck}^{2/3}$ in EC2.

If the amount of reinforcement given in Table 3.14 was provided as a few large bars spaced widely apart it would not be effective in controlling cracking, especially cracking of concrete remote from a bar. Restrictions on the maximum bar spacing and/or the maximum bar size for beams and slabs are given in Tables 3.15 and 3.16.

Table 3.15: Maximum bar size or maximum bar spacing for 0.3-mm crack width limit for load-induced cracking in beams and in slabs more than 200 mm thick

Steel stress (N/mm^2): see note 1	**160**	**200**	**240**	**280**	**320**	**360**	**400**
Max. bar size	H32	H25	H16	H12	H10	H8	H6
Max. bar spacing (mm)	300	250	200	150	100	50	–

Notes:

(1) These rules do not apply to secondary or distribution reinforcement.

(2) The steel stress can be taken as $435(G_k + 0.8Q_k)/(1.35G_k + 1.50Q_k)$ N/mm^2, or conservatively as 320 N/mm^2.

(3) Cracks may be controlled by meeting either the max. bar spacing requirement *or* the max. bar size requirement. It is not necessary to meet both requirements. For example, if the steel stress is 280 N/mm^2 then either bars of size H12 or smaller can be used at any spacing or bars of any size can be used at a spacing of 150 mm or less.

(4) Data are from Tables 7.2N and 7.3N of EC2 Part 1-1.

Table 3.16: Maximum bar spacing: other provisions

Location	**Maximum bar spacing**
Reinforcement in slabs 200 mm thick or less	Main reinforcement, $3d$ but not more than 400 mm Secondary reinforcement, $3.5d$ but not more than 450 mm
Additional side reinforcement in beams more than 1000 mm deep	See EC2
Shear links	See Section 3.9.5
Reinforcement in columns	See Section 3.11

3.7.2 Maximum Reinforcement and Minimum Bar Spacing

Proper compacting of the concrete will not be possible if there is too much reinforcement in a member or if the bars are too close together. The requirements are given in Tables 3.17 and 3.18.

Table 3.17: Maximum amount of reinforcement

Location	Maximum amount of reinforcement
Slab or beam, tension reinforcement	4% other than at laps
Slab or beam, compression reinforcement	4% other than at laps
Column	4%, or 8% at laps

Table 3.18: Minimum clear space between bars

Gap between bars (except at laps) should be at least:	• the maximum bar size ϕ • 20 mm • aggregate size + 5 mm (=25 mm when using the normal aggregate size of 20 mm)

3.8 Flexural Members

Flexural members are those subject to bending, e.g. beams and slabs. The design procedures for beams and slabs are essentially the same with some subtle differences, so beams will be studied first.

3.9 Beams

The design of reinforced concrete beams is considered under the following headings:

3.9.1 Effective span of beams

3.9.2 Deep beams

3.9.3 Slender beams

3.9.4 Design of beams for bending ULS

3.9.5 Design of beams for shear ULS

3.9.6 Design of beams for deflection SLS

3.9.1 Effective Span of Beams

The effective span of a simply supported beam may be taken as the lesser of:

(a) the distance between the centres of bearings,

(b) the clear distance between supports plus the overall depth h.

The effective length of a cantilever should be taken as its length to the face of the support plus half its overall depth *h*.

3.9.2 Deep Beams

Deep beams having a clear span of less than three times their overall depth require special consideration and are outside the scope of this manual. EC2 gives design guidance.

3.9.3 Slender Beams

Slender beams, where the breadth of the compression face *b* is small compared with the depth and length, have a tendency to fail by lateral torsional buckling. If this buckling is prevented by, for example, a slab attached to the compression face of the beam, then the beam is considered as laterally restrained and it will be satisfactory if the depth *h* is not more than 2.5 times the breadth *b*. If the compression face is unrestrained, then the unrestrained length should not be more than 35*b*.

In practice, slender beams are rare.

3.9.4 Design of Beams for Bending ULS

Downward loads on a simply supported beam induce sagging bending moments which cause compressive stress in the material fibres above the neutral axis and tensile stresses in those below. Concrete is good at resisting compressive stresses but its resistance to tension is so poor that it is ignored. Instead, steel reinforcing bars are introduced to resist the tension.

On this basis, simply supported beams are designed so that the concrete above the neutral axis is capable of resisting the induced compression, and reinforcement capable of resisting the induced tension is introduced below the neutral axis. This reinforcement is positioned near the bottom of the beam, subject to minimum cover requirements, where it will be most effective. Concrete beams designed in this way are described as singly reinforced. Some of the parameters used to describe the cross-section of a singly reinforced beam are shown in Figure 3.3.

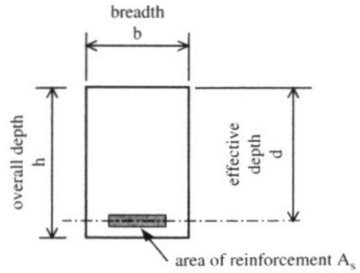


Figure 3.3: Cross-section of rectangular singly reinforced beam

A beam will be adequate in bending if its internal moment of resistance is not less than the externally applied bending moment. Therefore, the design ultimate resistance moment M of a concrete beam must be greater than or equal to the ultimate bending moment M_u.

$$M \geq M_u$$

The ultimate bending moment M_u is calculated under ultimate design loads and will be greatest at or near the centre of the beam. The method of determining M is described in Figure 3.4 and the text that follows.

Figure 3.4(a) shows a cross-section of the beam, and Figure 3.4(b) shows the strains in the beam when it is subject to a sagging bending moment equal to M. The neutral axis is at distance x from the top of the beam.

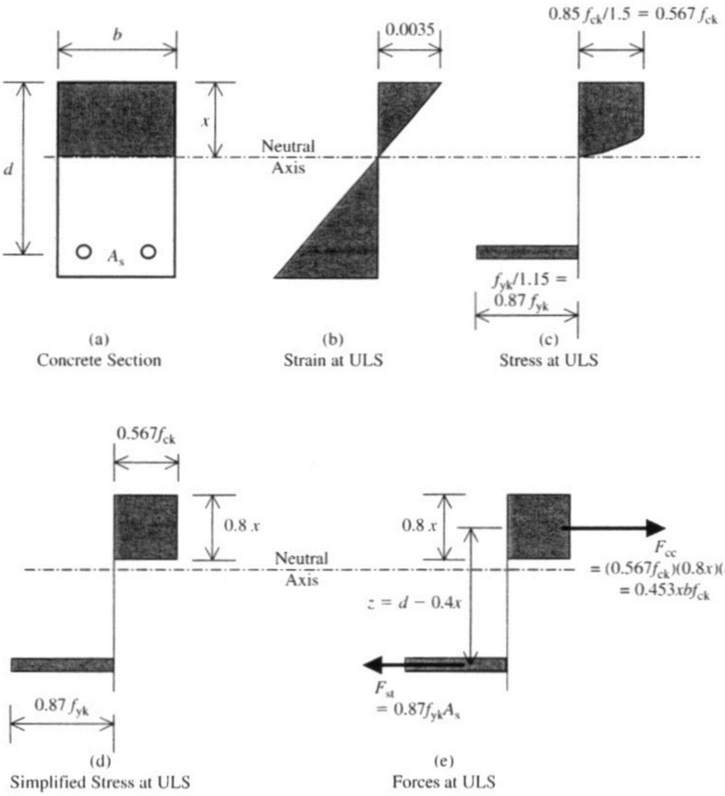


Figure 3.4: Stresses and forces in a rectangular concrete beam at ULS

In the UK it is considered good practice that x should not be more than $d/2$. This will ensure that the beam is under-reinforced and will fail in a ductile manner by yielding of the reinforcement giving a robust structural element with some warning of the failure. If x is much greater than $d/2$ then the beam is over-reinforced and may fail by crushing of the concrete, possibly leading to a sudden collapse. Limiting K to 0.167, as described later in this section, ensures that the beam is under-reinforced.

Figure 3.4(c) shows the design values of the stresses at the ULS.

- The concrete cylinder strength f_{ck} is multiplied by 0.85, a factor which converts compressive strength into bending strength, and divided by the partial safety factor $\gamma_m = 1.5$.
- The reinforcement strength f_{yk} is divided by the partial safety factor $\gamma_m = 1.15$.

Note that the stress in the concrete below the neutral axis is zero as it is assumed to be cracked.

Figure 3.4(d) is a simplified version of 3.4(c) with the curved stress block replaced by a rectangular stress block of depth $0.8\boldsymbol{x}$.

Figure 3.4(e) shows the stresses from 3.4(d) converted to forces.

- Concrete force F_{cc} = stress multiplied by area
  $= (0.567f_{ck})(0.8x)(b) = 0.453xbf_{ck}$
- Steel force F_{st} = stress multiplied by area
  $= 0.87f_{yk}A_s$

Since the beam is not carrying any axial force, F_{cc} and F_{st} are equal.

Taking $z = d - 0.4x$ as the distance between the two forces, the design ultimate resistance moment of the beam $M = zF_{cc}$, or alternatively $M = zF_{st}$.

Hence $\quad M = (d - 0.4x)(0.453xbf_{ck})$
or $\quad M = (d - 0.4x)(0.87f_{yk}A_s)$.

These formulas can be used to find M when A_s is known, but a designer normally needs to find A_s when M is known. The formulas are re-arranged to give the following method of finding the amount of reinforcement needed to give the beam a design ultimate resistance moment M.

1. Find the factor $K = M/f_{ck}bd^2$.

If the value of K for a singly reinforced beam is found to exceed $K_{lim} = 0.167$** then the beam is likely to fail by crushing of the concrete before the reinforcement yields. This should be avoided by either:

- using stronger concrete and so reducing K
- increasing the size of the beam and so reducing K
- providing compression reinforcement above the neutral axis to form a doubly reinforced beam.

The design of doubly reinforced beams is not covered by this manual.

** If the bending moments have been re-distributed during the analysis, the limit on K is lower. This manual does not cover design using re-distributed bending moments.

The units of the variables used to calculate K must be consistent. Since f_{ck}, b and d are normally expressed in N and mm, M must be converted to Nmm when it is used in the formula.

Note that 1 kNm = 10^6 Nmm

If M is given in kNm, then it should be multiplied by 10^6 when used in the formula for K.

2. Find $z/d = 0.5(1 + \sqrt{(1 - 3.53K)})$ but not more than 0.95. Table 3.19 can be used in place of this formula.

Table 3.19: Values of z/d for design of bending reinforcement in beams and slabs

K	**up to 0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**	**0.11**	**0.12**	**0.13**	**0.14**	**0.15**	**0.16**	**0.167**
z/d	0.950	0.944	0.934	0.924	0.913	0.902	0.891	0.880	0.868	0.856	0.843	0.830	0.820
x/d	0.125	0.140	0.165	0.191	0.217	0.245	0.272	0.301	0.331	0.361	0.393	0.425	0.449

Formulas: $z/d = 0.5(1 + \sqrt{(1 - 3.53K)})$ but not more than 0.95
$x/d = 2.5(1 - z/d)$

3. Find $z = d(z/d)$.
4. Find $A_s = M/(0.87zf_{yk})$.
5. Check that A_s is not less than the minimum percentage in Table 3.14.
6. Choose some reinforcing bars with an area of at least A_s.
7. Check that the area of bars provided does not exceed the maximum percentage in Table 3.17. If it does then a larger beam may be needed.

Concrete beams are often constructed monolithic with floor slabs, and it may be possible for some of the floor slab to be considered as part of the beam. This has the advantage of increasing the breadth b of the beam, and hence the lever arm z, at little or no additional cost, and this may reduce the amount of reinforcement required. The resulting L and T beams are shown in Figure 3.5. Note that design for shear reinforcement is based on the beam web width as shown later in Figure 3.11. This manual does not cover the design of L and T beams.

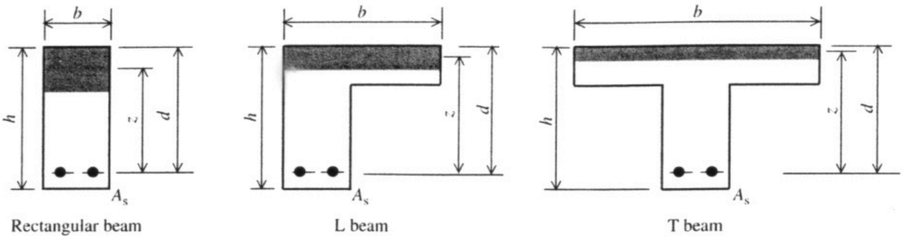


Figure 3.5: Rectangular, L and T beams at bending ULS

Example 3.1 Design of beam for bending ULS

The singly reinforced concrete beam shown in Figure 3.6 is made of C30/37 concrete and is required to resist an ultimate sagging bending moment of 525 kNm. Check the section size and determine the area of reinforcement required.

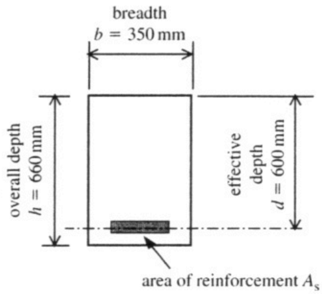


Figure 3.6: Beam cross-section for Example 3.1

Calculations for Example 3.1	
Data given	
Beam width b	$b = 350$ mm
Overall depth h	$h = 660$ mm
Effective depth d	$d = 600$ mm
Required M	$M = 525$ kNm $= 525 \times 10^6$ Nmm
Concrete grade	C30/37

Bending reinforcement calculations	
f_{ck} (characteristic cylinder strength of concrete)	$f_{ck} = 30\,N/mm^2$
f_{yk} (characteristic tensile strength of reinforcement, class H)	$f_{yk} = 500\,N/mm^2$
$K = M/bd^2f_{ck} = 525 \times 10^6/(350 \times 600^2 \times 30)$	$K = 0.139$
K should not be more than not more than $K_{lim} = 0.167$	Accept
$z/d = 0.5\times(1 + \sqrt{(1 - 3.53K)}) = 0.5\times(1 + \sqrt{(1 - 3.53 \times 0.139)})$	$z/d = 0.857$
z/d not more than 0.95	Accept
$z = d(z/d) = 600 \times 0.857$	$z = 514\,mm$
$A_s = M/(0.87zf_{yk}) = 525 \times 10^6/(0.87 \times 514 \times 500)$	$A_s = 2348\,mm^2$
From Table 3.14, min. $A_s = 0.15\%$ of the gross concrete area $= 0.15\% bh = 0.15\% \times 350 \times 660$	Min. $A_s = 346\,mm^2$
From Table 3.5, Choose bars to provide at least $2348\,mm^2$	Use 5no. H25 bars
	$A_{s,prov} = 2454\,mm^2$
Percentage of reinforcement $= 2454 \times 100/(350 \times 660) = 1.06\%$ which is less than 4% from Table 3.17	Accept

Example 3.2 Choice of beam size to satisfy bending ULS

The singly reinforced concrete beam shown in Figure 3.7 is made of C35/45 concrete and is required to resist an ultimate sagging bending moment of 150 kNm. The breadth b is 250 mm. Choose a suitable beam depth and determine the area of reinforcement required.

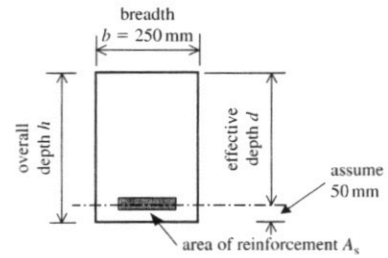


Figure 3.7: Beam cross-section for Example 3.2

Calculations for Example 3.2	
Data given	
Beam width b	$b = 250\,mm$
Required M	$M = 150\,kNm = 150 \times 10^6\,Nmm$
Concrete grade	C35/45

Continued on next page

Calculations for Example 3.2 (Continued from previous page)	
Calculations	
f_{ck} (characteristic cylinder strength of concrete)	$f_{ck} = 35\,N/mm^2$
f_{yk} (characteristic tensile strength of reinforcement, class H)	$f_{yk} = 500\,N/mm^2$
$K = M/bd^2f_{ck} = 150 \times 10^6/(250 \times d^2 \times 35)$	$K = 17143/d^2$
K should not more than 0.167, so $17143/d^2 \leq 0.167$	
$d^2 \geq 17143/0.167$	$d^2 \geq 102653$
$d \geq \sqrt{102653}$	$d \geq 320\,mm$ Adopt $d = 325\,mm$
$h = d + 50 = 325 + 50$	$h = 375\,mm$
$K = M/bd^2f_{ck} = 150 \times 10^6/(250 \times 325^2 \times 35)$	$K = 0.162$
$z/d = 0.5 \times (1 + \sqrt{(1 - 3.53K)}) = 0.5 \times (1 + \sqrt{(1 - 3.53 \times 0.162)})$	$z/d = 0.827$
z/d not more than 0.95	Accept
$z = d(z/d) = 325 \times 0.827$	$z = 269\,mm$
$A_s = M/(0.87zf_{yk}) = 150 \times 10^6/(0.87 \times 269 \times 500)$	$A_s = 1282\,mm^2$
From Table 3.14, min. $A_s = 0.15\% \times 250 \times 375$	Min. $A_s = 141\,mm^2$
From Table 3.5, choose bars to provide at least $1282\,mm^2$	Use 3no. H25 bars $A_{s,prov} = 1473\,mm^2$
Percentage of reinforcement = $1473 \times 100/(250 \times 375) = 1.6\%$ which is less than 4% from Table 3.17	Accept

Example 3.3 Design of beam for bending ULS

A simply supported reinforced concrete beam with an effective span of 7.0 m is 500 mm deep overall by 250 mm wide (see Figure 3.8). It supports the following characteristic loads:

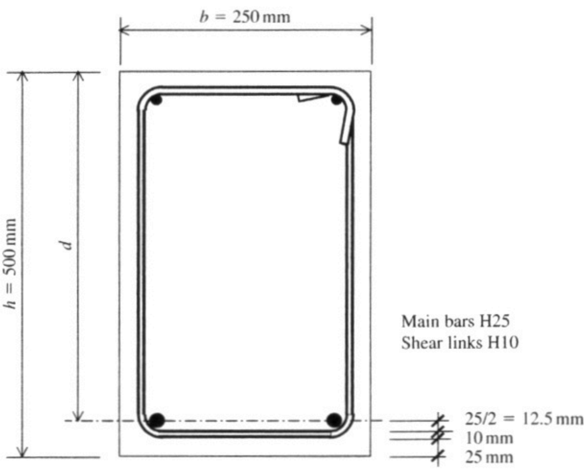


Figure 3.8: Beam cross-section for Example 3.3

Permanent dead loads: 12.0 kN/m plus beam self-weight
Variable imposed loads: 11.0 kN/m.

The concrete is grade C40/50, and 25-mm cover is required to all reinforcement. Assuming that the shear links are H10 and the main bars are H25, check whether the beam size is adequate for the bending ULS and determine suitable tension reinforcement.

Calculations for Example 3.3	
Data given	
Beam width b	$b = 250\,\text{mm}$
Beam overall height h	$h = 500\,\text{mm}$
Effective span L	$L = 7.0\,\text{m}$
Dead load excluding beam self weight	$g_k = 12.0\,\text{kN/m}$
Imposed load	$q_k = 11.0\,\text{kN/m}$
Concrete grade	C40/50
Min. cover to all steel	25 mm
f_{ck} (characteristic cylinder strength of concrete)	$f_{ck} = 40\,\text{N/mm}^2$
f_{yk} (characteristic tensile strength of reinforcement, class H)	$f_{yk} = 500\,\text{N/mm}^2$
Loading	
Unit weight of concrete = $25\,\text{kN/m}^3$, so beam self-weight = $0.25 \times 0.50 \times 25 \times 7.0$	Beam self-weight = 21.9 kN
Total dead load = $21.9 + 12.0 \times 7.0$	$G_k = 105.9\,\text{kN}$
Total imposed load = 11.0×7.0	$Q_k = 77.0\,\text{kN}$
Using $\gamma_f = 1.35$ for dead loads and $\gamma_f = 1.50$ for imposed loads, ultimate load	Ultimate load
$F = 1.35G_k + 1.50Q_{k.} = 1.35 \times 105.9 + 1.50 \times 77.0$	$F = 258.5\,\text{kN}$
Bending ULS	
$M_u = FL/8 = 258.5 \times 7.0/8$	$M_u = 226.2\,\text{kNm} = 226.2 \times 10^6\,\text{Nmm}$
From Figure 3.7, effective depth $d = 500 - 25 - 10 - 12.5$	$d = 452.5\,\text{mm}$
$K = M/bd^2f_{ck} = 226.2 \times 10^6/(250 \times 452.5^2 \times 40)$	$K = 0.110$
K should not be more than 0.167	Accept
$z/d = 0.5 \times (1 + \sqrt{(1 - 3.53K)}) = 0.5 \times (1 + \sqrt{(1 - 3.53 \times 0.110)})$	$z/d = 0.891$
z/d not more than 0.95	Accept
$z = d(z/d) = 452.5 \times 0.891$	$z = 403\,\text{mm}$
$A_s = M/(0.87zf_{yk}) = 226.2 \times 10^6/(0.87 \times 403 \times 500)$	$A_s = 1291\,\text{mm}^2$
From Table 3.14, min. $A_s = 0.19\% \times 250 \times 500$	Min. $A_s = 238\,\text{mm}^2$

Continued on next page

Calculations for Example 3.3 (Continued from previous page)	
From Table 3.5, choose bars to provide at least 1291 mm^2	Use 2no. H25 bars plus 1no. H20 bar
	$A_{s,prov} = 1296\,mm^2$
Percentage of reinforcement = 1296 × 100/(250 × 500) = 1.04%, which is less than 4% from Table 3.17	Accept
Note: Checks on the shear capacity and span/effective depth ratio of this beam are also required; see Sections 3.9.5 and 3.9.6	

3.9.5 Design of Beams for Shear ULS

The effect of shear at the ULS must be examined for all reinforced concrete beams. Members of minor structural importance such as lintels do not normally need shear reinforcement and should be checked using the procedure given in Section 3.10.2 for slabs. All other beams should be provided with shear reinforcement, usually in the form of vertical links though inclined links and bent-up bars are sometimes used. This manual covers only design with vertical links.

Shear failure in reinforced concrete beams is complex and can occur in several ways. A typical failure mode for a simply supported beam is illustrated in Figure 3.9, which also shows how reinforcement can assist in resisting the shear.

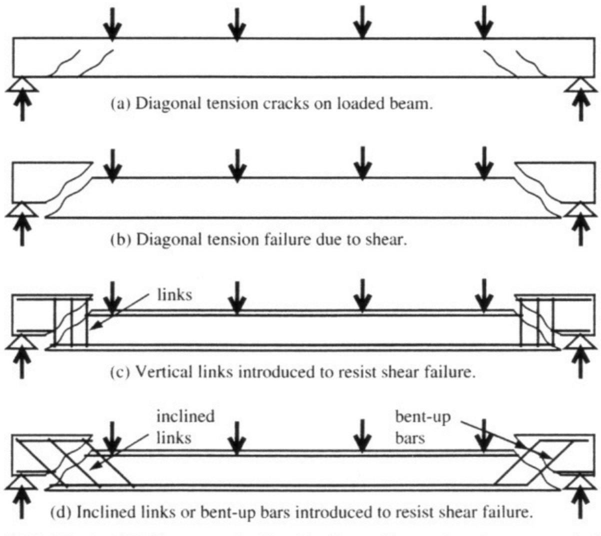


Figure 3.9: Typical failure mode due to shear for a simply supported beam

In a reinforced concrete beam with vertical links, shear forces are considered to be carried by the links in tension acting with diagonal concrete struts in compression, as shown in Figure 3.10.

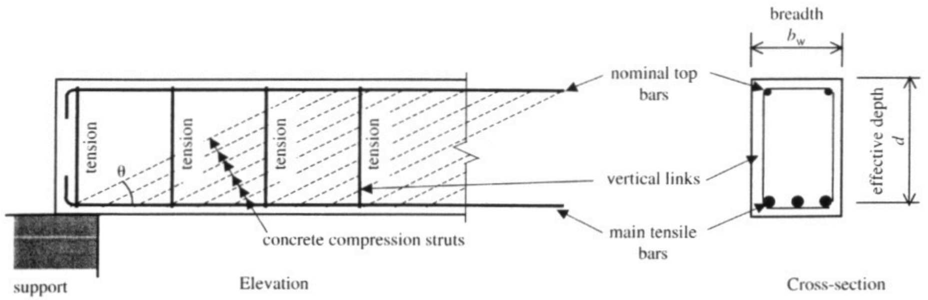


Figure 3.10: Beam carrying shear: links in tension and concrete in compression

EC2 allows the designer to vary the angle θ of the strut to obtain the most economical solution. However an angle θ of 22° (the minimum allowed in EC2) will give practical designs in most cases, and this approach is adopted in this manual.

Note that for a rectangular beam the breadth b_w used in the shear calculation is equal to the overall breadth b. For T and L beams b_w is the breadth of the web, as shown in Figure 3.11.

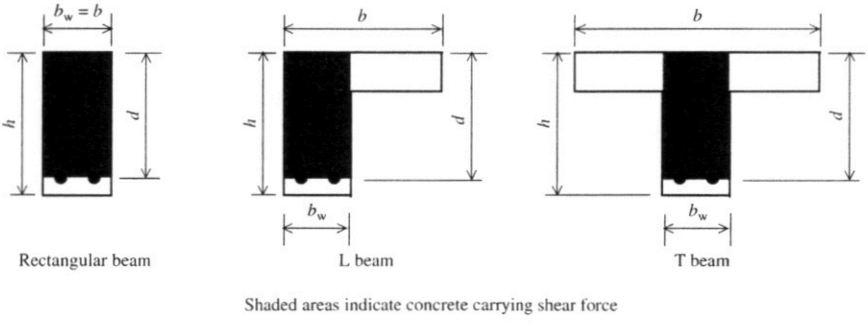


Figure 3.11: b and b_w for rectangular, L and T beams in shear

The procedure for checking the shear resistance of a concrete beam involves first verifying that the concrete has sufficient capacity at the face of the support. Reinforcement design is based on the shear force at a distance equal to one effective depth from the face of the support as shown in Figure 3.12. Finally the requirements for minimum reinforcement should be checked and a suitable arrangement of links should be chosen.

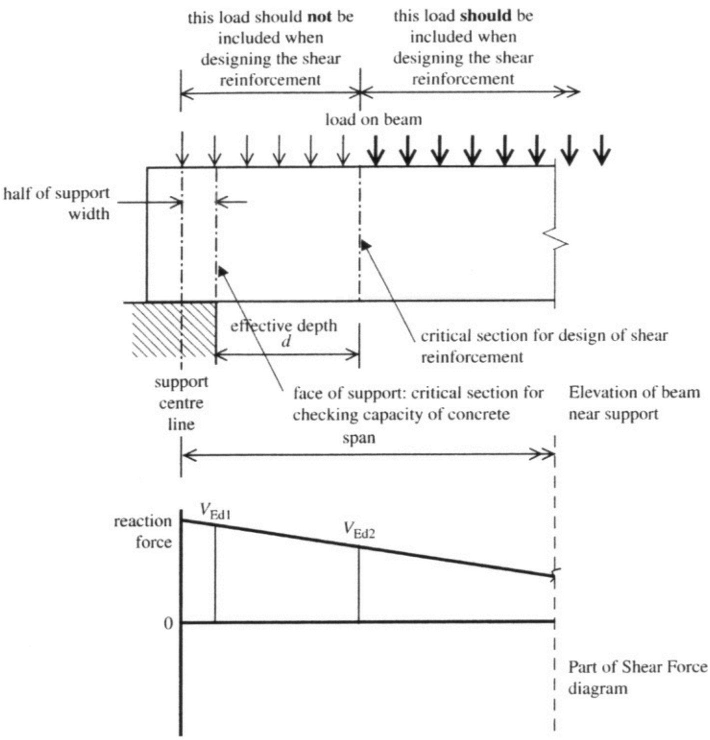


Figure 3.12: Critical sections for a beam carrying shear

The procedure is as follows.

1. Find V_{Ed1} = shear force at the face of the support.
2. Find v_{Ed1} = shear stress at the face of the support = $V_{Ed1}/(0.9b_w d)$.
3. Find the concrete strut capacity $v_{Rd,}$ from Table 3.20.

Table 3.20: v_{Rd} concrete strut capacities for calculations of shear in beams

f_{ck} (N/mm^2)	25	28	30	32	35	40	45	50
v_{Rd} (N/mm^2)	3.10	3.43	3.64	3.84	4.15	4.63	5.08	5.51

Formula: $v_{Rd} = 0.36(1 - f_{ck}/250)f_{ck}/(\cot\theta + \tan\theta)$ with $\theta = 22°$.

4. If v_{Ed1} is greater than v_{Rd} then see EC2 for use of other values of θ up to 45°. In some cases it may be necessary to use a larger beam or a higher class of concrete.

5. If v_{Ed1} is not greater than v_{Rd} then use $\theta = 22°$ and follow steps 6–11.
6. Find V_{Ed2} = shear force at a distance d from the face of the support.
7. Find v_{Ed2} = shear stress at a distance d from the face of the support = $V_{Ed2}/(0.9b_w d)$.
8. Calculate the area of shear reinforcement required: $A_{sw}/s = 0.4v_{Ed2}b_w/0.87f_{yk}$.
   Since f_{yk} is always 500 N/mm², this gives $A_{sw}/s = 0.00092v_{Ed2}b_w$.
9. Find the minimum A_{sw}/s from Table 3.21.

Table 3.21: Minimum shear reinforcement in beams

f_{ck} (N/mm²)	25	28	30	32	35	40	45	50
Minimum A_{sw}/s	$0.0008b_w$	$0.00085b_w$	$0.00088b_w$	$0.00091b_w$	$0.00095b_w$	$0.00101b_w$	$0.00107b_w$	$0.00113b_w$

Formula: minimum ratio = $0.08\sqrt{(f_{ck})}/f_{yk}$

10. Consider the following limits to the spacing of the links along the beam:

    Minimum spacing — 75 mm

    Maximum spacing — $0.75d$ but not more than 600 mm.

    Economy will be achieved by having as few links as possible, so in step 11 it is best to choose a spacing s close to the maximum permitted value.
11. Choose a link size A_{sw} and link spacing s so that A_{sw}/s is not less than the values from steps 8 and 9. Table 3.22 may be used for single links, and other arrangements using multiple links are shown in Figure 3.13.

Table 3.22: Area of shear links A_{sw}/s (mm²/mm) for various link sizes and spacings (based on two legs per link)

Bar size	**Spacing of links (mm)**								
	75	**100**	**125**	**150**	**175**	**200**	**250**	**300**	**350**
H6	0.754	0.565	0.452	0.377	0.323	0.283	0.226	0.188	0.162
H8	1.340	1.005	0.804	0.670	0.574	0.503	0.402	0.335	0.287
H10	2.094	1.571	1.257	1.047	0.898	0.785	0.628	0.524	0.449
H12	3.016	2.262	1.81	1.508	1.293	1.131	0.905	0.754	0.646
H16	5.362	4.021	3.217	2.681	2.298	2.011	1.608	1.340	1.149

This procedure determines the shear reinforcement needed at the support. In many beams it is possible to use less shear reinforcement towards the centre of the span where the shear force is lower, and this can often be achieved by using the same size of links but increasing the links spacing s. This manual does not cover the calculations required to do this, and the chosen link arrangement should be used for the full length of the beam.

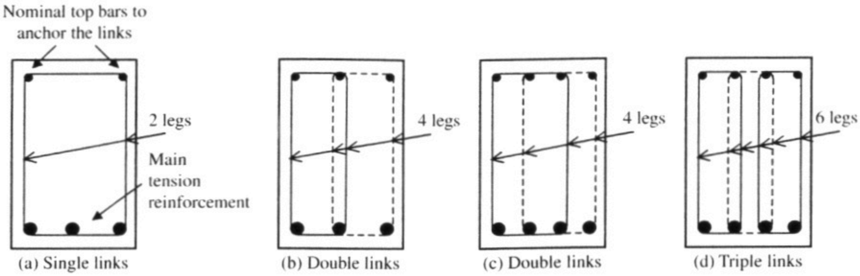
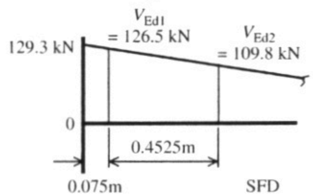


Figure 3.13: Examples of shear reinforcement in the form of links.

Example 3.4 Design of a beam for shear ULS

Taking the beam from Example 3.3 and assuming a bearing width of 150 mm, determine the shear reinforcement required.

Calculations for Example 3.4	
Data taken from Example 3.3	
Beam width b	$b_w = 250\,\text{mm}$
Beam overall height h	$h = 500\,\text{mm}$
Effective span L	$L = 7.0\,\text{m}$
Ultimate load on beam F	$F = 258.5\,\text{kN}$
Concrete strength f_{ck}	$f_{ck} = 40\,\text{N/mm}^2$
Effective depth d	$d = 452.5\,\text{mm}$
Shear calculations	
The figure below shows part of the Shear Force diagram. The values in the diagram are calculated below	



Reaction = $F/2$ = 258.5/2	Reaction = 129.3 kN
Width of support = 150 mm, so distance from centre of support to face of support = 0.15/2	Dist. = 0.075 m
$V_{Ed1} = 129.3 - 258.5 \times 0.075/7.0$	$V_{Ed1} = 126.5\,\text{kN} = 126.5 \times 10^3\,\text{N}$

$v_{Ed1} = V_{Ed1}/(0.9b_w d) = 126.5 \times 10^3/(0.9 \times 250 \times 452.5)$	$v_{Ed1} = 1.24\,\text{N/mm}^2$
From Table 3.20 with $f_{ck} = 40\,\text{N/mm}^2$	$v_{Rd} = 4.63\,\text{N/mm}^2$
Check v_{Ed1} is not more than v_{Rd}	Accept
Effective depth $d = 0.4525\,\text{m}$, so $V_{Ed2} = 126.5 - 258.5 \times 0.4525/7.0$	$V_{Ed2} = 109.8\,\text{kN}$
$v_{Ed2} = V_{Ed2}/(0.9b_w d) = 109.8 \times 10^3/(0.9 \times 250 \times 452.5)$	$v_{Ed2} = 1.08\,\text{N/mm}^2$
$A_{sw}/s = 0.00092 v_{Ed2} b_w = 0.00092 \times 1.08 \times 250$	$A_{sw}/s = 0.248\,\text{mm}^2/\text{mm}$
From Table 3.21, with $f_{ck} = 40\,\text{N/mm}^2$, min. $A_{sw}/s = 0.0010 b_w = 0.0010 \times 250$	Min. $A_{sw}/s = 0.25\,\text{mm}^2/\text{mm}$
Max. link spacing $= 0.75d = 0.75 \times 452.5$	Max. link spacing $= 339\,\text{mm}$
From Table 3.22, with A_{sw}/s not less than $0.25\,\text{mm}^2/\text{mm}$, by interpolation	Use H8 links at 325 mm centres (see Figure 3.14)
	$A_{sw}/s = 0.309\,\text{mm}^2/\text{mm}$

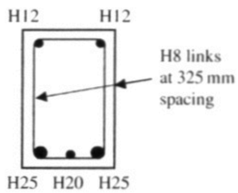


Figure 3.14: Cross-section of beam in Examples 3.3 and 3.4

3.9.6 Design of Beams for Deflection SLS

Reinforced concrete beams should be made sufficiently stiff that excessive deflections, which would impair the efficiency or appearance of the structure, will not occur. The deflections allowed should be commensurate with the capacity of movement of any services, finishes, partitions, glazing, cladding and so on that the member may support or influence.

The direct calculation of deflections is laborious and seldom leads to accurate predictions of real deflections because the properties of the concrete vary with time and with its history. A simpler method is to restrict the span to effective depth ratio of the beam. Limiting factors are given in the IStructE *Manual for the design of reinforced concrete structures to Eurocode 2* and reproduced in Tables 3.23, 3.24 and 3.25. Use of these factors should ensure that:

- Under dead load + imposed load, deflection is not more than span/250
- Under imposed load only, deflection is not more than span/500.

These limits will be satisfactory for most structures.

Note that the factors vary according to the percentage of reinforcement in the beam. Generally a beam with a higher percentage of reinforcement ρ will have a deeper stress block of concrete in compression which will cause more curvature of the beam and more deflection, so the limiting span to effective depth ratio limits are lower for such a beam.

Table 3.23: Basic ratios of span/effective depth for reinforced concrete beams or slabs

Structural system	Concrete lightly stressed $\rho_l = 0.35\%$	$\rho_l = 0.50\%$	Concrete highly stressed $\rho_l = 1.5\%$
1. Simply supported beam, one- or two-way spanning simply supported slab	30	20	14
2. End span of continuous beam or one-way continuous slab, or two-way spanning slab continuous over one long edge	39	26	18
3. Interior span of continuous beam or one-way or two-way continuous slab	45	30	20
4. Slab supported on columns without beams (flat slab), based on longer span	36	24	17
5. Cantilever	12	8	6

Table 3.24: Basic ratios of span/effective depth for simply supported beams and slabs

Percentage of main reinforcement ρ_l	up to 0.35%	0.4%	0.5%	0.6%	0.7%	0.8%	0.9%	1.0%	1.1%	1.2%	1.3%	1.4%	1.5% or more
Basic span/ effective depth ratio	30.0	26.6	20	19.4	18.8	18.2	17.6	17.0	16.4	15.8	15.2	14.6	14

Table 3.25: Modification factors for use with basic ratios from Table 3.23 or Table 3.24

Situation	Modification factor
When more reinforcement is provided ($A_{s,prov}$) than is required for the Ultimate Limit State ($A_{s,req}$)	Multiply factor by $A_{s,prov}/A_{s,req}$, but not more than 1.5
For a T or L beam with flange breadth 0.3 × table breadth	Multiply factor by 0.8

Note: When beam spans more than 7 m and supports partitions which are liable to be damaged by deflection, lower ratios should be used: see EC2.

The beam and slab designs covered in this manual are all simply supported so Structural System 1 of Table 3.23 applies. Table 3.24 shows values interpolated between the figures given in Table 3.23.

The basic ratio from Table 3.23 or Table 3.24 should be multiplied by a modification factor from Table 3.25 if appropriate.

Example 3.5 Design of a beam for deflection SLS

Check whether the beam from Example 3.3 meets the span/effective depth limits.

Calculations for Example 3.5	
Data taken from Example 3.3	
Effective span L	$L = 7.0\,\text{m}$
Beam breadth b	$b = 250\,\text{mm}$
Effective depth d	$d = 452.5\,\text{mm}$
Area of reinf. required	$A_{s,req} = 1291\,\text{mm}^2$
Area of reinf. provided	$A_{s,prov} = 1296\,\text{mm}^2$
Span/effective depth ratio calculations for deflection SLS	
Percentage of main reinf. $\rho_l = 100\,A_{s,req}/bd = 100 \times 1291/(250 \times 452.5)$	$\rho_l = 1.14\%$
From Table 3.24, by interpolation	Basic ratio = 16.2
From Table 3.25, modification factor = 1296/1291	Mod. factor = 1.004
Modification factor should not be not more than 1.5	Accept
Permitted ratio = basic ratio × mod. factor = 16.2 × 1.004	Permitted ratio = 16.3
Actual ratio = span/effective depth = 7.0/0.4525	Actual ratio = 15.5
Check actual ratio not more than permitted ratio	Accept

Example 3.6 Beam design for bending ULS, deflection SLS and shear ULS

A simply supported reinforced concrete beam 650 mm deep and 300 mm wide has an effective span of 8.50 m onto supports which are 300 mm wide. In addition to its own self-weight, the beam carries the following loads:

Dead load	22 kN/m
Imposed load	17 kN/m.

The beam is in grade C35/45 concrete and will be inside a building where a fire resistance of 1 hour is required. The main reinforcing bars are size H32 and the links are size H12.

Determine the reinforcement required in the beam and check whether the deflection of the beam will be acceptable.

Calculations for Example 3.6

Data given	
Beam width b	$b = 300\,\text{mm}$
Beam overall height h	$h = 650\,\text{mm}$
Effective span L	$L = 8.5\,\text{m}$
Dead load excluding beam SW	$g_k = 22.0\,\text{kN/m}$
Imposed load	$q_k = 17.0\,\text{kN/m}$
Concrete grade	C35/45
Cover to bars	
Fire resistance: From Table 3.12 with a beam width of 300 mm	Min. axis distance = 25 mm
Durability: From Table 3.9, exposure class is XC1. From Table 3.10	Min. cover to all bars = 25 mm
Placing of concrete: Min. cover to H32 bars = 32 + 10	Min. cover to main bars = 42 mm
Min cover to H12 links = 12 + 10	Min. cover to links = 22 mm
These three can be achieved by specifying a cover of 30 mm to the links, which will give 30 + 12 = 42 mm cover to the main bars and 30 + 12 + 32/2 = 58 mm axis distance to the main bars	Provide 30 mm cover to all bars
Material strengths	
f_{ck} (characteristic cylinder strength of concrete)	$f_{ck} = 35\,\text{N/mm}^2$
f_{yk} (characteristic tensile strength of reinforcement, class H)	$f_{yk} = 500\,\text{N/mm}^2$
Loading	
Unit weight of concrete = $25\,\text{kN/m}^3$, so beam self-weight $= 0.30 \times 0.65 \times 25 \times 8.5$	Beam self-weight = 41.4 kN
Total permanent load = $41.4 + 22.0 \times 8.5$	$G_k = 228.4\,\text{kN}$
Total imposed load = 17.0×8.5	$Q_k = 144.5\,\text{kN}$
Using $\gamma_f = 1.35$ for permanent loads and $\gamma_f = 1.50$ for variable loads, ultimate load	Ultimate load
$F = 1.35G_k + 1.50Q_k = 1.35 \times 228.4 + 1.50 \times 144.5$	$F = 525.1\,\text{kN}$
Bending ULS	
$M_u = FL/8 = 525.1 \times 8.5/8$	$M_u = 557.9\,\text{kNm} = 557.9 \times 10^6\,\text{Nmm}$
With H12 links, H32 main bars and cover to all bars of 30 mm	
Effective depth $d = 650 - 30 - 12 - 32/2$	$d = 592\,\text{mm}$
$K = M/bd^2f_{ck} = 557.9 \times 10^6/(300 \times 592^2 \times 35)$	$K = 0.152$
K should not be more than 0.167	Accept
$z/d = 0.5\,(1 + \sqrt{(1 - 3.53K)}) = 0.5\,(1 + \sqrt{(1 - 3.53 \times 0.152)})$	$z/d = 0.840$
z/d not more than 0.95	Accept
$z = d(z/d) = 592 \times 0.840$	$z = 497.3\,\text{mm}$

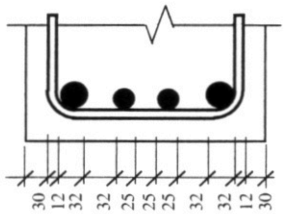
$A_s = M/(0.87zf_{yk}) = 557.9 \times 10^6/(0.87 \times 497.3 \times 500)$	$A_s = 2579\,\text{mm}^2$
From Table 3.14, min. A_s = 0.17% of $bh = 0.17 \times 300 \times 650/100$	Min. $A_s = 332\,\text{mm}^2$
From Table 3.5, choose bars to provide at least 2579 mm²	Use 2no. H32 bars plus 2no. H25 bars
	$A_{s,prov} = 2590\,\text{mm}^2$
Percentage of reinforcement = $A_{s,prov} \times 100/bd = 2590 \times 100/(300 \times 592)$ = 1.46%, which is less than 4% from Table 3.17	Accept

Span/effective depth ratio calculations for deflection SLS	
Percentage of main reinf. $\rho_l = 100\,A_{s,req}/bd = 100 \times 2579/(300 \times 592)$	$\rho_l = 1.45\%$
From Table 3.24, by interpolation	Basic ratio = 14.3
From Table 3.25, modification factor = 2590/2579	Mod. factor = 1.004
Modification factor should not be not more than 1.5	Accept
Permitted ratio = basic ratio × mod. factor = 14.3 × 1.004	Permitted ratio = 14.4
Actual ratio = span/effective depth = 8.5/0.592	Actual ratio = 14.4
Actual ratio is not more than permitted ratio	Accept

Check whether two No. H32 + two No. H25 bars will fit in the width of the beam

Minimum gaps between bars = bar size

The sketch opposite shows bar sizes, cover and minimum bar spacing.
The minimum beam width required
= 30 + 12 + 32 + 32 + 25 + 25 + 25 + 32 + 32 + 12 + 30
= 287 mm



Min. beam width = 287 mm

Accept

Check maximum bar spacing

From Note 2 of Table 3.15,

$$\text{steel stress} = 435(G_k + 0.8Q_k)/(1.35G_k + 1.50Q_k)$$
$$= 435(228.4 + 0.8 \times 144.5)/(1.35 \times 228.4 + 1.50 \times 144.5)$$

Steel stress = 285 N/mm²

From Table 3.15, since the bar size is more than 12 mm, we must meet the requirement for maximum bar spacing:

- If the steel stress was 280 N/mm² the maximum spacing would be 150 mm
- If the steel stress was 320 N/mm² the maximum spacing would be 100 mm

By interpolation, a steel stress of 285 N/mm² gives a maximum spacing of 144 mm

Accept

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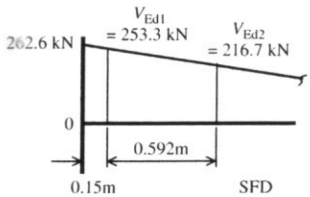
Calculations for Example 3.6 (Continued from previous page)

Shear ULS

The figure below shows part of the Shear Force diagram.
The values in the diagram are calculated below.



As beam and loading are symmetric, reaction = $F/2 = 525.1/2$	Reaction = 262.6 kN
Width of support = 300 mm, so distance from centre of support to face of support = 0.30/2	Dist. = 0.15 m
$V_{Ed1} = 262.6 - 525.1 \times 0.15/8.5$	$V_{Ed1} = 253.3\,kN = 253.3 \times 10^3\,N$
$v_{Ed1} = V_{Ed1}/(0.9 b_w d) = 253.3 \times 10^3/(0.9 \times 300 \times 592)$	$v_{Ed1} = 1.58\,N/mm^2$
From Table 3.20, with $f_{ck} = 35\,N/mm^2$	$v_{Rd} = 4.15\,N/mm^2$
Check v_{Ed1} is not more than v_{Rd}	Accept
Effective depth $d = 0.592$ m, so $V_{ed2} = 253.3 - 525.1 \times 0.592/8.5$	$V_{Ed2} = 216.7\,kN$
$v_{Ed2} = V_{Ed2}/(0.9 b_w d) = 216.7 \times 10^3/(0.9 \times 300 \times 592)$	$v_{Ed2} = 1.36\,N/mm^2$
$A_{sw}/s = 0.00092 v_{Ed2} b_w = 0.00092 \times 1.36 \times 300$	$A_{sw}/s = 0.375\,mm^2/mm$
From Table 3.21, with $f_{ck} = 35\,N/mm^2$,	
min. $A_{sw}/s = 0.00095 b_w = 0.00095 \times 300$	Min. $A_{sw}/s = 0.29\,mm^2/mm$
Max. link spacing = $0.75d = 0.75 \times 592$	Max. link spacing = 444 mm
From Table 3.22, with A_{sw}/s not less than $0.375\,mm^2/mm$	Use H10 links at 400 mm centres (see Figure 3.15)
	$A_{sw}/s = 0.393\,mm^2/mm$

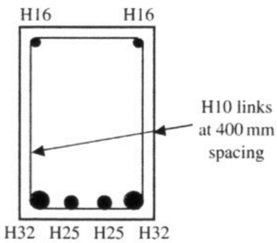


Figure 3.15: Cross-section of beam in Example 3.6

3.9.7 Design Summary for Beams

The design procedure for simply supported, singly reinforced concrete beams may be summarised as follows.

- Calculate the ultimate loads on the beam. Find the maximum ultimate bending moment in the beams and the ultimate reactions at the supports.
- Determine the cover required for durability, concrete placing and fire resistance. Determine the effective depth.
- Check the bending ULS using the K factor method in Section 3.9.4. This will determine whether the beam has adequate size and the area of tension reinforcement required. Choose suitable main bars.
- Ensure that the cracking SLS is satisfied by compliance with the requirements for minimum reinforcement content and bar spacing.
- Check the shear ULS by providing link reinforcement as explained in Section 3.9.5.
- Check the deflection SLS using the method in Section 3.9.6.

3.10 Slabs

This manual covers the design of suspended slabs; that is, slabs supported by beams, walls or columns. Guidance on the design of ground-bearing slabs can be found in publications from the Concrete Centre.

Suspended slabs may be designed to span in either one or two directions depending on how they are supported at the edges. Slabs can be classified into three types.

Solid slabs

These, as the name implies, consist of solid concrete reinforced where necessary to resist tension (Figure 3.16).

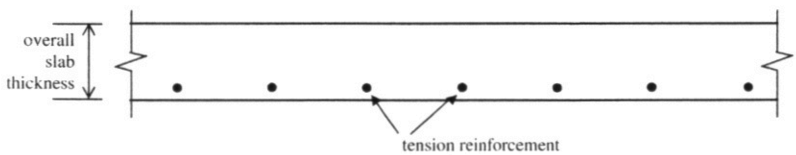


Figure 3.16: Cross-section through a solid slab

A common variant is a slab cast *in situ* on profiled metal decking, where the decking is both permanent formwork and tensile reinforcement (Figure 3.17).

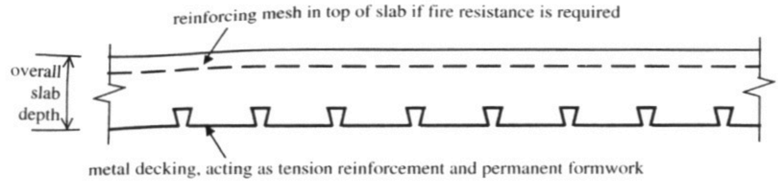


Figure 3.17: Cross-section through a solid slab on metal decking

Ribbed slabs

Solid slabs exceeding 4 m in span may be uneconomic because of their self-weight. Ribbed slabs may be more economical because they can achieve the same structural strength with less concrete. These may be formed as:

- a series of *in-situ* concrete ribs cast monolithically with the concrete topping on removable formers (Figure 3.18);
- a hollow slab containing permanent void formers (Figure 3.19).

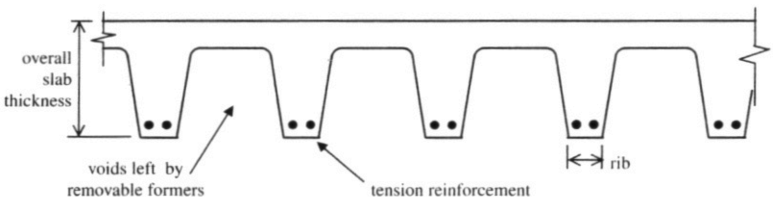


Figure 3.18: Cross-section through a ribbed slab cast on removable formers

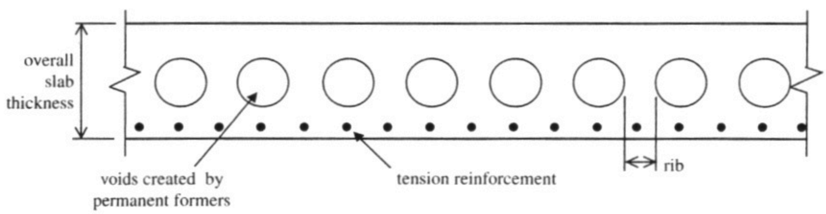


Figure 3.19: Cross-section through a hollow slab cast with permanent void formers

Flat slabs

This title is something of a misnomer. It describes slabs which are supported on columns without the use of beams, and hence have a flat soffit (Figure 3.20). However, they may have a thickened section, known as a drop, which forms a stiffening band between columns (Figure 3.21). In addition the top of the column may be flared out to give a column head which reduces the maximum shear stresses in the slab (Figure 3.22).

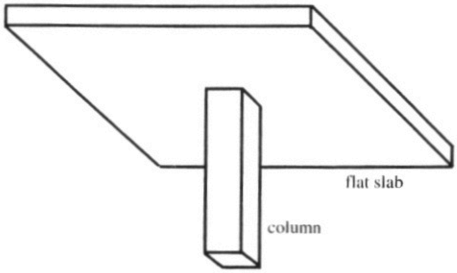


Figure 3.20: Flat slab supported on a column

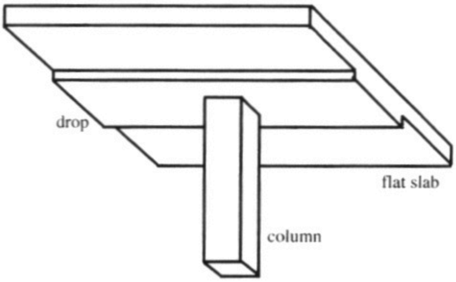


Figure 3.21: Flat slab with drop

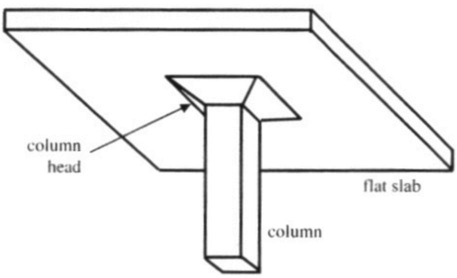


Figure 3.22: Flat slab with column head

Flat slabs may be solid, or may have void formers in the soffit to give a series of ribs in both directions, in which case they are referred to as waffle or coffered slabs.

Suspended slabs are usually found as floors or roofs in buildings, although sloping slabs used to form ramps and staircases are often made as cranked slabs.

This manual covers only one-way-spanning solid slabs, and many of the design considerations for these are very similar to those given in Section 3.9 for beams. The following sections describe the design of slabs for bending ULS, shear ULS, deflection SLS and cracking SLS.

3.10.1 Design of Slabs for Bending ULS

Since a solid slab may be considered for design purposes to be a series of 1 m wide beams, design for bending ULS is the same as described for beams in Section 3.9.4 with the width $b = 1.0$ m or 1000 mm.

3.10.2 Design of Slabs for Shear ULS

It is not practical to provide shear reinforcement in solid slabs supported on walls or beams. To determine whether a slab is safe without shear reinforcement, follow this procedure:

1. Find V_{Ed} = shear force at the face of the supporting beam or wall.
2. Find v_{Ed} = shear stress = $V_{Ed}/(0.9bd)$. Normally $b = 1000$ mm.
3. Find $v_{Rd,c}$ from Table 3.26.
4. Multiply $v_{Rd,c}$ by the modification factor from Table 3.27.
5. If v_{Ed} is not greater than the modified $v_{Rd,c}$ then the slab is safe without shear reinforcement.

Table 3.26: $v_{Rd,c}$ shear resistance of solid slabs in class C25/30 concrete without shear reinforcement (N/mm²)

Reinforcement ratio $\rho_1 = A_s/bd$	Effective depth d (mm)								
	200 mm or less	225	250	275	300	400	500	600	750
0.25%	0.49	0.47	0.46	0.44	0.43	0.39	0.37	0.35	0.34
0.50%	0.56	0.54	0.53	0.52	0.51	0.48	0.45	0.44	0.42
0.75%	0.64	0.62	0.60	0.59	0.58	0.54	0.52	0.50	0.48
1.00%	0.70	0.68	0.66	0.65	0.64	0.60	0.57	0.55	0.53
1.25%	0.76	0.73	0.72	0.70	0.69	0.65	0.62	0.60	0.57
1.50%	0.80	0.78	0.76	0.74	0.73	0.69	0.66	0.63	0.61
1.75%	0.85	0.82	0.80	0.78	0.77	0.72	0.69	0.67	0.64
2.00%	0.88	0.86	0.84	0.82	0.80	0.75	0.72	0.70	0.67

Formula: $v_{Rd,c} = 0.12k(100\rho_1 f_{ck})^{1/3}$ where $k = 1 + \sqrt{(200/d)}$ but not more than 2.0.
Source: Equation 6.2a of EC2 Part 1-1.

Table 3.27: Concrete strength modification factors for use with factors from Table 3.26

f_{ck} (N/mm^2)	25	28	30	32	35	40	45	50
Mod. factor	1.00	1.04	1.06	1.09	1.12	1.17	1.22	1.26

Formula: Modification factor = $(f_{ck}/25)^{1/3}$.

If this check shows that a slab does require shear reinforcement, then it is best to increase the slab thickness and repeat the calculations.

By contrast shear reinforcement is often required in flat slabs, and EC2 contains guidance on the design of this.

3.10.3 Design of Slabs for Deflection SLS

Deflection of slabs is controlled by restricting the span to effective depth ratio using the same method as for beams; see Section 3.9.6.

3.10.4 Design of Slabs for Cracking SLS

The rules for minimum bar areas and maximum bar spacing are given in Section 3.7.1. To control cracking, and also to allow concentrated loads on slabs to be distributed onto the main reinforcement, bars equal to at least the minimum area should be provided in both directions in the slab. This distribution reinforcement will normally be on top of the main tension reinforcement, as shown in Figure 3.23.

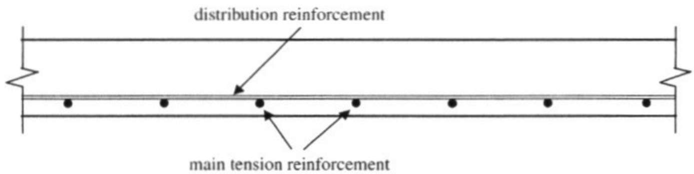


Figure 3.23: Cross section through a one-way solid slab showing main reinforcement and distribution reinforcement

Example 3.7 Design of a sold slab for bending ULS

A reinforced concrete slab carries loads, including its self-weight, which cause an ultimate moment M_u of 44 kNm per metre width. The overall thickness h of the slab is 200 mm, the concrete is grade C30/37 and the main reinforcing bars are H16. Cover of 30 mm is required to all bars. Check the adequacy of the slab and determine the spacing of the main bars and the size and spacing of the distribution reinforcement.

Calculations for Example 3.7	
Effective depth $d = 200 - 30 - 16/2$	$d = 162\,\text{mm}$
Calculate K for a 1-m-wide slab strip: $b = 1000\,\text{mm}$	
$K = M/bd^2f_{ck} = 44 \times 10^6/(1000 \times 162^2 \times 30)$	$K = 0.056$
K should not be more than 0.167	Accept
$z/d = 0.5 \times (1 + \sqrt{(1 - 3.53K)}) = 0.5 \times (1 + \sqrt{(1 - 3.53 \times 0.056)})$	$z/d = 0.95$
z/d not more than 0.95	Accept
$z = d(z/d) = 162 \times 0.95$	$z = 153.9\,\text{mm}$
$A_s = M/(0.87zf_{yk}) = 44 \times 10^6/(0.87 \times 153.9 \times 500)$	$A_s = 657\,\text{mm}^2/\text{m}$
From Table 3.14, min. $A_s = 0.15\% \times 1000 \times 200$	Min. $A_s = 300\,\text{mm}^2/\text{m}$
From Table 3.5, area of 1no H16 bar is 201 mm^2, so bar spacing required = 201 × 1000/657 = 306 mm (Note: Table 3.6 can be used in place of this calculation)	Main reinforcement: use H16 bars at 300 mm centres $A_{s,prov} = 670\,\text{mm}^2/\text{m}$
Distribution reinforcement should not be less than the minimum A_s found previously, 300 mm^2/m (from Table 3.6)	Distribution reinforcement: use H12 bars at 350 mm centres $A_{s,prov} = 323\,\text{mm}^2/\text{m}$
Check maximum bar spacing	
Slab is not more than 200 mm thick, so the provisions of Table 3.15 do not apply. Use Table 3.16	
Max. spacing of main reinforcement $= 3d = 3 \times 162 = 486\,\text{mm}$	Accept
Max. spacing of distribution reinforcement $= 3.5d = 3.5 \times 162 = 567\,\text{mm}$	Accept

Example 3.8 Design of a solid slab for bending ULS, deflection SLS and shear ULS

A 250-mm-thick, simply supported, reinforced concrete slab spans 5.0 m. The slab is made of C32/40 concrete and carries the following loads:

Imposed load 7.5 kN/m^2

Slab self-weight to be determined

Finishes 0.5 kN/m^2

The slab is part of a highway structure away from direct spray, and there is no requirement for fire resistance.

Determine a suitable arrangement of reinforcement for the slab.

Calculations for Example 3.8

Cover required	
From Table 3.9	Exposure class is XD1
From Table 3.10	Cover to all steel to be 40 mm
From Table 3.10: Additional requirements	Water/cement ratio not more than 0.55
	Cement content not less than 320 kg/m^3
Loading	
Slab self-weight = 0.25 × 25	Slab self-weight = 6.25 kN/m^2
Total permanent load = 0.5 + 6.25	g_k = 6.75 kN/m^2
Variable load	q_k = 7.5 kN/m^2
Total load for ULS = 1.35 × 6.75 + 1.5 × 7.5	At ULS, total load = 20.4 kN/m^2
Bending ULS	
At ULS, load F on, 1 m wide strip = 20.4 × 5.0	F = 102 kN/m
$M_u = FL/8 = 102 \times 5.0/8$	$M_u = 63.8\,\text{kNm} = 63.8 \times 10^6\,\text{Nmm}$
Assume H16 main reinforcement	
Effective depth d = 250 – 40 – 16/2	d = 202 mm
Calculate K for a 1 m wide slab strip: b = 1000 mm	
$K = M/bd^2f_{ck} = 63.8 \times 10^6/(1000 \times 202^2 \times 32)$	$K = 0.049$
K should not be more than 0.167	Accept
$z/d = 0.5 \times (1 + \sqrt{(1 - 3.53K)}) = 0.5\times (1 + \sqrt{(1 - 3.53 \times 0.049)})$	$z/d = 0.96$
z/d not more than 0.95	$z/d = 0.95$
$z = d(z/d) = 202 \times 0.95$	z = 191.9 mm
$A_{s,req} = M/(0.87zf_{yk}) = 63.8 \times 10^6/(0.87 \times 191.9 \times 500)$	$A_{s,req}$ = 764 mm^2/m
From Table 3.14, min. A_s = 0.16% ×1000 × 250	Min. A_s = 400 mm^2/m
Area of one No. H16 bar = 201 mm^2, so bar spacing = 1000 × 201/764 = 263 mm	Provide H16 bars at 260 mm centres
From the formula in Table 3.15, steel stress = $435(g_k + 0.8q_k)/(1.35g_k + 1.50q_k)$	$A_{s,prov}$ = 773 mm^2/m
= 435(6.75 + 0.8 × 7.5)/(1.35 × 6.75 + 1.50 × 7.5)	steel stress = 272 N/mm^2
From Table 3.15 by interpolation, bar spacing should not exceed 160 mm for bars larger than H12	
Instead of using H16 bars at 160-mm centres it will be more economical to use H12 bars:	
Area of one No. H12 bar = 113 mm^2, so bar spacing = 1000 × 113/764 = 147 mm centres	Use H12 bars at 145-mm
	$A_{s,prov}$ = 779 mm^2/m
Change in effective depth	
The diameter of the main bars is 12 mm, not 16 mm as assumed	
The effective depth d is now 250 – 40 –12/2	d = 204 mm

Continued on next page

Calculations for Example 3.8 (Continued from previous page)

Span/effective depth ratio check for deflection SLS	
Percentage of main reinforcement $\rho_l = 100A_{s,req}/bd = 100 \times 764/(1000 \times 204)$	$\rho_l = 0.37\%$
From Table 3.24, by interpolation	Basic ratio = 28.7
From Table 3.25, modification factor = $A_{s,prov}/A_{s,req} = 779/764$	Mod. factor = 1.02
Permitted ratio = basic ratio × mod. factor = 28.7 × 1.02	Permitted ratio = 29.2
Actual ratio = span/effective depth = 5.0/0.204	Actual ratio = 24.5
Check actual ratio not more than permitted ratio	Accept
Shear ULS	
The width of support is not given, so take V_{Ed}, the shear force at the face of supporting beam or wall, equal to the reaction. This is a conservative assumption	
$V_{Ed} = F/2 = 102.0/2$	$V_{Ed} = 51.0\,\text{kN/m}$
Shear stress $v_{Ed} = V_{Ed}/(0.9bd) = 51.0 \times 10^3/(0.9 \times 1000 \times 204)$	$v_{Ed} = 0.28\,\text{N/mm}^2$
The following values can be read from Table 3.26	

	$d = 200\,\text{mm}$	$d = 225\,\text{mm}$
$\rho_l = 0.25\%$	$v_{Rd,c} = 0.49$	$v_{Rd,c} = 0.47$
$\rho_l = 0.50\%$	$v_{Rd,c} = 0.56$	$v_{Rd,c} = 0.54$

By interpolation with $d = 204\,\text{mm}$ and $\rho_l = 0.37\%$	$v_{Rd,c} = 0.52$
From Table 3.27	Mod. factor = 1.09
Revised $v_{Rd,c} = 1.09 \times 0.52$	$v_{Rd,c} = 0.57$
Check that v_{Ed} is not greater than the revised $v_{Rd,c}$	Accept
	Slab does not require shear reinforcement

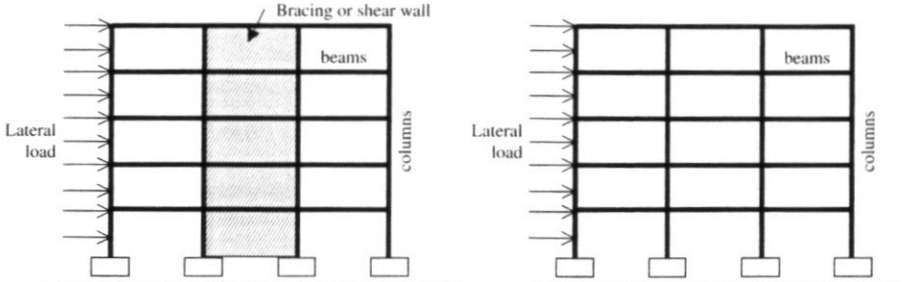
3.11 Columns

The provisions for column design to EC2 apply to vertical load-bearing members whose greater cross-section dimension does not exceed four times its smaller dimension. Compression members with a larger aspect ratio are considered to be walls, and slightly different provisions apply. Columns may be square, rectangular, circular, elliptical, cruciform or of other shapes. Initial dimensions are normally determined by taking into account requirements for durability and fire resistance (Table 3.11), and it is not practical to cast vertical columns smaller than 200 mm × 200 mm. Normally columns should be short not slender (see Section 3.11.2) and this requirement may control the minimum cross-section dimension.

3.11.1 Braced and Unbraced Columns

Reinforced concrete columns are classified as either braced or unbraced. The difference relates to the manner in which the structure carries lateral loads, as shown in Figure 3.24.

Most concrete building structures are designed with braced frames, and unbraced frames are only used when there is a requirement for open unobstructed floor areas.



A braced frame: lateral loads are transmitted down to the foundations through bracing or a shear wall

An unbraced frame: lateral loads are transmitted down to the foundations by interaction between the beams and the columns.

Figure 3.24: Braced and unbraced frames

Only braced columns are considered in this manual.

3.11.2 Short and Slender Columns

Columns may be short or slender, depending on the ratio of their effective height to their lateral dimension. The effective height l_0 can be found by multiplying the column clear height (Figure 3.25) by a factor from Table 3.28 which depends on the degree of fixity at the top and bottom of the column (Figure 3.26).

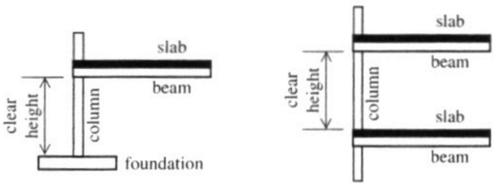


Figure 3.25: Clear height of columns

Reinforced concrete columns are classified as slender if their slenderness ratio $\lambda = l_0$/(radius of gyration) is greater than a limiting value given by Equation 5.13N of EC2 Part 1-1:

$$\lambda_{\text{lim}} = 20\, ABC / \sqrt{n}$$

where $n = N_{Ed}/(0.67bhf_{ck})$ = ratio of the axial column load to the strength of the concrete.

Table 3.28: Effective height factors for braced columns

l_0 = clear height × factor from this table

End condition at bottom	**End condition at top**		
	1	**2**	**3**
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

End fixity conditions, see Figure 3.26.
Condition 1: Column is connected monolithically to beams on each side that are at least as deep as the overall depth of the column in the plane considered. Where column is connected to a foundation this should be designed to carry moment.
Condition 2: Column connected monolithically to beams or slabs on each side that are shallower than the overall depth of the column in the plane considered but generally not less than half the column depth.
Condition 3: Column connected to members that do not provide more than nominal restraint to rotation.
Values taken from *IStructE Manual for the Design of Reinforced Concrete Building Structures to Eurocode 2.*

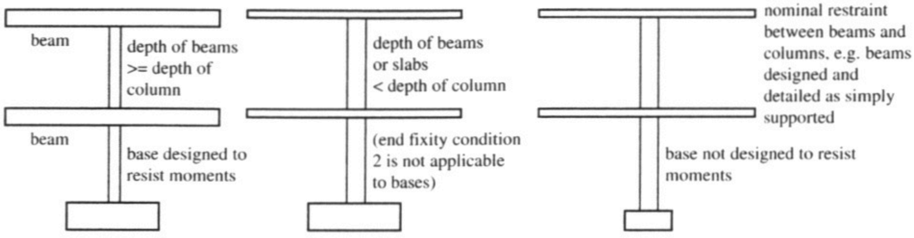


Figure 3.26: End fixity conditions for braced columns

For a rectangular section the radius of gyration is $b/3.46$, so the limit can be given as

$$\text{Limiting } l_0/b \text{ ratio} = (2.0/3.46)ABC\sqrt{n} = 5.78\ ABC/\sqrt{n}$$

For braced columns conservative values of the parameters are $A = 0.7$, $B = 1.1$, $C = 1.7$, which give:

$$\text{Limiting } l_0/b \text{ ratio} = 5.78 \times 0.7 \times 1.1 \times 1.7/\sqrt{(N_{Ed}/(0.67bhf_{ck}))} = 6.19\sqrt{(bhf_{ck}/N_{Ed})}$$

Note that the more complex calculation methods in EC2 may give a higher limiting ratio.

When the actual l_0/b ratio is not more than the limiting value, then the column is short not slender. In practice, most reinforced columns are short not slender, and this manual does not cover the design of slender columns.

Example 3.9

A rectangular column 300 mm × 400 mm in C30/37 concrete carries at ULS an axial load of 1100 kN. If the column is braced and supports 550 mm deep beams, and the floor-to-floor height is 3.5 m, determine whether the column is slender.

Calculations for Example 3.9	
The clear height is 3500 – 550	Clear height = 2950 mm
As the column is braced and the beams are deeper than the size of the column, then both ends of the column have end condition 1 (Figure 3.26), so from Table 3.28:	
$l_0 = 0.75 \times \text{clear height} = 0.75 \times 2950$	$l_0 = 2213\,\text{mm}$
$l_0/b = 2213/300$	$l_0/b = 7.4$
Limiting $l_0/b = 6.19\sqrt{(bhf_{ck}/N_{Ed})} = 6.19\sqrt{((300 \times 400 \times 30)/(1100 \times 10^3))}$	Limiting $l_0/b = 11.2$
	The column is not slender

3.11.3 Maximum and Minimum Reinforcement in Columns

Sufficient reinforcement must be provided to control cracking of the concrete. A maximum steel content is also specified to allow proper placing and compaction of the concrete. The limits on longitudinal reinforcement are given in Table 3.29 (see also Figure 3.27).

Table 3.29: Limits on longitudinal reinforcement in columns

Area of longitudinal reinforcement.	Not less than or Not more than or	$0.12N/f_{yk}$ $0.002bh$ $0.04bh$ generally $0.08bh$ at laps
Size of longitudinal reinforcement	Not less than	12 mm
Number of longitudinal bars	Not fewer than	4 in a square column 6 in a circular column

Lateral reinforcement in columns is commonly referred to as ties, links or binders. Their purpose is to prevent lateral buckling of the main bars when they carry compressive force, as illustrated in Figure 3.28.

Limits on size and spacing of ties are given in Table 3.30.

Typical arrangements of ties in reinforced concrete columns are shown in Figure 3.29. Every corner bar should be restrained by a tie, and intermediate bars should be restrained if they are more than 150 mm from a restrained bar.

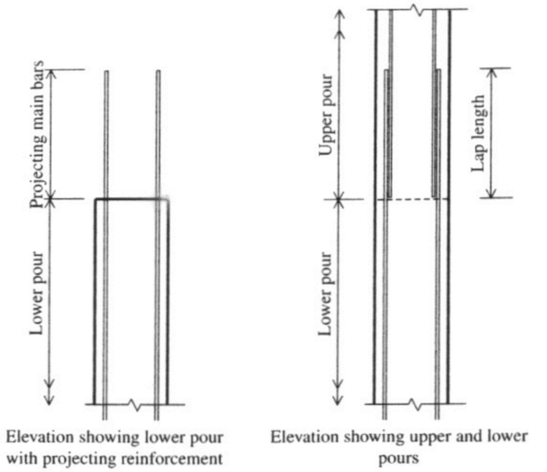


Figure 3.27: Lapped longitudinal reinforcement in a concrete column

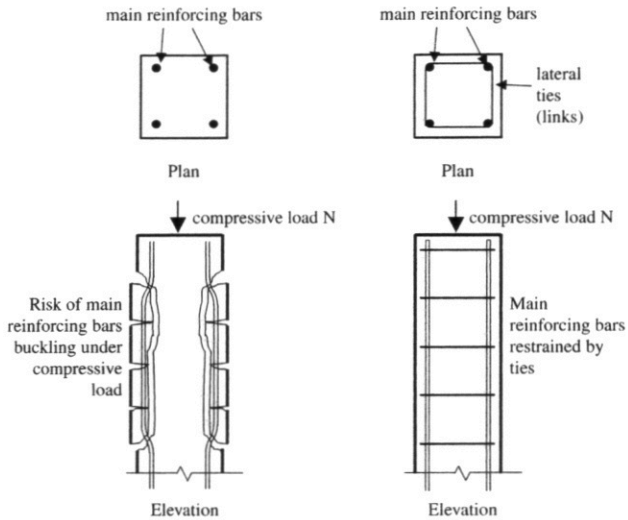


Figure 3.28: Function of lateral ties in a reinforced concrete column

3.11.4 Short, Axially Loaded Columns

In rare cases where a column is loaded through a properly designed pinned joint it may be possible to consider the column as axially loaded. In this case the longitudinal reinforcement can be determined from Table 3.31, which will give conservative results in most cases.

Table 3.30: Limits on ties in reinforced concrete columns

Size of ties	Not less than or	main bar size/4 6 mm
Spacing of ties generally	Not less than or or	20 × main bar diameter the least column dimension 400 mm
Spacing of ties over a height equal to the larger dimension of the column above or below a beam or slab	Not more than or or	12 × main bar diameter 0.6 × the least column dimension 240 mm
Spacing of ties where longitudinal bars exceeding 12 mm in size are lapped	Not more than or or	12 × main bar diameter 0.6 × the least column dimension 240 mm

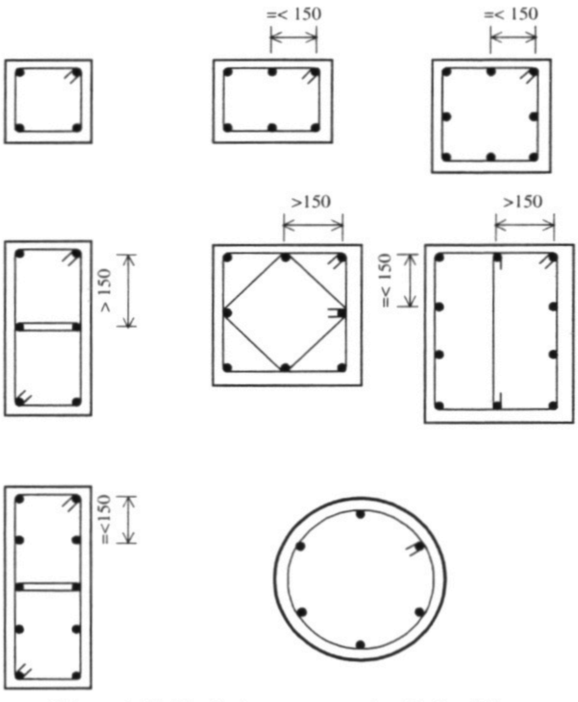


Figure 3.29: Typical arrangements of lateral ties

Table 3.31: Longitudinal reinforcement in braced rectangular columns with axial load

N/bhf_{ck}	up to 0.45	0.5	0.6	0.7	0.8	0.9	1
A_s/bhf_{ck}	0	0.17×10^{-3}	0.51×10^{-3}	0.85×10^{-3}	1.19×10^{-3}	1.53×10^{-3}	1.87×10^{-3}

Based on the design chart for $d_2/h = 0.25$ and a nominal eccentricity of $0.1h$ with $f_{yk} = 500$ N/mm^2.

Example 3.10

A braced rectangular column 250 mm × 300 mm in C30/37 concrete carries at ULS an axial load of 1200 kN. The column is not slender. Determine the longitudinal reinforcement required.

Calculations for Example 3.10	
From data given	$b = 250$ mm
	$h = 300$ mm
$N/bhf_{ck} = 1200 \times 10^3/(250 \times 300 \times 30)$	$N/bhf_{ck} = 0.53$
From Table 3.31, by interpolation	$A_s/bhf_{ck} = 0.27 \times 10^{-3}$
$A_s = 0.27 \times 10^{-3} \times 250 \times 300 \times 30$	$A_s = 608$ mm^2
From Table 3.29, longitudinal reinforcement should not be less than	
$0.12N/f_{yk} = 0.12 \times 1200 \times 10^3/500 \quad = 288$ mm^2	
or $0.002bh = 0.002 \times 250 \times 300 \quad = 150$ mm^2	
	Provide four No. H16 bars, $A_{s,prov} = 804$ mm^2

It should be emphasised that axially loaded columns are unusual, and for most columns it will be necessary to use structural analysis to find the bending moments caused by the loads on the frame as demonstrated in Example 3.11.

3.11.5 Columns with Bending Moment

Columns usually have to carry both compressive force and bending moment. The bending moment to be carried is calculated by structural analysis of the column and the slabs or beams that it supports, taking into account possible uneven distributions of load on the beams. This moment is then increased by a nominal amount to allow for the possibility that the compressive load N is not at the centre of the column. The additional moment is calculated by multiplying N by a nominal eccentricity e, where e is the greatest of

- $h/30$, where h = size of the column
- $l_0/400$, where l_0 = effective length of the column
- 20 mm

3.11.6 Use of Column Design Charts

In a column carrying bending moment and axial force, calculation of the amount of longitudinal reinforcement requires an iterative solution involving a search for the correct neutral axis position, and is beyond the scope of this manual. Design charts for rectangular columns with symmetric reinforcement can be found on the website www.eurocode2.info, and these allow the reinforcement to be found without this calculation. A typical chart is in Figure 3.30.

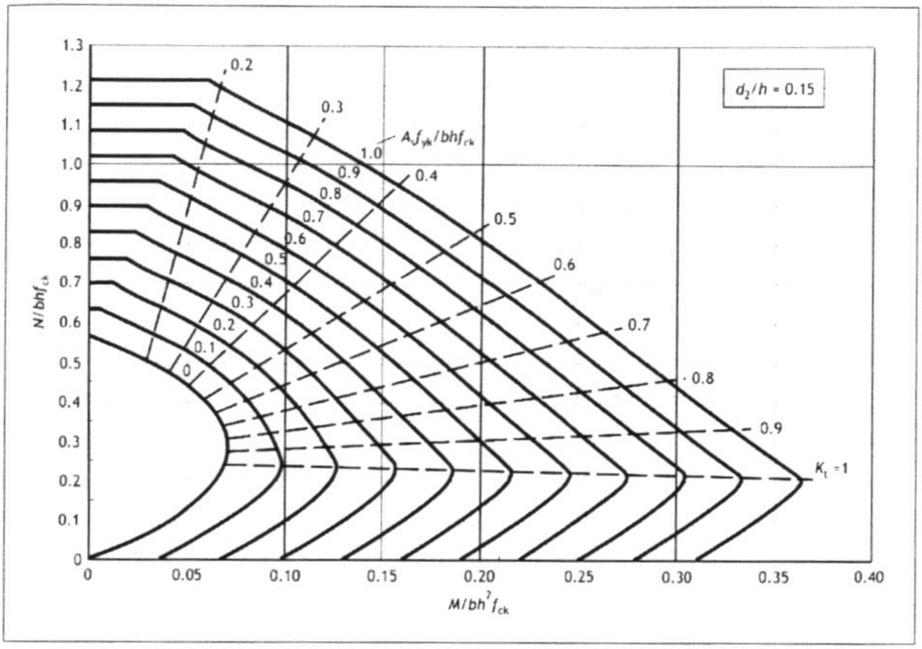


Figure 3.30: Column design chart for d_2/h = 0.15 from www.eurocode2.info

All longitudinal reinforcement will carry the vertical load, but reinforcement near the centre of the column will be less efficient at carrying bending moment than reinforcement near the surface (i.e. further away from the neutral axis of the section). For this reason different charts are available for different values of d_2/h (see Figure 3.31), and the charts available on the website noted above are for d_2/h values of 0.10, 0.15, 0.20 and 0.25. It is always safe to use a chart for a d_2/h value higher than the actual value required.

The dimension d_2 = (cover to the ties) + (size of ties) + (half of main bar diameter)

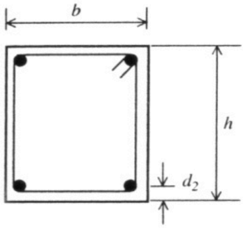


Figure 3.31: Notation used in column design charts

The chart in Figure 3.30 shows dashed lines labelled for different values of K_t. These relate to the position of the neutral axis and are not required for the examples used in this manual.

The design method is best illustrated by an example.

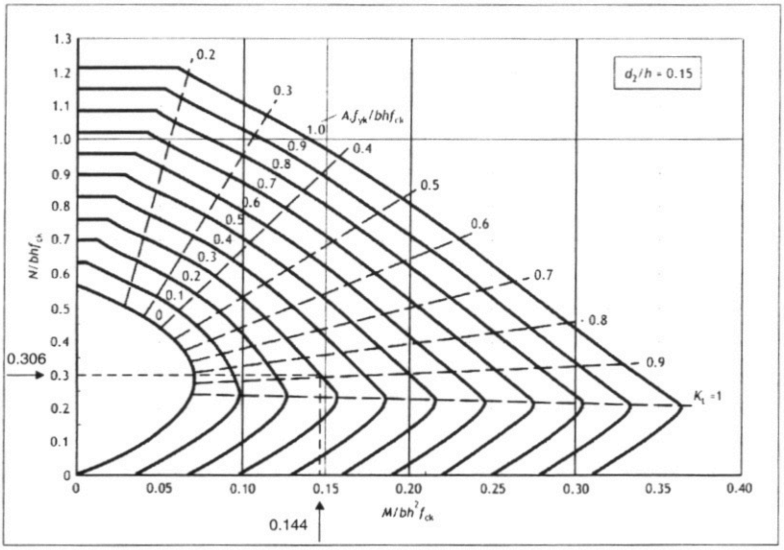
Example 3.11 Determining the longitudinal reinforcement in a column

A braced rectangular column 300 mm × 400 mm in C30/37 concrete carries at ULS an axial load of 1100 kN and a maximum bending moment of 185 kNm in the direction of the 400 mm dimension. Determine the longitudinal reinforcement required. The column has an effective length of 2213 mm and is not slender.

The main bars are H25, the ties are H8 and the cover to all reinforcement is 30 mm (see Figure 3.32 for cross-section of column).

Calculations for Example 3.11

Axial load	$N = 1100\,\text{kN}$
From Section 3.11.5 of this manual the nominal eccentricity e is the greatest of	
$400/30 = 13\,\text{mm}$	
$2213/400 = 5.5\,\text{mm}$	
$20\,\text{mm}$	Adopt $e = 20\,\text{mm} = 0.02\,\text{m}$
Additional bending moment $= Ne = 1100 \times 0.02$	$= 22\,\text{kNm}$
Total bending moment $M = 185 + 22$	$M = 207\,\text{kNm}$
Dimension $d_2 = 30 + 8 + 25/2$	$d_2 = 50.5\,\text{mm}$
$d_2/h = 50.5/400$	$d_2/h = 0.13$
	Use chart for $d_2/h = 0.15$
For use in the design chart	
$M/bh^2f_{ck} = 207 \times 10^6/(300 \times 400^2 \times 30)$	$M/bh^2f_{ck} = 0.144$
$N/bhf_{ck} = 1100 \times 10^3/(300 \times 400 \times 30)$	$N/bhf_{ck} = 0.306$
From the chart for $d_2/h = 0.15$ (Figure 3.30), by reading onto the curved lines,	$A_sf_{yk}/bhf_{ck} = 0.30$



$A_s = 0.30bhf_{ck}/f_{yk} = 0.30 \times 300 \times 400 \times 30/500$ $\qquad A_s = 2160\,\text{mm}^2$

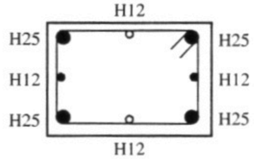
From Table 3.29, area of longitudinal reinforcement should not be less than

$0.12N/f_{yk} = 0.12 \times 1100 \times 10^3/500$ $\qquad = 264\,\text{mm}^2$

$0.002bh = 0.002 \times 300 \times 400$ $\qquad = 240\,\text{mm}^2$

Use four No. H25 + two No. H12 bars

$A_{s,prov} = 2189\,\text{mm}^2$



Note the additional H12 side bars, shown as open circles. These are not required for strength but are provided so that the maximum spacing between longitudinal bars does not exceed 150 mm

Figure 3.32: Cross-section of column from Example 3.11

Example 3.12 Design of columns in a frame

Figure 3.33 shows part of a multi-storey reinforced concrete building frame. The columns are 300 mm square, and the concrete grade is C35/45. Also shown are three load patterns which may be critical for the design of the columns. A structural analysis of the frame under the different load patterns gave the axial loads and bending moments shown in Table 3.32. Determine the reinforcement required in columns A and B.

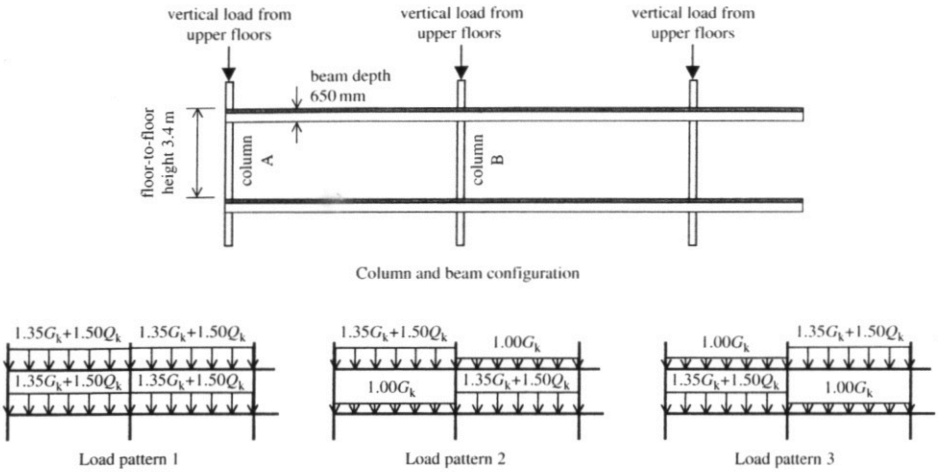


Figure 3.33: Configuration and load patterns for Example 3.12

Table 3.32: Axial forces and bending moments from structural analysis of the frame in Example 3.12

	Load pattern 1		**Load pattern 2**		**Load pattern 3**	
	Axial Force	**Max. BM**	**Axial Force**	**Max. BM**	**Axial Force**	**Max. BM**
Column A	708 kN	98.2 kNm	712 kN	87.5 kNm	604 kN	54.4. kNm
Column B	1476 kN	7.6 kNm	1352 kN	38.7 kNm	1353 kN	33.1 kNm

As we do not know which load case may be critical for each column, we must check all of them.

Calculations for Example 3.12

Slenderness (both columns)

The clear height is 3400 – 650 = 2750 mm

As the columns are braced and the beams are deeper than the size of the column, then both ends of the column have end condition 1 (Figure 3.26); so from Table 3.28

$l_0 = 0.75 \times$ clear height $= 0.75 \times 2750$ = 2063 mm

$l_0/b = 2063/300$ = 6.88

Maximum axial load $N = 1476$ kN

Limiting l_0/b ratio $= 6.19\sqrt{(bhf_{ck}/N)} = 6.19\sqrt{((300 \times 300 \times 35)/(1476 \times 10^3))}$ = 9.04

So the columns are not slender

Nominal eccentricity for both columns, see Section 3.11.5

The nominal eccentricity e is the greatest of

300/30 = 10 mm

2063/400 = 5.1 mm

20 mm

Adopt e = 20 mm = 0.02 m

Column A	Load pattern 1	Load pattern 2	Load pattern 3
Axial load from structural analysis N	708 kN	712 kN	604 kN
Additional bending moment = Ne = $N \times 0.02$	14.2 kNm	14.2 kNm	12.1 kNm
Bending moment from structural analysis M_0	98.2 kNm	87.5 kNm	54.4 kNm
Total design bending moment $M = M_0 + Ne$	112.4 kNm	101.7 kNm	66.5 kNm
For use in the design chart			
$M/bh^2f_{ck} = M \times 10^6/(300 \times 300^2 \times 35)$	0.119	0.108	0.070
$N/bhf_{ck} = N \times 10^3/(300 \times 300 \times 35)$	0.225	0.226	0.192
From the chart for $d_2/h = 0.15$ (Figure 3.30), by interpolation between the curved lines, A_sf_{yk}/bhf_{ck}=	0.18	0.14	0.02

Adopt the largest value A_sf_{yk}/bhf_{ck} = 0.18

$A_s = 0.18bhf_{ck}/f_{yk} = 0.18 \times 300 \times 300 \times 35/500$ $A_s = 1134\,\text{mm}^2$

From Table 3.29, longitudinal reinforcement should not be less than

$0.12N/f_{yk} = 0.12 \times 712 \times 10^3/500$ $= 171\,\text{mm}^2$

$0.002bh = 0.002 \times 300 \times 300$ $= 180\,\text{mm}^2$

Use four No. H20 bars

$A_{s,prov} = 1257\,\text{mm}^2$

At laps the area of reinforcement will be $2 \times 1257 = 2514\,\text{mm}^2$

From Table 3.29, max. allowed area = $0.08bh = 0.08 \times 300 \times 300 = 7200\,\text{mm}^2$ Accept

Ties: from Table 3.30. Min. size = 20/4 = 5 mm Use H6 ties

Tie spacing generally: not more than

- 20 × bar diameter = 20 × 20 = 400 mm
- least column dimension = 300 mm
- 400 mm

Spacing generally: 300 mm

Tie spacing at laps or within 300 mm of beams: not more than

- 12 × bar diameter = 12 × 20 = 240 mm
- 0.6 × least column dimension = 180 mm
- 240 mm

Spacing at laps or within 300 mm of beams: 180 mm

Continued on next page

Calculations for Example 3.12 (Continued from previous page)

Column B	Load pattern 1	Load pattern 2	Load pattern 3
Axial load from structural analysis N	1476 kN	1352 kN	1353 kN
Additional bending moment $= Ne = N \times 0.02$	29.5 kNm	27.0 kNm	27.1 kNm
Bending moment from structural analysis M_0	7.6 kNm	38.7 kNm	33.1 kNm
Total design bending moment $M = M_0 + Ne$	37.1 kNm	65.7 kNm	60.2 kNm
For use in the design chart			
$M/bh^2f_{ck} = M \times 10^6/(300 \times 300^2 \times 35)$	0.039	0.069	0.064
$N/bhf_{ck} = N \times 10^3/(300 \times 300 \times 35)$	0.469	0.429	0.430
From the chart for $d_2/h = 0.15$ (Figure 3.30), by interpolation between the curved lines, $A_s f_{yk}/bhf_{ck} =$	0.0	0.07	0.05

Adopt the largest value $A_s f_{yk}/bhf_{ck} =$ 0.07

$A_s = 0.07bhf_{ck}/f_{yk} = 0.07 \times 300 \times 300 \times 35/500$ $A_s = 441\,\text{mm}^2$

From Table 3.29, longitudinal reinforcement should not be less than

$0.12N/f_{yk} = 0.12 \times 1476 \times 10^3/500$ $= 354\,\text{mm}^2$

$0.002bh = 0.002 \times 300 \times 300$ $= 180\,\text{mm}^2$

Use four No. H12 bars

$A_{s,prov} = 452\,\text{mm}^2$

Note that although Column B carries the greater vertical load, Column A requires more reinforcement.

Ties: from Table 3.30. Min. size = 12/4 = 3 mm Use H6 ties

Tie spacing generally: not more than

- 20 × bar diameter = 20 × 12 = 240 mm
- least column dimension = 300 mm
- 400 mm

Spacing generally: 240 mm

Tie spacing at laps or within 300 mm of beams: not more than

- 12 × bar diameter = 12 × 12 = 144 mm
- 0.6 × least column dimension = 180 mm
- 240 mm

Spacing at laps or within 300 mm of beams: 140 mm

3.11.7 Design Summary for Short Braced Columns

The design procedure for short braced columns may be summarized as follows:

- Ensure that the remainder of the structure is designed for any lateral loads, so that the column can be considered as braced.
- By reference to its slenderness ratio, ensure that the column is short and not slender.
- Analyse the whole structure, or part of the structure local to the column, to determine the axial force and the bending moment in the column.

- By use of the column design charts, determine the longitudinal reinforcement required at the ULS. Ensure that the requirements for minimum and maximum reinforcement are met.
- By reference to Table 3.30 determine the tie size and tie spacing.

3.12 References

BS EN 1992-1-1: 2004 Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings.

BS EN 1992-1-2: 2004 Eurocode 2: Design of concrete structures – Part 1-2: General rules – structural fire design.

BS EN-206:2000 Concrete – Part 1: Specification, performance, production and conformity.

BS 4449: 2005 Steel for the reinforcement of concrete.

BS 4483: 2005 Steel fabric for the reinforcement of concrete – specification.

BS 8500: 2006 Concrete – complimentary British Standard to BS EN 206-1.

IStructE Manual for the design of reinforced concrete structures to Eurocode 2, Institution of Structural Engineers, 2006.

Concise Eurocode 2 for the design of *in-situ* concrete framed buildings to BS EN 1991-1-1:2004 and its UK National Annex: 2005, Narayanan R S and Goodchild CH, The Concrete Centre, 2006.

'How to…' sheets and other information from www.eurocode2.info

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CHAPTER 4

Masonry Elements

Contents

4.1 Structural Design of Masonry

Guidance on the design of all unreinforced and reinforced masonry structures is given in BS EN 1996 (EC6). This is published in several sections, as shown in Table 4.1.

Table 4.1: Codes relating to the design of structural masonry

	BSI reference	**Title**
EC6 Part 1-1	**BS EN 1996 Part 1-1**	**General – Rules for reinforced and unreinforced masonry**
EC6 Part 1-2	BS EN 1996 Part 1-2	General rules – Structural fire design
EC6 Part 2	BS EN 1996 Part 2	Design considerations, selection of materials and execution of masonry
EC6 Part 3	**BS EN 1996 Part 3**	**Simplified calculation methods for unreinforced masonry structures**

Each code should be read with the appropriate National Annex. Titles shown in **bold** are relevant to this manual.

This manual is based on the design methods in EC6 Part 1-1 and EC6 Part 3. Much useful additional information, including guidance on UK practice, is given in the *Manual for the design of plain masonry in building structures to Eurocode 6*, IStructE, February 2008.

When an unreinforced masonry wall is found to be inadequate consideration may be given to adding reinforcement or pre-stressing the masonry. Design methods for reinforced and pre-stressed masonry are given in EC6 but are not covered by this manual.

EC6 gives guidance on the design of walls to resist lateral loading from wind or from earth pressure, as well as vertical loading. This manual does not cover the design of walls for lateral loads.

4.2 Symbols and Definitions

Table 4.2 lists some of the symbols used in the design process. Where relevant the units commonly used for the quantity are shown.

Table 4.2: Symbols used in the design of structural masonry

Symbol	Normal units	Meaning	Comment
A	mm^2	Horizontal cross-section area of a wall	
e	mm	Eccentricity of load on a wall leaf	
f_d	N/mm^2	Design compressive strength of masonry	$f_d = f_k/\gamma_m$
f_k	N/mm^2	Characteristic compressive strength of masonry	See Table 4.5 and Table 4.6
f_m	N/mm^2	Characteristic compressive strength of mortar	
f_b	N/mm^2	Normalized mean compressive strength of masonry unit	Manufacturer's literature may state f_b. Otherwise $f_b = \delta \times$ actual mean compressive strength
h	mm	Height of wall panel	
h_{ef}	mm	Effective height of wall	
t	mm	Thickness of a wall	
t_1, t_2	mm	Thicknesses of the leaves of a cavity wall	
t_{eff}	mm	Effective thickness of a wall, or of one leaf of a cavity wall	See Section 4.5.5
α		Load ratio	Used in the simplified method of EC6 Part 3
δ		Shape factor for a masonry unit	See Table 4.4
η		Moment reduction factor	η should not be less than 0.5
γ_m		Partial factor of safety for strength of masonry	γ_m takes values from 2.3 to 3.0, see Table 4.8
λ		slenderness ratio h_{ef}/t_{ef}	For loadbearing walls λ should not be more than 27
Φ_A		Capacity reduction factor for walls of area less than $0.1\,m^2$	
Φ_s		Capacity reduction factor for slenderness	
ρ		Effective height reduction factor	
ρ_t		Effective thickness coefficient for a wall stiffened by piers	See Table 4.10

Definitions

Many specialist terms are used in masonry construction, and most are defined where they are first used in this chapter. The following terms are also used:

bed joint: a horizontal joint between courses of masonry; normally filled with mortar.

perpend joint: a vertical joint between masonry units; normally filled with mortar.

stretcher: the longer face of a masonry unit when it is exposed on the face of the wall.

header: the shorter face of a masonry unit when it is exposed on the face of the wall.

The terms *group*, *category* and *execution class* have special meanings in the context of masonry design. The terms are explained later in this chapter, and are summarised here:

group: Masonry units are grouped according to the volume of voids in the unit

Group 1: solid or up to 25% voids

Group 2: over 25% voids.

category: Masonry units of category I are made under stricter quality control conditions than units of category II, and the manufacturer's literature will state which category is applicable.

execution class: Execution control class 1 denotes a higher degree of quality control on site than execution control class 2.

Execution Class 2: All work on site is properly supervised and carried out in accordance with BS EN 1996 Part 2.

Execution Class 1: As above. In addition, the work is properly inspected and the mortar is regularly sampled and tested for strength.

4.3 Materials

The fundamental properties of the individual materials that comprise a masonry wall are well understood and documented, but an understanding of their combined behaviour is vital to a successful design. The designer must specify bricks or blocks of appropriate quality and appearance, choose suitable mortar, specify correct use and devise appropriate details.

This manual covers methods of ensuring that the masonry can carry the design vertical loads. Design for resisting lateral loads is covered by EC6 but is beyond the scope of this manual. Where relevant, the following other factors should also be considered:

- appearance
- durability and frost resistance, including water absorption and sulphate content

- fire resistance
- thermal insulation
- sound insulation
- resistance to damp penetration
- provision for thermal and shrinkage movement
- workmanship and supervision of the execution of the masonry.

In many cases the factors listed above will be more critical than the need for structural strength. For instance in a two-storey building an external brick wall which is durable and frost resistant is likely to be stronger than is necessary to carry the weight of the building.

4.3.1 Bricks and Blocks

UK practice is to distinguish between blocks and bricks by face size. A unit smaller than 300 mm × 100 mm is a brick, larger sizes are blocks. However the EN standards do not make this distinction and give requirements for 'masonry units', a term that includes both bricks and blocks. The following types of masonry unit are described in standard BS EN 771.

Clay masonry units to BS EN 771-1: These are made from clay or other argillaceaous material, with or without sand, fuel or other additives, fired at a sufficiently high temperature to achieve a ceramic bond.

Calcium silicate masonry units to BS EN 771-2: These are made predominantly from lime and sand, hardened by high-pressure steam.

Aggregate concrete masonry units to BS EN 771-3: These are made from concrete, comprising cement, water, aggregate and admixtures. Lightweight or normal weight aggregate may be used, though lightweight aggregate blocks are considerably more dense than aerated concrete blocks.

Autoclaved aerated concrete units to BS EN 771-4: These are made from cement and/or lime with some fine aggregate with water and foaming agents. The blocks have a foam-like structure with many small air voids. These blocks have low density, generally fairly low strength and good thermal resistance.

Manufactured stone masonry units to BS EN 771-5: These are concrete units with at least one exposed face having a close structure intended to resemble natural stone. Generally they are moulded using pressure as well as vibration, and they can be made in complex shapes.

Natural stone masonry units to BS EN 771-6: This standard describes various tests that can be carried out on natural stones to determine their suitability as building materials, and gives permissible deviations for dimensions.

Bricks

Bricklaying is a manual skill with a long history. The bricklayer picks up the brick in one hand, uses the other hand to spread the mortar, then places the brick. Thus a fundamental requirement is that each brick is small enough and light enough to be lifted in one hand. Historically bricks have been made in a wide variety of sizes and it is often difficult to match old bricks when altering or extending historic structures. However, UK practice for new construction is now standardized on brick CB.1.5 from BS 4729, which is shown in Figure 4.1. Figure 4.2 shows how, for a standard brick, two header faces plus one mortar joint coordinate with the length of one stretcher face.

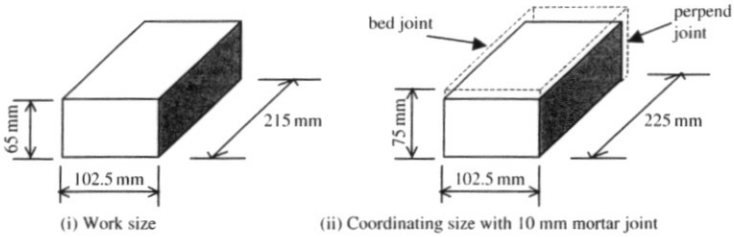


Figure 4.1: UK standard-format brick: Brick CB.1.5 to BS4729

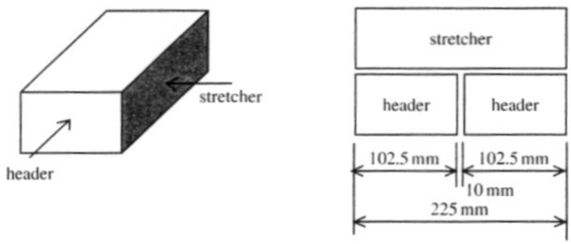


Figure 4.2: UK standard-format brick: relation of header to stretcher

Bricks are often categorized as *facing bricks, common bricks* or *engineering bricks*. These terms are not included in the defining standards but generally have the following meanings:

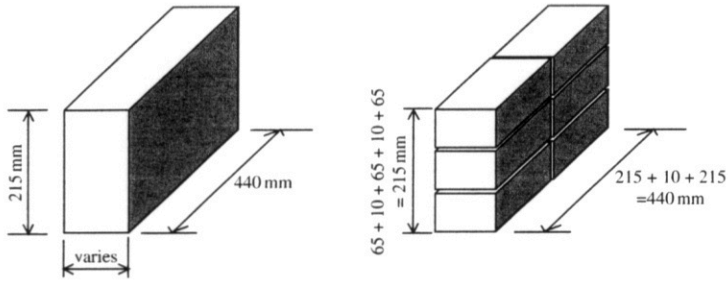
Facing bricks: These are clay, concrete or calcium silicate bricks manufactured to satisfy aesthetic requirements. They are available in a wide range of strengths, colours and textures.

Common bricks: These are clay or concrete bricks produced for general building work where the appearance is not important. The term is not a guide to structural quality, and many common bricks have excellent strength properties.

Engineering bricks: These are clay bricks with good strength and durability suitable for use in conditions requiring good resistance to weathering or in corrosive conditions. Typical uses include manholes and retaining walls.

Blocks

Blocks are available in a wide variety of sizes, but the UK standard block is 440 mm long and 215 mm high. Figure 4.3 shows how the work size of one block coordinates with the size of six standard bricks. The width of the block varies, and widths between 75 mm and 300 mm can be obtained to order. Standard widths of 100, 140, 190, 200, 215 and 300 mm are generally available from stock.



(i) Work size

(ii) Illustration that the face size of one block equals the face size of six bricks

The block width may be 75, 90, 100, 125, 130, 140, 150, 190, 200, 215, 265 or 300 mm
Normal widths available are 100, 140, 190, 200, 215 and 300 mm

Figure 4.3: UK standard-format block

The size and weight of blocks as compared to bricks means that the bricklayer often has first to spread the mortar and then place the block with both hands.

Designers should be aware that other sizes are sometimes supplied, particularly a 'metric brick' with a face size of 190 mm × 90 mm and a 'metric block' with a face size of 390 mm × 190 mm. Outside the UK many other sizes are in common use, so the specification of imported bricks or blocks requires particular care.

Grouping of masonry units: holes and perforations

Bricks and blocks are rarely solid. Bricks often have frogs or vertical holes, which reduce the volume of clay (and thus the weight of the unit) and also speed up the process of drying and firing the bricks. Concrete block are often hollow to reduce weight and cost, and hand-holds may be provided to facilitate lifting. Voids may be filled with non-structural foam to improve thermal insulation. Some common hole patterns are shown in Figure 4.4.

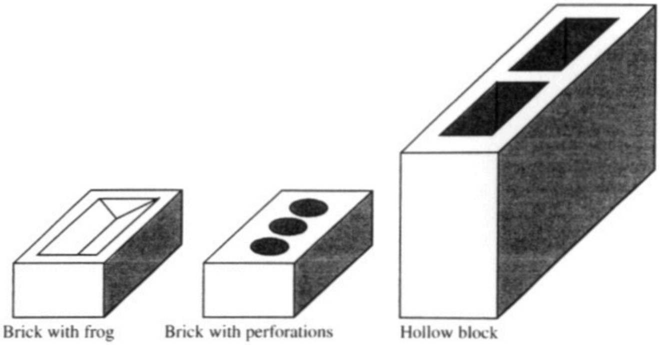


Figure 4.4: Common patterns of holes in masonry units

For strength calculations masonry units are divided into groups according to the volume and distribution of holes.

Units are classified as Group 1 if the total volume of holes is not more than 25% of the unit and the volume of any single hole is not more than 12.5%. Group 2 units may have up to 55% of holes by volume, subject to other limits given in EC6 Part 1-1, Table 3.1. Groups 3 and 4 are also defined with higher permitted percentages of holes, but these are rarely used in the UK.

In practice the Group category for a unit is found by consulting the manufacturer's literature, and will be:

Group 1: solid or up to 25% voids

Group 2: over 25% voids.

Manual handling

By its nature bricklaying requires repetitive manual handling of bricks, blocks and mortar, and it is important to protect operatives from musculoskeletal disorders arising from this handling. Good access including proper scaffolding and correct placing of materials are important factors. The mass to be lifted is also important.

Standard-format bricks have a mass below 5 kg and pose a low risk. A somewhat higher risk is posed by 140 mm thick aerated concrete blocks which have a mass of approximately 10 kg. Repetitive lifting of units with a mass exceeding 20 kg, for example 300 mm thick aerated concrete blocks or 140 mm thick dense concrete blocks, poses a high risk and should not be undertaken for extended periods. The provision of hand-holds in large blocks can reduce the risk of injury. Other techniques are team-lifting and the use of mechanical lifting aids, although these change the nature of the bricklaying process and it may be more appropriate to consider other techniques such as off-site prefabrication into larger units.

Advice on minimizing the risks involved in manual handling is available from the Health and Safety Executive, www.hse.gov.uk.

4.3.2 Mortar

Mortar is used in all masonry walls whether they are constructed from bricks, blocks or natural stone. The mortar serves several purposes, and must satisfy a number of requirements in both the newly mixed and the hardened state.

During construction mortar should have good workability to ensure efficient use by the bricklayer. It must be easily spread using a trowel or nozzle to provide an even bed on which the unit can be lined and levelled. When used with absorbent units the mortar should retain moisture to avoid premature drying and stiffening. Finally the mortar should stiffen in a reasonable time so that it is not squeezed out by the weight of units laid above.

In the hardened state the mortar must allow even transfer of the load across the joint. Ideally the mortar should be weaker than the units so that any movement, which may be caused by drying shrinkage, thermal effects or foundation settlement, is accommodated in the joints without cracking the units.

Prior to about 1920 most mortar was made using lime and sand. These mortars have excellent workability and are easy for the bricklayer to use, and they harden to a soft mortar which can accommodate movement without cracking. However the hardening process which involves carbonation of the lime is very slow, so construction rates are limited. Modern mortars generally contain cement which hardens much faster than lime, though renovation work on old buildings may still require lime mortar.

Classes of mortar and their constituent proportions are given in Table 4.3. In general, mortar requires a good proportion of fine material in the mix to give sufficient workability. If a strong mortar is required then the fine material can all be ordinary Portland cement (OPC), for example 1:3 cement:sand for a type (i) mortar, but if a weaker mortar is required then some of the OPC must be removed. Workability can be restored in one of these ways:

- by adding lime
- by using masonry cement, a mixture of OPC with an inorganic filler which may be lime or some other product
- by using a proprietary plasticizer additive which induces many very small air bubbles in the mortar and thereby increases the workability. The harsher mixes in Table 4.3

(e.g. 1:6 cement:sand) will almost certainly require a plasticizer additive to make them workable.

Table 4.3: Mortar classes, constituents and proportions

Compressive strength class	Compressive strength f_m	Mortar class	Mortar constituents and proportions			
			Cement: Lime: Sand	**Cement: Sand**	**Masonry cement[a]: Sand**	**Masonry cement[b]: Sand**
M12	12 N/mm^2	(i)	1:0:3 to 1:¼:3	1:3		
M6	6 N/mm^2	(ii)	1:½:4 to 1:½:4½	1:3 to 1:4	1: $2\frac{1}{2}$ to 1: $3\frac{1}{2}$	1:3
M4	4 N/mm^2	(iii)	1:1:5 to 1:1:6	1:5 to 1:6	1:4 to 1:5	1: 3½ to 1: 4
M2	2 N/mm^2	(iv)	1:1:8 to 1:1:9	1:7 to 1:8	1: 5½ to 1: 6½	1: 4½

[a]Masonry cement with inorganic filler other than lime.
[b]Masonry cement with lime.
When the sand portion is given as, for example, 5 to 6, the lower figure should be used with sands containing a higher proportion of fines while the higher proportion should be used with sands containing a lower proportion of fines.
Source: Table NA2 of UK National Annex to EC6 Part 1-1

For all but the smallest projects mortar is supplied to site pre-mixed, and suppliers can provide a wide variety of mortars to suit the project's requirements for strength and colour. Two methods of supply to site are common.

- Pre-mixed mortar dosed with a retarder which maintains workability for a specified period, normally 36 to 72 hours. Coloured containers are used to identify the ages of the batches.
- Dry mortar is delivered by tanker, stored on site in a silo and mixed on demand by the silo discharge mechanism. This method is only suitable for large sites because a concrete slab with access for tankers is required to hold the silo and piped water supply to the silo is also needed.

EC6 permits the use of proprietary lightweight mortars, although standard proportions for these are not given. Lightweight mortar is easier to handle and increases the thermal resistance of the wall. The fine aggregate used in lightweight mortar is weaker than normal sand, so the strength of the mortar joints is not greatly increased by the confinement effect discussed in Section 4.4.3. As a result, masonry made with lightweight mortar is not as strong as masonry made with normal mortar (see Table 4.5).

4.3.3 Thin-Joint Masonry

Thin-joint techniques, usually with a nominal joint thickness of between 1 mm and 3 mm, can give significant economies in the construction process. The advantages of these techniques over conventional construction are:

- less mortar is needed
- the mortar can be applied using a pump and a special nozzle, which is a quicker process than conventional trowel application
- the design strength of the wall may be slightly higher (see Table 4.5)
- the thermal resistance of the wall may be better
- the wall can be quickly built to any height without waiting for the mortar to stiffen.

The following points should also be noted.

- The techniques will only be successful if the units are manufactured to close tolerances, and manufacturers can advise on the suitability of their products.
- Although thin-joint masonry may be quick to build, the conventional skills of the bricklayer are still required to maintain the coursing and the verticality of the wall.
- Normal practice is to use standard-format blocks which are nominally 215 mm high. Thus the nominal course height will be around 218 mm, not 225 mm as in normal construction. For quick construction without excessive cutting of blocks, floor heights and opening sizes should be determined on this basis.
- If thin-joint blockwork is used as the inner leaf of a cavity wall with standard-format bricks for the outer skin, the bed joints for the two leaves will not line up.

The general requirements for mortar in thin-joint construction are the same as in conventional work, though the application method may be different. These mortars are generally proprietary products and are used with specialist equipment for mixing and spreading.

4.3.4 Cavity Wall Ties

Ties complying with BS EN 845-1 should be installed in cavity walls at not less than 2.5 ties per square metre, normally at 900 mm centres horizontally and 450 mm centres vertically. Additional ties around openings and adjacent to movement joints should be installed at 300 mm centres or closer. Typical tie locations in a wall made from standard-format bricks and blocks are shown in Figure 4.5.

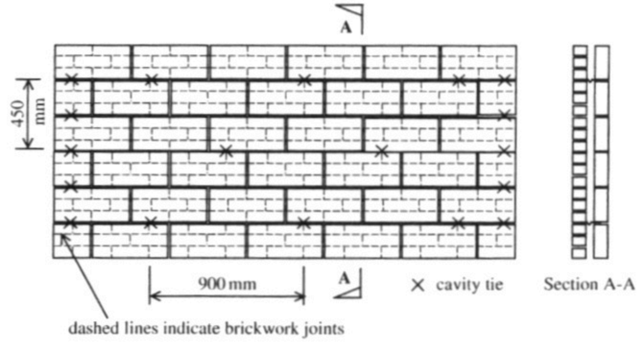


Figure 4.5: Typical cavity tie positions in a wall made from standard format bricks and blocks

In normal construction with 10 mm mortar joints every third bed joint in the brick leaf is at the same level as a bed joint in the block leaf, and the cavity ties are laid in the mortar. In thin mortar construction the bed joints do not line up in this way and thin bed joints are too narrow to accommodate cavity ties, so instead ties are screwed into the blockwork units to line up with the bed joints in the brick leaf.

4.3.5 Damp Proof Courses

While the main purpose of a damp proof course (DPC) is to provide a moisture barrier, a DPC in a structural wall must be stiff enough to resist squeezing out under vertical load. If the wall is carrying significant horizontal load then the shear resistance and bond of the DPC may also be important.

4.4 Material Properties

4.4.1 Compressive Strength of Masonry Units

Masonry units are tested for compressive strength by crushing them between steel platens using the test methods in EN 772-1. The strength is calculated as

Actual compressive strength = (maximum load achieved)/(gross area of loaded face)

Any voids in the units are ignored in this calculation, so for hollow units the value quoted is not a measure of the strength of the material used to make the unit.

Structural design is based on the normalised mean compressive strength f_b which is the actual mean strength converted to the air-dried strength of an equivalent 100 mm wide × 100 mm high unit using the shape factor δ from EN 772-1 Annex A, reproduced in Table 4.4.

$f_b = \delta \times$ actual mean compressive strength

Table 4.4: Shape factor δ to allow for the tested dimension of the unit

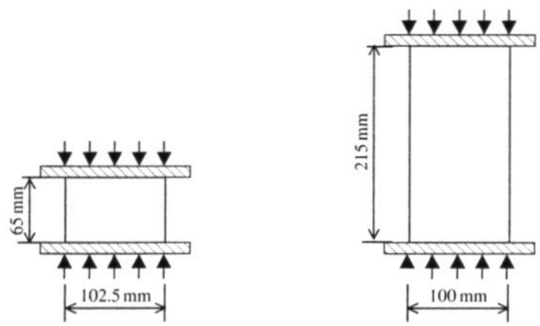
Height of unit (mm)	Width of unit (mm)							
	50	100	140	150	190	200	215	250 or more
40	0.80	0.70	–	–	–	–	–	–
50	0.85	0.75	0.71	0.70	–	–	–	–
65	0.95	0.85	0.77	0.75	0.71	0.70	0.685	0.65
100	1.15	1.00	0.92	0.90	0.82	0.80	0.785	0.75
150	1.30	1.20	1.12	1.10	1.02	1.00	0.985	0.95
200	1.45	1.35	1.27	1.25	1.17	1.15	1.135	1.10
215	1.48	1.38	1.30	1.28	1.20	1.18	1.168	1.115
250 or more	1.55	1.45	1.37	1.35	1.27	1.25	1.22	1.15

Linear interpolation between values is permitted.
Source: Table A1 of BS EN 772-1 with interpolation.

The steel platens of the compression testing machine exert a greater lateral restraint on a stocky unit than they do on a more slender unit, so

- for a stocky unit such as a standard brick 102.5 mm wide × 65 mm high, the normalized strength is lower than the actual strength.
- for a slender unit such as a block 100 mm wide × 215 mm high, the normalized strength is higher than the actual strength.

These are illustrated in Figure 4.6.



(i) Standard format brick.
Normalised compressive strength
$f_b = 0.85$ x actual compressive strength

(ii) 100 mm block.
Normalised compressive strength
$f_b = 1.38$ x actual compressive strength

Figure 4.6: Normalized compressive strengths

Designers should take care when using manufacturer's data on compressive strength. At the time of writing (early 2009) most manufacturers were stating the actual mean compressive strength of their products, and the normalized strength can be found from:

$$f_b = \delta \times \text{actual mean compressive strength}$$

In future it is likely that manufacturers will state the normalized compressive strength f_b directly, and this can be used without conversion.

4.4.2 Ultimate Compressive Strength of Masonry

As a generalisation, masonry is weaker than the individual masonry unit but stronger than the mortar used. It may seem odd that the masonry can be stronger than the mortar because all the load in the wall is carried by the bed joints, but this effect is explained by noting that mortar strengths are measured by crushing cubes (height = width) but a bed joint in which the height is much less than the width provides much greater lateral restraint to the mortar and so develops more strength. At an extreme, bricks laid in damp sand have a small but significant strength while a cube of damp sand has negligible strength. This effect is more significant for thin-layer mortar joints which are about 3 mm thick than for normal mortar joints which are about 10 mm thick.

Formulas for finding the characteristic masonry strengths f_k, derived from the normalized unit strength f_b and the mortar strength f_m, are given in Table 4.5. The f_k values are characteristic (95%) values while the unit and mortar strengths are mean values, so the relation between strengths discussed in the preceding paragraph is not apparent.

Table 4.5: Characteristic compressive strength of masonry f_k

	General-purpose mortar	Thin-layer mortar	Lightweight mortar
Clay units group 1	$f_k = 0.50 f_b^{0.7} f_m^{0.3}$	$f_k = 0.75 f_b^{0.85}$	$f_k = 0.30 f_b^{0.7} f_m^{0.3}$
Clay units group 2	$f_k = 0.40 f_b^{0.7} f_m^{0.3}$	$f_k = 0.70 f_b^{0.70}$	$f_k = 0.25 f_b^{0.7} f_m^{0.3}$
Calcium silicate units group 1	$f_k = 0.50 f_b^{0.7} f_m^{0.3}$	$f_k = 0.80 f_b^{0.85}$	–
Calcium silicate units group 2	$f_k = 0.40 f_b^{0.7} f_m^{0.3}$	$f_k = 0.70 f_b^{0.85}$	–
Aggregate concrete block units group 1			
Autoclaved aerated block units group 1	$f_k = 0.55 f_b^{0.7} f_m^{0.3}$	$f_k = 0.80 f_b^{0.85}$	$f_k = 0.45 f_b^{0.7} f_m^{0.3}$
Aggregate concrete block units group 2	$f_k = 0.52 f_b^{0.7} f_m^{0.3}$	$f_k = 0.76 f_b^{0.85}$	$f_k = 0.45 f_b^{0.7} f_m^{0.3}$

Source: Clause NA.2.4 of UK National Annex to EC6. f_m is taken to be not greater than $2f_b$.

Table 4.6 gives values of f_k for bricks and blocks in general purpose mortar. These values are based on the appropriate formula from Table 4.5.

Table 4.6: Values of the characteristic compressive strength of masonry f_k

Mortar	Normalised compressive strength of unit f_b											
	2	**4**	**6**	**8**	**10**	**12**	**16**	**20**	**25**	**30**	**50**	**75**
	Clay units and calcium silicate units group 1											
M2	1.0	1.6	2.2	2.6	3.1	3.5	4.3	5.0	5.9	6.7	9.5	12.6
M4	1.2	2.0	2.7	3.2	3.8	4.3	5.3	6.2	7.2	8.2	11.7	15.6
M6	1.2	2.3	3.0	3.7	4.3	4.9	6.0	7.0	8.1	9.3	13.2	17.6
M12	1.2	2.5	3.7	4.5	5.3	6.0	7.3	8.6	10.0	11.4	16.3	21.6
	Clay units and calcium silicate units group 2											
M2	0.8	1.3	1.7	2.1	2.5	2.8	3.4	4.0	4.7	5.3	7.6	10.1
M4	1.0	1.6	2.1	2.6	3.0	3.5	4.2	4.9	5.8	6.6	9.4	12.5
M6	1.0	1.8	2.4	2.9	3.4	3.9	4.8	5.6	6.5	7.4	10.6	14.1
M12	1.0	2.0	3.0	3.6	4.2	4.8	5.9	6.9	8.0	9.1	13.0	17.3
	Aggregate concrete block units and autoclaved aerated block units group 1											
M2	1.1	1.8	2.4	2.9	3.4	3.9	4.7	5.5	6.4	7.3	10.5	–
M4	1.4	2.2	2.9	3.6	4.2	4.7	5.8	6.8	7.9	9.0	12.9	–
M6	1.4	2.5	3.3	4.0	4.7	5.4	6.6	7.7	9.0	10.2	14.6	–
M12	1.4	2.7	4.1	5.0	5.8	6.6	8.1	9.4	11.0	12.5	17.9	–
	Aggregate concrete block units group 2											
M2	1.0	1.7	2.2	2.7	3.2	3.6	4.5	5.2	6.1	6.9	9.9	–
M4	1.3	2.1	2.8	3.4	4.0	4.5	5.5	6.4	7.5	8.5	12.2	–
M6	1.3	2.3	3.1	3.8	4.5	5.1	6.2	7.2	8.5	9.6	13.8	–
M12	1.3	2.6	3.8	4.7	5.5	6.2	7.6	8.9	10.4	11.9	16.9	–

Modification to characteristic compressive strength for shell bedding

Hollow concrete blocks are sometimes laid on a mortar bed consisting of two strips along the outer edges of the block. This is termed 'shell bedding' and is illustrated in Figure 4.7. The width of each mortar strip should not be less than 30 mm, and the ratio g/t should not be less than 0.4.

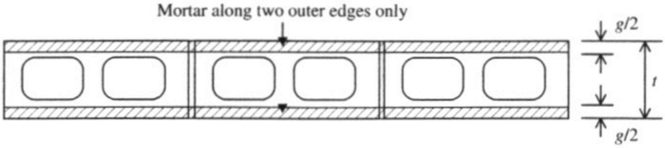


Figure 4.7: Shell bedding to hollow blocks

If this is to be used, the designer should ascertain whether the block strengths quoted by the manufacturer were obtained from tests on shell-bedded blocks. If they were, then the characteristic compressive strengths from Tables 4.5 and 4.6 can be used without modification. If the quoted block strengths were obtained from tests on fully bedded blocks then, by EC6 Part 1-1 Clause 3.6.1.3, the strength values should be reduced thus:

$$\text{Shell bedded } f_k = f_k \text{ from table} \times \left(0.17 + 0.83\frac{g}{t}\right)$$

where g is the total of the widths of the mortar strips and t is the block thickness.

4.4.3 Partial Safety Factors for Materials

Separate partial safety factors for masonry units and for mortar are not given; instead a single partial factor is used for the masonry. The design compressive strength of masonry f_d is given by

$$f_d = f_k/\gamma_m$$

where f_k is the characteristic compressive strength of masonry (see Tables 4.5 and 4.6) and γ_m is the partial safety factor from Table 4.7.

Table 4.7: Partial safety factors γ_m for masonry in compression

	Execution control class 1	Execution control class 2
Masonry units of category I	$\gamma_m = 2.3$	$\gamma_m = 2.7$
Masonry units of category II	$\gamma_m = 2.6$	$\gamma_m = 3.0$

Source: Table NA1 of the UK National Annex to EC6 Part 3.

The value of γ_m depends on the degree of quality control exercised in the manufacture of the masonry units, and on the supervision and control exercised on site. It is expected that better quality control and supervision will result in masonry which is more consistent, so the probability of defects is reduced and a lower value of γ_m can be used.

Masonry units of category I are made under stricter quality control conditions than units of category II, and the manufacturer's literature will state which category is applicable.

The quality of work on site is assumed to depend on the execution control class, and these are defined in Table 4.8.

Table 4.8: Execution control classes

Execution control class 2	All work on site is properly supervised and carried out in accordance with BS EN 1996 Part 2 with attention to: • setting out • storage of materials • batching, mixing and use of mortar • laying of masonry units • construction details • protection during construction
Execution control class 1	As above. In addition, the work is properly inspected and the mortar is regularly sampled and tested for strength

4.5 Factors Influencing the Loadbearing Capacity of Masonry Members

4.5.1 Minimum Wall Thickness

The minimum thickness of a loadbearing wall is 90 mm for a single leaf wall or 75 mm for the leaves of a cavity wall. In practice, it is unusual for the thickness of a loadbearing wall in the UK to be less than 100 mm.

4.5.2 Capacity Reduction Factor for Walls of Small Cross-Section Area Φ_A

The way in which masonry is laid means it is not uncommon for some mortar to be missing from bed joints. This defect may have a significant effect on the strength of a small area of wall, but has little effect on a larger wall. To allow for this the design compressive strength is reduced for walls of small cross-section area.

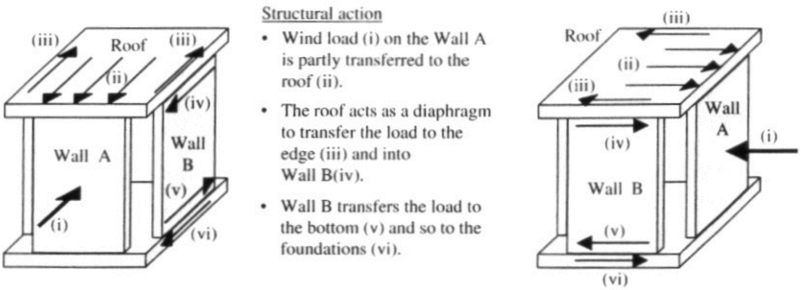
Where the cross-section area A of a wall is less than 0.1 m^2, the design compressive strength should be reduced by a factor Φ_A

$$\Phi_A = (0.7 + 3A)$$

where A is in m^2.

4.5.3 Lateral Restraint

Masonry walls generally require lateral restraint at the top and bottom, as they have little strength for horizontal loads perpendicular to their plane. Lateral restraint is normally provided by walls or roofs which must be effectively anchored to the wall, and those floors or roofs must themselves be restrained by perpendicular walls. This ensures that lateral loads on the structure are carried as forces in the plane of walls, and is illustrated in Figure 4.8.

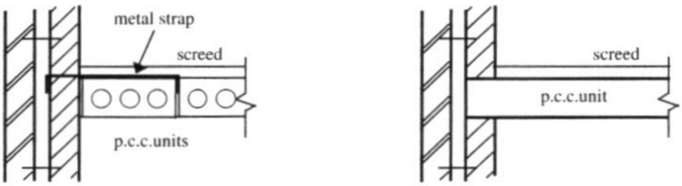


Note that for clarity the walls are shown separately. The structural action is the same if the walls meet at the corners of the building.

Figure 4.8: Lateral restraint to wall provided by roof and by other walls

Effective height

As illustrated in Figure 1.15 of Chapter 1, the effective length of a strut depends on the way that it is held at the ends. A strut which has rotation restraint at the ends has a shorter effective length than a strut which has simple restraints. For a masonry wall, metal ties to a floor or roof provide simple restraint while an adequate bearing of a concrete slab onto the wall provides rotation restraint. These are illustrated in Figure 4.9.



(i) Example of simple restraint

(ii) Example of rotation restraint

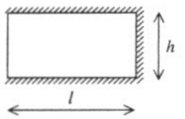
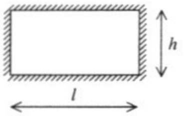
Figure 4.9: Examples of simple restraint and rotation restraint

The effective height h_{ef} of a wall is given by

$$h_{ef} = \rho h$$

where h is the clear storey height and ρ is a reduction factor which depends on the perimeter restraint of the wall. Table 4.9 shows how ρ is determined.

Table 4.9: Effective height reduction factor ρ

<table>
<tr><th colspan="2">Condition</th><th>ρ</th></tr>
<tr><td rowspan="2">Wall restrained at the top and the bottom by reinforced concrete floors or roofs spanning from both sides at the same level, or by a reinforced concrete floor* spanning from one side only and having a bearing of at least 2/3 of the thickness of the wall</td><td>Eccentricity of the load at the top of the wall not greater than 0.25 times the thickness of the wall</td><td>0.75</td></tr>
<tr><td>Eccentricity of the load at the top of the wall greater than 0.25 times the thickness of the wall</td><td>1.0</td></tr>
<tr><td colspan="2">Wall restrained at the top and the bottom by timber floors or roofs spanning from both sides at the same level, or by a timber floor* spanning from one side only and having a bearing of at least 2/3 of the thickness of the wall but not less than 85 mm</td><td>1.0</td></tr>
<tr><td>Wall laterally restrained at top and bottom and at one vertical edge</td><td></td><td>See EC6 Part 1-1 Clause 5.5.1.2 (11) (iii) and Annex D</td></tr>
<tr><td>Wall laterally restrained at top and bottom and at two vertical edges</td><td></td><td>See EC6 Part 1-1 Clause 5.5.1.2 (11)(iv) and Annex D</td></tr>
</table>

Source: EC6 Part 1-1 Clauses 5.5.1.2 (11) (i) and (ii).

*Note that although EC6 refers here to floors, the IStructE *Manual for the design of plain masonry in building structures to Eurocode 6* suggests that the same value can be used for roofs provided they are adequately fixed to the wall.

4.5.4 *Effective Length*

The slenderness of a short wall restrained at one or both vertical edges many be determined by the length of the wall as well as by its height, as illustrated in Figure 4.10.

The effective height of walls with significant edge restraint can be found from EC6 Part 1-1 Clauses 5.5.1.2 (11) (iii) and (iv) and Annex D, but is not covered by this manual.

4.5.5 *Effective Thickness*

The effective thickness of a wall is a measure of its resistance to buckling when it is carrying a vertical load. For a plain single-skin wall, the effective thickness t_{ef} is equal to the actual wall thickness t. If the wall is stiffened by piers built into the wall, or if it is one leaf of a cavity wall built with adequate cavity ties, then t_{ef} is more than t and can be found as follows.

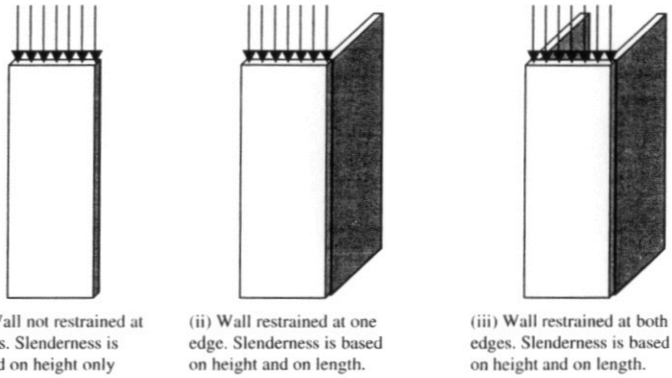


Figure 4.10: Effect of wall length and edge restraint on slenderness

Effective thickness of a wall stiffened by piers

A wall stiffened by piers has an effective thickness given by

$$t_{ef} = \rho_t t$$

where ρ_t is determined as shown in Figure 4.11 and Table 4.10. Figure 4.12 shows an example.

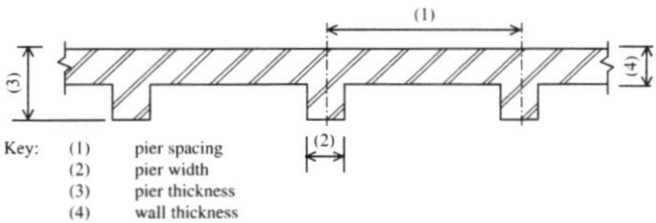


Figure 4.11: Definition of dimensions used in Table 4.10

Effective thickness of a leaf in a cavity wall

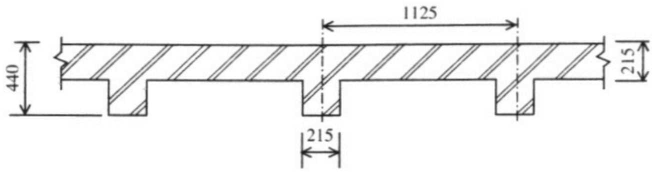
The effective thickness of a leaf in a cavity wall with adequate cavity ties (at least 2.5 ties per square metre of wall) is given by

$$t_{ef} = \sqrt[3]{t_1^3 + t_2^3}$$

Table 4.10: Stiffness coefficient ρ_t for wall stiffened by piers (see Figure 4.11)

Ratio of centre-to-centre pier spacing (1) to pier width (2)	Ratio of pier thickness (3) to wall thickness (4)		
	1	2	3
6	1.0	1.4	2.0
10	1.0	1.2	1.4
20	1.0	1.0	1.0

Linear interpolation between these values is permissible, but not extrapolation outside the ranges given.
Source: Table 5.1 of EC6 Part 1-1.



Pier spacing (1) = 1125 mm
Pier width (2) = 215 mm — (1)/(2) = 1125/215 = 5.2
Pier thickness (3) = 440 mm
Wall thickness (4) = 215 mm — (3)/(4) = 440/215 = 2.05

From Table 4.10, ρ_t = 1.43
t_{ef} = 215 x 1.43 = 307 mm

Figure 4.12: Example calculation of effective thickness for a wall stiffened by piers

where t_1 and t_2 are the actual thicknesses of the leaves, or their effective thicknesses if they are stiffened by piers.

Example: In a cavity wall with a 140 mm loadbearing inner leaf and a 102.5 mm outer leaf, the effective thickness of the loadbearing leaf is $t_{ef} = \sqrt[3]{(140^3 + 102.5^3)} = 156\,\text{mm}$.

4.5.6 Slenderness Ratio

As explained in Chapter 1, the ability of a structural element to carry compressive force is related to its slenderness ratio. A wall with a high slenderness ratio will not be able to carry as much vertical load as a similar wall with a low slenderness ratio.

The slenderness ratio of a wall is taken as h_{ef}/t_{ef}, where

h_{ef} is the effective height of the wall

and

t_{ef} is the effective thickness of the wall

The slenderness ratio of a wall carrying vertical load should not exceed 27.

4.5.7 Load Eccentricity

Eccentricity of loading reduces the capacity of the wall, as shown in Figure 4.13.

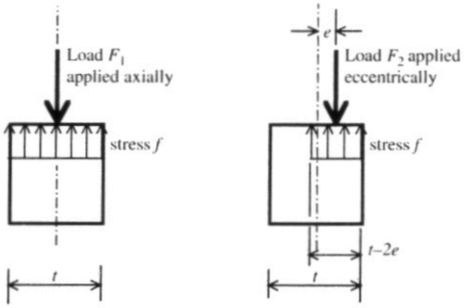


Figure 4.13: Effect of load eccentricity on wall capacity

The maximum stress f is limited by the strength of the masonry and is the same for both cases, so for a unit length of wall

$$F_1 = f \times t \qquad F_2 = f \times (t - 2e)$$

So

$$F_2/F_1 = (t - 2e)/t = 1 - 2e/t.$$

In this case the capacity reduction factor Φ_s is $1 - 2e/t$.

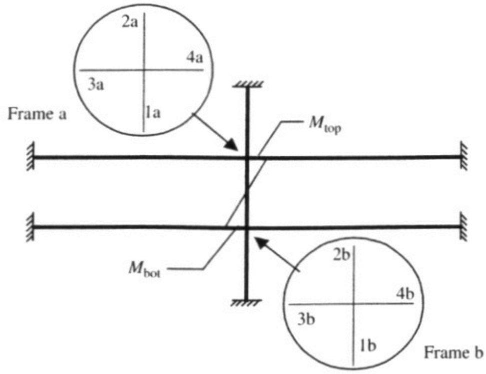
Calculation of the eccentricity e requires calculation of the moments in the walls caused by the loads on the floors or roofs, as shown in the following sections.

Moments in walls

Annex C of EC6 Part 1-1 gives the following method for the calculation of moments at the top or bottom of a wall panel. The structure is simplified to the sub-frame shown in Figure 4.14 which is reproduced from EC6 Part 1-1 Figure C1. Where fewer than four members meet at a joint those not existing should be ignored, as shown in Tables 4.11(a) and 4.11(b).

The moment M_{top} is found from the formula

$$M = \eta \frac{n_1 \dfrac{E_1 I_1}{h_1}}{n_1 \dfrac{E_1 I_1}{h_1} + n_2 \dfrac{E_2 I_2}{h_2} + n_3 \dfrac{E_3 I_3}{l_3} + n_4 \dfrac{E_4 I_4}{l_4}} \left[\frac{w_3 l_3^2}{4(n_3 - 1)} - \frac{w_4 l_4^2}{4(n_4 - 1)} \right]$$



Note: Moment M_{top} is found from Frame a and Moment M_{bot} is found from Frame b

Figure 4.14: Simplified frame diagram from Figure C1 of EC6 Part 1-1

using loads and dimensions as shown in Table 4.11(a)(a), and a similar formula is used for M_{bot}.

n_1, n_2, n_3 and n_4 are the stiffness factors of members, taken as 4 for members fixed at both ends and otherwise 3. The remote ends of members can be considered as fixed unless they are known to take no moment at all, so for most cases it will be satisfactory to take all the n values as 4 and the equation becomes:

$$M = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}} \left[\frac{w_3 l_3^2}{12} - \frac{w_4 l_4^2}{12} \right]$$

The η factor in the formula is used to reduce the eccentricity, and hence also the moment, by taking into account that the wall/floor junction will not be fully rigid. η is given by

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

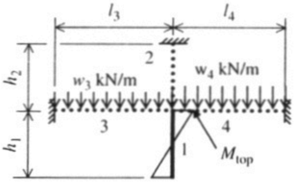
Different configurations of walls, floors and roofs require different versions of this equation. The different versions are shown in Tables 4.11(a) and 4.11(b), and Figure 4.15 shows which version is appropriate for finding the top and bottom moments in wall panels in a multi-storey building. It should be noted that a wall supporting a symmetric arrangement of floors or roofs will have $w_3 = w_4$ and $l_3 = l_4$ so that $M = 0$.

E values for walls and for concrete floors or roofs: for calculating the moments in the wall.

EC6 Part 1-1 Clause NA 2.9 gives the E value for a masonry wall as $f_k \times 10^3$.

Table 4.11(a): Formulas for the moment at the top of a wall panel (the dimensions and loads shown in the first diagram apply to all the diagrams)

(a)

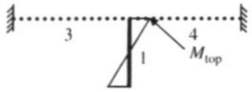


$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}} \left[\frac{w_3 l_3^2}{12} - \frac{w_4 l_4^2}{12}\right]$$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

Note that if $l_3 = l_4$ and $w_3 = w_4$ then $M_{top} = 0$

(b)

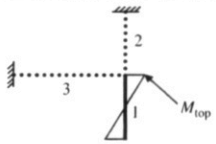


$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}} \left[\frac{w_3 l_3^2}{12} - \frac{w_4 l_4^2}{12}\right]$$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1}\right)} \text{ but not less than 0.5}$$

Note that if $l_3 = l_4$ and $w_3 = w_4$ then $M_{top} = 0$

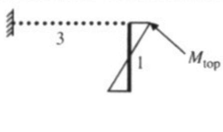
(c)



$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_3 I_3}{l_3}} \left[\frac{w_3 l_3^2}{12}\right]$$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

(d)



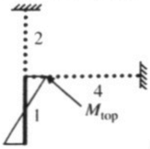
$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_3 I_3}{l_3}} \left[\frac{w_3 l_3^2}{12}\right]$$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3}}{4\left(\dfrac{E_1 I_1}{h_1}\right)} \text{ but not less than 0.5}$$

(*Continued*)

Table 4.11(a): (Continued)

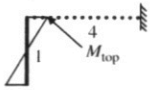
(e)



$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_4 I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right]$$

$$\eta = 1 - \frac{\dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

(f)

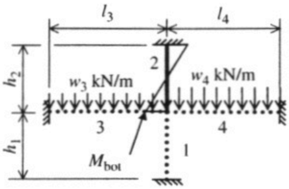


$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_4 I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right]$$

$$\eta = 1 - \frac{\dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1}\right)} \text{ but not less than 0.5}$$

Table 4.11(b): Formulas for the moment at the bottom of a wall panel (the dimensions and loads shown in the first diagram apply to all the diagrams)

(a)

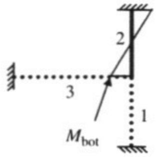


$$M_{bot} = \eta \frac{\dfrac{E_2 I_2}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}} \left[\frac{w_3 l_3^2}{12} - \frac{w_4 l_4^2}{12} \right]$$

Note that if $l_3 = l_4$ and $w_3 = w_4$ then $M_{bot} = 0$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3} + \dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

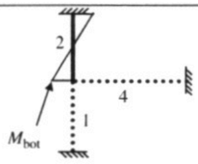
(b)



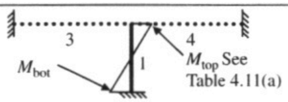
$$M_{bot} = \eta \frac{\dfrac{E_2 I_2}{h_2}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_3 I_3}{l_3}} \left[\frac{w_3 l_3^2}{12} \right]$$

$$\eta = 1 - \frac{\dfrac{E_3 I_3}{l_3}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} \text{ but not less than 0.5}$$

(c)

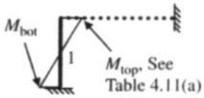
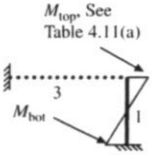
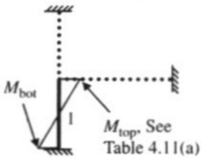
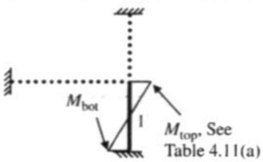


(d)



(Continued)

Table 4.11(b): (Continued)

$$M_{bot} = \eta \frac{\frac{E_2I_2}{h_2}}{\frac{E_1I_1}{h_1} + \frac{E_2I_2}{h_2} + \frac{E_4I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right]$$ $$\eta = 1 - \frac{\frac{E_4I_4}{l_4}}{4\left(\frac{E_1I_1}{h_1} + \frac{E_2I_2}{h_2}\right)}$$ but not less than 0.5	$$M_{bot} = \frac{M_{top}}{2}$$ Note that if $l_3 = l_4$ and $w_3 = w_4$ then $M_{bot} = 0$
(e)  $$M_{bot} = \frac{M_{top}}{2}$$	(f)  $$M_{bot} = \frac{M_{top}}{2}$$
(g)  $$M_{bot} = \frac{M_{top}}{2}$$	(h)  $$M_{bot} = \frac{M_{top}}{2}$$

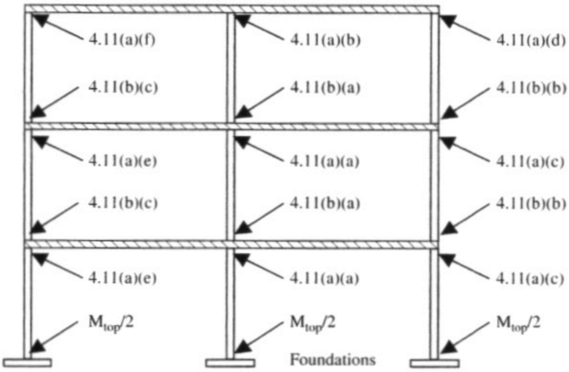


Figure 4.15: Reference to Tables 4.11(a) and 4.11(b) for top and bottom moments in wall panels in a multi-storey building

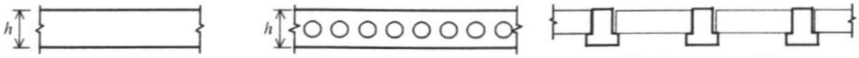
E values for structural concrete in floor or roof slabs can be obtained from Table 4.12.

Table 4.12: E values for structural concrete floors and roofs

Concrete grade	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60
E (N/mm^2)	30 000	31 000	33 000	34 000	35 000	36 000	37 000

Source: Table 3.1 of EC2 Part 1-1, formula $E = 22\,000(0.8 + f_{ck}/10)^{0.3}$.

***I* values for walls and for concrete floors or roofs**: for calculating the moments in the wall (Figure 4.16).



Solid slab
$I = 1000h^3/12\,\text{mm}^4/\text{m}$

Precast hollow-cored slab
I from manufacturer
or
$I = 0.85 \times 1000h^3/12\,\text{mm}^4/\text{m}$
for units not more than
200 mm thick

Block and beam slab
I from manufacturer

Figure 4.16: *I* values for concrete slabs

The I value for a masonry wall can be taken as $1000t^3/12\,\text{mm}^4$ per metre width, where t is the actual wall thickness in millimetres (not the effective thickness).

For solid reinforced concrete floor or roof slabs the I value is $1000h^3/12\,\text{mm}^4$ per metre width, where h is the overall slab thickness in millimetres. If hollow-cored pre-cast concrete floor or roof slabs are used then the I value can be obtained from the manufacturer, or alternatively for units not exceeding 200 mm deep an approximate $I = 0.85 \times 1000h^3/12\,\text{mm}^4$ per metre width can be used. For other types of concrete floor or roof the I value should be obtained from the manufacturer.

Calculation of the load eccentricities e and the corresponding slenderness reduction factors Φ_s is based on EC6 Part 1-1 Clause 6.1.2.2, and assumes that all walls are built with an initial eccentricity e_{init} which can be taken as $h_{ef}/450$. Design values of eccentricity are given by

$$e = \frac{M}{N} + e_{hm} \pm e_{init}$$

where M is the moment, N is the axial load and e_{hm} is the eccentricity due to horizontal forces on the wall panel. As previously stated this manual does not cover lateral loads on walls, so we can write

$$e = \frac{M}{N} \pm \frac{h_{ef}}{450}$$

The $\pm$ symbol means that the initial eccentricity $\frac{h_{ef}}{450}$ is assumed to add to, not reduce, the eccentricity $\frac{M}{N}$.

The clause also requires that the design value of e is always at least $0.05t$. If e is more than $0.45t$ then the method described here is not suitable and the method given in EC6 Part 1-1 Annex C Clause (5) should be used.

The critical section of a wall panel may be at the top or the bottom where the moments are large, or at the centre where the moments are usually smaller but wall buckling effects are more significant. As a result it is necessary to find the slenderness reduction factors Φ_s at top, middle and bottom of the wall panel and to use the smallest value.

At top of the wall $e_{top} = \frac{M_{top}}{N} \pm \frac{h_{ef}}{450}$ but not less than $0.05t$, and the slenderness reduction factor $\Phi_{s,top} = 1 - \frac{2e_{top}}{t}$ which is never more than 0.9.

Similarly at the bottom of the wall $e_{bot} = \frac{M_{bot}}{N} \pm \frac{h_{ef}}{450}$ but not less than $0.05t$ and $\Phi_{s,bot} = 1 - \frac{2e_{bot}}{t}$.

At the middle of the wall $e_{mid} = \frac{M_{mid}}{N} \pm \frac{h_{ef}}{450}$ but not less than $0.05t$. The slenderness reduction factor takes into account buckling effects and is given by $\Phi_{s,mid} = A_1 e^{-\frac{u^2}{2}}$ where

$$u = \frac{\frac{h_{ef}}{t_{ef}} - 2}{23 - 37\frac{e_{mid}}{t}} \text{ and } A_1 = 1 - 2\frac{e_{mid}}{t}.$$

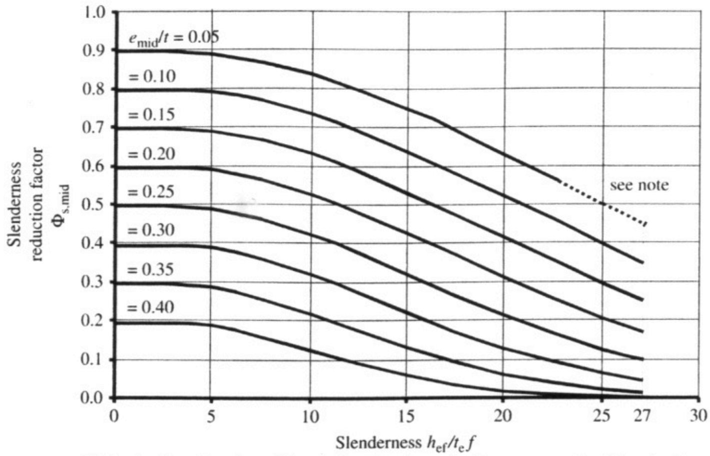
Note that in the formula for $\Phi_{s,mid}$, e is 2.718, the base of natural logarithms, not an eccentricity.

Figure 4.17 is a graph which allows $\Phi_{s,mid}$ to be read off against h_{ef}/t_{ef} and e_{mid}/t, and Table 4.13 gives values of $\Phi_{s,mid}$ when $M_{mid} = 0$.

The use of these formulas is shown in Section 4.6.

4.6 Calculation of Unit Strength and Mortar Grade Required to Carry a Vertical Load

The following design summary and Example 4.1 show how the principles explained in the previous sections are applied to a practical design situation. In some cases, for example step



Note: the dotted portion of the top line is not required because a wall with a slenderness exceeding 22.5 will always have e_{mid}/t greater than 0.05

Figure 4.17: Mid-height slenderness reduction factor $\Phi_{s,mid}$

Table 4.13: Mid-height slenderness reduction factor $\Phi_{s,mid}$ for walls carrying vertical load only and no moment

h_{ef}/t_{ef}	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.0
e_{mid}/t	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.056	0.060
$\Phi_{s,mid}$	0.900	0.900	0.891	0.870	0.838	0.796	0.746	0.689	0.628	0.565	0.495	0.439

(g)(ii) in the design summary, it is necessary to assume a parameter in order to proceed with the calculations. If completion of the calculations shows that the assumed parameter was incorrect then the designer will have to judge whether it is necessary to repeat the calculations with a revised parameter. Repeat of the calculations is not normally required unless the incorrect assumption compromises safety or economy.

Design summary for a vertically loaded wall or column using EC6 Part 1-1

(a) Find the ultimate applied load N kN per metre length of wall.

(b) Determine the effective height h_{ef} using Table 4.9.

(c) If the wall is part of a cavity wall or is stiffened by piers, calculate the effective thickness t_{ef}.

(d) Calculate the slenderness ratio h_{ef}/t_{ef} and check that it does not exceed 27.

(e) If the cross-section area of the wall A is less than $0.1\,m^2$, find $\Phi_A = (0.7 + 3A)$. Otherwise $\Phi_A = 1.0$.

(f) If the wall carries a symmetric arrangement of floors, roofs and loads, determine the slenderness reduction factor. $\Phi_s = \Phi_{s,mid}$ from Table 4.13.

(g) If the wall carries an asymmetric arrangement of floors, roofs or loads, then, taking t as the actual thickness of the wall loadbearing leaf (not the effective thickness):

(i) Determine EI values for the floors or roofs. E values for concrete can be taken from Table 4.12. I for a solid concrete floor of thickness h mm is $1000h^3/12\,\text{mm}^4/\text{m}$. I for hollow-core precast floor units not exceeding 200 mm deep can be taken as $0.85 \times 1000h^3/12\,\text{mm}^4/\text{m}$.

(ii) Assume a masonry compressive strength f_k and take E for the walls as $f_k \times 10^3\,\text{N/mm}^2$.

(iii) Take I for the wall as $1000t^3/12\,\text{mm}^4$. Using the appropriate formula from Table 4.11(a), determine M_{top} the moment at the top of the wall.

(iv) Using the appropriate formula from Table 4.11(b), determine M_{bot} the moment at the bottom of the wall.

(v) Calculate the moment at mid-height of the wall $M_{mid} = (M_{top} - M_{bot})/2$.

(vi) At the top of the wall calculate

$$\text{the eccentricity } e_{top} = \frac{M_{top}}{N} \pm \frac{h_{ef}}{450} \text{ but not less than } 0.05t$$

$$\text{and the slenderness reduction factor } \Phi_{s,top} = 1 - \frac{2e_{top}}{t}.$$

(vii) At mid-height of the wall calculate

$$\text{the eccentricity } e_{mid} = \frac{M_{mid}}{N} \pm \frac{h_{ef}}{450} \text{ but not less than } 0.05t$$

and find the slenderness reduction factor $\Phi_{s,mid}$ from Figure 4.17.

(viii) At the bottom of the wall calculate

$$\text{the eccentricity } e_{bot} = \frac{M_{bot}}{N} \pm \frac{h_{ef}}{450} \text{ but not less than } 0.05t$$

$$\text{and the slenderness reduction factor } \Phi_{s,bot} = 1 - \frac{2e_{bot}}{t}.$$

(ix) If any of the eccentricities are more than $0.45t$ then this method cannot be used. Refer to EC3 Part 1-1 Annex C Clause (5).

(x) Take the slenderness factor Φ_s as the smallest of $\Phi_{s,top}$, $\Phi_{s,mid}$ and $\Phi_{s,bot}$.

(h) Calculate the masonry design strength f_d required from $f_d = (N/A)/(\Phi_A \Phi_s)$.

(i) Select the material partial safety factor γ_m from Table 4.7.

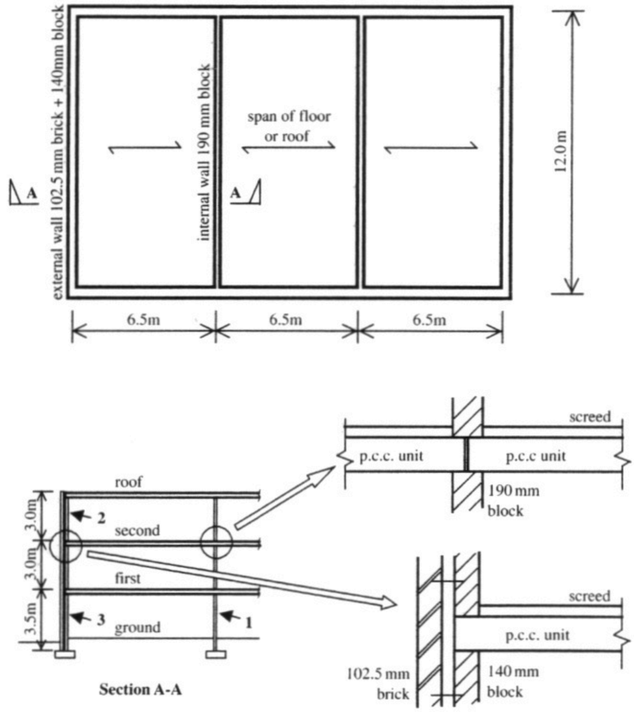
(j) Calculate the masonry characteristic strength f_k required from $f_k = f_d \times \gamma_m$.

(k) Using Table 4.6, choose a type of unit, normalized unit strength f_b and a grade of mortar to provide at least this strength.

(l) If required, find the actual mean strength $= f_b/\delta$, where δ is from Table 4.4.

Example 4.1

Find appropriate block strengths and mortar grades for the wall panels noted as 1, 2 and 3 in the building shown in Figure 4.18. In relation to the external cavity walls, all vertical loads are carried by the inner leaf. As this manual does not cover the design of walls for lateral loads, wind loading calculations are not included.



Note: **1**, **2** and **3** are the critical wall panels for vertical load design.

Figure 4.18: Building for Example 4.1

Structure data

Internal walls	190 mm solid blockwork
External walls	102.5 mm brickwork + cavity + 140 mm solid blockwork
Masonry units	Category I
Execution control	Class 2
Weight of 140 mm solid block walls:	2.3 kN/m^2
Weight of 190 mm solid block walls:	2.6 kN/m^2

Roof and floors: 200 mm thick pre-cast hollow-cored concrete units, weight 3.1 kN/m^2, Grade C45/55 concrete

50 mm concrete screed, weight 1.2 kN/m^2

Imposed loads: roof 1.5 kN/m^2

floors 3.5 kN/m^2

Calculations for Example 4.1

From Table 4.7 with manufacturing category I and execution control class 2 $\gamma_m = 2.7$

Wall Panel 1: supports 6.5 m spans of two floors and a roof

Loads per metre length of wall		Dead loads	Imposed loads
pcc units	$6 \times (6.5/2) \times 3.1 =$	60.5 kN/m	–
Screed	$6 \times (6.5/2) \times 1.2 =$	23.4 kN/m	–
Imposed load: roof	$2 \times (6.5/2) \times 1.5 =$	–	9.8 kN/m
Imposed load: floors	$4 \times (6.5/2) \times 3.5 =$	–	45.5 kN/m
Self-weight of blockwork (at mid-height)	$(3.0 + 3.0 + 3.5/2) \times 2.6 =$	20.2 kN/m	–
	Totals	104.1 kN/m	55.3 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$

$= 1.35 \times 104.1 + 1.50 \times 55.3$ $N = 223.5$ kN/m

Clear height $h = 3500 - 200$ $h = 3300$ mm

The concrete floors are built into the supporting walls, so from Table 4.9 $\rho = 0.75$

Effective height $h_{ef} = \rho h = 0.75 \times 3300$ $h_{ef} = 2475$ mm

Effective thickness t_{ef} = actual thickness t $t_{ef} = 190$ mm

Slenderness $h_{ef}/t_{ef} = 2475/190$ $h_{ef}/t_{ef} = 13.0$

Check that slenderness is not more than 27 Accept

The area of the wall is more than 0.1 m^2 $\Phi_A = 1.0$

The wall carries a symmetric arrangement of floors, roofs and loads, so from Table 4.13 (see Section 4.7) with $h_{ef}/t_{ef} = 13.0$, by interpolation $\Phi_s = \Phi_{s,mid} = 0.786$

Load/area $= N/A = 223.5 \times 10^3/(1000 \times 190) = 1.18$ N/mm^2

Required value of $f_d = (F/A)/(\Phi_A\Phi_s) = 1.18/(1.0 \times 0.786)$ $f_d = 1.50$ N/mm^2

Required value of $f_k = f_d \times \gamma_m = 1.50 \times 2.7$ $f_k = 4.05$ N/mm^2

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Calculations for Example 4.1 (Continued from previous page)

From Table 4.6 use Group 1 concrete blocks with $f_b = 8.3\,N/mm^2$ in M6 mortar giving $f_k = 4.1\,N/mm^2$

Use Group 1 concrete blocks with normalised compressive strength $f_b = 8.3\,N/mm^2$ in M6 mortar

From Table 4.4 the normalised compressive strength $f_b = \delta \times$ actual compressive strength. For a 215 mm high × 190 mm wide block $\delta = 1.20$, so the required actual compressive strength $= f_b/\delta = 8.3/1.20$

Actual compressive strength of unit $= 6.9\,N/mm^2$

Wall Panel 2: supports $6.5/2 = 3.25$ m span of roof

Load per metre length of wall on inner leaf only

Note that the outer leaf of brickwork is self-supporting so its weight is not included

		Dead loads	Imposed loads
pcc units	$(6.5/2) \times 3.1 =$	10.1 kN/m	–
Screed	$(6.5/2) \times 1.2 =$	3.9 kN/m	–
Imposed load: roof	$(6.5/2) \times 1.5 =$	–	4.9 kN/m
Self-weight of blockwork (at mid-height)	$(3.0/2) \times 2.3 =$	3.5 kN/m	–
	Totals	17.5 kN/m	4.9 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$
$= 1.35 \times 17.5 + 1.50 \times 4.9$ — $N = 31.0\,kN/m$

Clear height $h = 3000 - 200$ — $h = 2800\,mm$

From Table 4.9, with concrete roof bearing on the full width of the wall — $\rho = 0.75$

Effective height $h_{ef} = \rho h = 0.75 \times 2800$ — $h_{ef} = 2100\,mm$

Effective thickness $t_{ef} = \sqrt[3]{(t_1^3 + t_2^3)} = \sqrt[3]{(140^3 + 102.5^3)}$ — $t_{ef} = 156\,mm$

Slenderness $h_{ef}/t_{ef} = 2100/156$ — $h_{ef}/t_{ef} = 13.5$

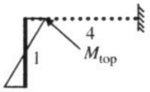
Check that slenderness is not more than 27 — Accept

The area of the wall is more than $0.1\,m^2$ — $\Phi_A = 1.0$

The wall carries an asymmetric arrangement of floors, roofs or loads, so we must use the formulas in Tables 4.11(a) and 4.11(b)

Moment at the top of the wall

Using diagram (f) from Table 4.11(a)



$$M_{top} = \eta \frac{\frac{E_1 I_1}{h_1}}{\frac{E_1 I_1}{h_1} + \frac{E_4 I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right]$$

$w_4 = 1.35(3.1 + 1.2) + 1.50 \times 1.50$ — $w_4 = 8.1\,kN/m$

<u>For the wall</u>

In order to proceed with the calculations we need a value for E_1, the E value of the masonry in the wall. Make the initial assumption that the strength of the masonry in this wall panel is the same as the strength of the masonry in wall panel 1, $f_k = 4.05\,N/mm^2$

$E_1 = f_k \times 10^3 = 4.05 \times 10^3$ — Assumed $E_1 = 4050\,\text{N/mm}^2$

$I_1 = 1000 \times 140^3/12$ — $I_1 = 229 \times 10^6\,\text{mm}^2\text{/m}$

h_1 = clear height — $h_1 = 2800\,\text{mm}$

$E_1\,I_1/h_1 = 4050 \times 229 \times 10^6/2800 = 331 \times 10^6\,\text{Nmm}$ — $E_1\,I_1/h_1 = 331\,\text{kNm}$

<u>For the roof</u>

From Table 4.12 with concrete grade C45/55 — $E_4 = 36\,000\,\text{N/mm}^2$

$I_4 = 0.85 \times 1000 \times 200^3/12$ — $I_4 = 567 \times 10^6\,\text{mm}^2\text{/m}$

l_4 = span of roof — $l_4 = 6500\,\text{mm}$

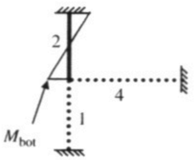
$E_4\,I_4/l_4 = 36\,000 \times 567 \times 10^6/6500 = 3140 \times 10^6\,\text{Nmm}$ — $E_4\,I_4/l_4 = 3140\,\text{kNm}$

$$\eta = 1 - \frac{\dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1}\right)} = 1 - \frac{3140}{4 \times 331} = -1.37,\ \text{but } \eta \text{ must not be less than 0.5 so}$$ — $\eta = 0.5$

$$M_{\text{top}} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_4 I_4}{l_4}}\left[-\frac{w_4 l_4^2}{12}\right] = 0.5 \times \frac{331}{331 + 3140}\left[-\frac{8.1 \times 6.5^2}{12}\right] = -1.3\ \text{kNm}$$ — $M_{\text{top}} = -1.3\,\text{kNm}$

Moment at the bottom of the wall

Using diagram (e) from Table 4.11(b)



$$M_{\text{bot}} = \eta \frac{\dfrac{E_2 I_2}{h_2}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_4 I_4}{l_4}}\left[-\frac{w_4 l_4^2}{12}\right]$$

$w_4 = 1.35(3.1 + 1.2) + 1.50 \times 3.50$ — $w_4 = 11.1\,\text{kN/m}$

Parameters for the walls are the same as for the top moment calculation — $E_1 I_1/h_1 = 331\,\text{kNm}$; $E_2 I_2/h_2 = 331\,\text{kNm}$

Parameters for the floor are the same as for the roof in the top moment calculation — $E_4 I_4/l_4 = 3140\,\text{kNm}$

$$\eta = 1 - \frac{\dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} = 1 - \frac{3140}{4(331 + 331)} = -0.18\ \text{but } \eta \text{ must not be less than 0.5, so}$$ — $\eta = 0.5$

$$M_{\text{bot}} = \eta \frac{\dfrac{E_2 I_2}{h_2}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_4 I_4}{l_4}}\left[-\frac{w_4 l_4^2}{12}\right] = 0.5 \times \frac{331}{331 + 331 + 3140}\left[-\frac{11.1 \times 6.5^2}{12}\right] = -1.7\ \text{kNm}$$

$M_{\text{bot}} = -1.7\,\text{kNm}$

Moment at mid-height of the wall

Moment at mid-height $M_{\text{mid}} = (M_{\text{top}} - M_{\text{bot}})/2 = (-1.3 - (-1.7))/2 =$ — $M_{\text{mid}} = 0.20\,\text{kNm}$

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Calculations for Example 4.1 (Continued from previous page)

Slenderness reduction factors

Top eccentricity $e_{top} = \dfrac{M_{top}}{N} \pm \dfrac{h_{ef}}{450} = \dfrac{-1.3 \times 10^6}{31.0 \times 10^3} \pm \dfrac{2100}{450}$

$e_{top} = -41.9 \pm 4.7$. Take the larger actual value — $e_{top} = 46.6\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ — Accept

Top slenderness reduction factor $\Phi_{s,top} = 1 - \dfrac{2e_{top}}{t} = 1 - \dfrac{2 \times 46.6}{140}$ — $\Phi_{s,top} = 0.33$

Mid-height eccentricity $e_{mid} = \dfrac{M_{mid}}{N} \pm \dfrac{h_{ef}}{450} = \dfrac{0.20 \times 10^6}{31.0 \times 10^3} \pm \dfrac{2100}{450}$

$e_{mid} = +6.4 \pm 4.7\,\text{mm}$, take the larger actual value — $e_{mid} = 11.1\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ — Accept

Mid-height slenderness reduction factor $\Phi_{s,mid}$ from Figure 4.17 with $h_{ef}/t_{ef} = 13.5$ and $e_{mid}/t = 11.1/140 = 0.080$ — $\Phi_{s,mid} = 0.72$

Bottom eccentricity $e_{bot} = \dfrac{M_{bot}}{N} \pm \dfrac{h_{ef}}{450} = \dfrac{-1.7 \times 10^6}{31.0 \times 10^3} \pm \dfrac{2100}{450}$

$e_{bot} = -54.8 \pm 4.7\,\text{mm}$, take the larger actual value — $e_{bot} = 59.5\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ — Accept

Bottom slenderness reduction factor $\Phi_{s,bot} = 1 - \dfrac{2e_{bot}}{t} = 1 - \dfrac{2 \times 59.5}{140}$ — $\Phi_{s,bot} = 0.15$

Actual slenderness reduction factor = the smallest of $\Phi_{s,top}$, $\Phi_{s,mid}$ and $\Phi_{s,bot}$ — $\Phi_s = 0.15$

Load/area = $F/A = 31.0 \times 10^3/(1000 \times 140) = 0.22\,\text{N/mm}^2$

Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 0.22/(1.0 \times 0.15)$ — $f_d = 1.5\,\text{N/mm}^2$

As before — $\gamma_m = 2.7$

Required value of $f_k = 1.5 \times 2.7$ — $f_k = 4.05\,\text{N/mm}^2$

From Table 4.6 use Group 1 concrete blocks with $f_b = 8.3\,\text{N/mm}^2$ in M6 mortar giving $f_k = 4.1\,\text{N/mm}^2$ — Use Group 1 concrete blocks with normalised compressive strength $f_b = 8.3\,\text{N/mm}^2$ in M6 mortar

From Table 4.4 for a 215-mm-high × 140-mm-wide block the normalised compressive strength $f_b = 1.30 \times$ actual compressive strength
Required actual compressive strength = 8.3/1.30 — Actual compressive strength of unit = $6.4\,\text{N/mm}^2$

Wall Panel 3: supports 6.5/2 = 3.25-m spans of two floors and one roof
Loads per metre length of wall on inner leaf only
Note that the outer leaf of brickwork is self-supporting so its weight is not included

		Dead loads	Imposed loads
pcc units	3 × (6.5/2) × 3.1 =	30.2 kN/m	–
Screed	3 × (6.5/2) × 1.2 =	11.7 kN/m	–
Imposed load: roof	(6.5/2) × 1.5 =	–	4.9 kN/m
Imposed load: floors	2 × (6.5/2) × 3.5 =	–	22.8 kN/m
Self-weight of blockwork	(3.0 + 3.0 + 3.5/2) × 2.3 =	17.8 kN/m	–
	Totals	59.7 kN/n	27.7 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$
$= 1.35 \times 59.7 + 1.50 \times 27.7$ $N = 122.1\,\text{kN/m}$

Clear height $h = 3500 - 200$ $h = 3300\,\text{mm}$

From Table 4.9, with concrete floor bearing on the full width of the wall $\rho = 0.75$

Effective height $h_{ef} = \rho h = 0.75 \times 3300$ $h_{ef} = 2475\,\text{mm}$

Effective thickness $t_{ef} = \sqrt[3]{(t_1^3 + t_2^3)} = \sqrt[3]{(140^3 + 102.5^3)}$ $t_{ef} = 156\,\text{mm}$

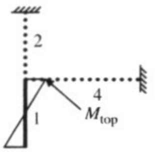
Slenderness ratio $h_{ef}/t_{ef} = 2475/156$ $h_{ef}/t_{ef} = 15.9$

The area of the wall is more than $0.1\,\text{m}^2$ $\Phi_A = 1.0$

The wall carries an asymmetric arrangement of floors, roofs or loads, so we must use the formulas in Tables 4.11(a) and 4.11(b)

Moment at the top of the wall

Using diagram (e) from Table 4.11(a)



$$M_{top} = \eta \frac{\dfrac{E_1 I_1}{h_1}}{\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2} + \dfrac{E_4 I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right]$$

$w_4 = 1.35(3.1 + 1.2) + 1.50 \times 3.50$ $w_4 = 11.1\,\text{kN/m}$

For the walls

In order to proceed with the calculations we need a value for E_1, the E value of the masonry in the wall. Make the initial assumption that the strength of the masonry in this wall panel is the same as the strength of the masonry in wall panel 1, $f_k = 4.05\,\text{N/mm}^2$

$E_1 = E_2 = f_k \times 10^3 = 4.05 \times 10^3$ $E_1 = E_2 = 4050\,\text{N/mm}^2$
$I_1 = I_2 = 1000 \times 140^3/12$ $I_1 = I_2 = 229 \times 10^6\,\text{mm}^2/\text{m}$
h_1 = clear height of lower panel $h_1 = 3300\,\text{mm}$
h_2 = clear height of upper panel $h_2 = 2800\,\text{mm}$
$E_1 I_1/h_1 = 4050 \times 229 \times 10^6/3300 = 281 \times 10^6\,\text{Nmm}$ $E_1 I_1/h_1 = 281\,\text{kNm}$
$E_2 I_2/h_2 = 4050 \times 229 \times 10^6/2800 = 331 \times 10^6\,\text{Nmm}$ $E_2 I_2/h_2 = 331\,\text{kNm}$

For the floor

From Table 4.12 with $f_{ck} = 45\,\text{N/mm}^2$ $E_4 = 36\,000\,\text{N/mm}^2$
$I_4 = 0.85 \times 1000 \times 200^3/12$ $I_4 = 567 \times 10^6\,\text{mm}^2/\text{m}$
l_4 = span of roof $l_4 = 6500\,\text{mm}$
$E_4 I_4/l_4 = 36\,000 \times 567 \times 10^6/6500 = 3140 \times 10^6\,\text{Nmm}$ $E_4 I_4/l_4 = 3140\,\text{kNm}$

$$\eta = 1 - \frac{\dfrac{E_4 I_4}{l_4}}{4\left(\dfrac{E_1 I_1}{h_1} + \dfrac{E_2 I_2}{h_2}\right)} = 1 - \frac{3140}{4(281 + 331)} = -0.28,$$ but η must not be less than 0.5 so $\eta = 0.5$

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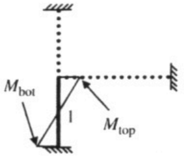
Calculations for Example 4.1 (Continued from previous page)

$$M_{top} = \eta \frac{\frac{E_1 I_1}{h_1}}{\frac{E_1 I_1}{h_1} + \frac{E_2 I_2}{h_2} + \frac{E_4 I_4}{l_4}} \left[-\frac{w_4 l_4^2}{12} \right] = 0.5 \times \frac{281}{281 + 331 + 3140} \left[\frac{-11.1 \times 6.5^2}{12} \right]$$

$M_{top} = -1.5\,\text{kNm}$

Moment at the bottom of the wall

Using diagram (g) from Table 4.11(b)



$$M_{bot} = \frac{M_{top}}{2} = \frac{-1.5}{2} = -0.75\,\text{kNm}$$

Moment at mid-height of the wall

Moment at mid-height $M_{mid} = (M_{top} - M_{bot})/2 = (-1.5 + 0.75)/2 =$ $M_{mid} = -0.38\,\text{kNm}$

Slenderness reduction factors

Top eccentricity $e_{top} = \dfrac{M_{top}}{N} \pm \dfrac{h_{ef}}{450} = \dfrac{-1.5 \times 10^6}{122.1 \times 10^3} \pm \dfrac{2475}{450}$

$e_{top} = -12.3 \pm 5.5$. Take the larger actual value $e_{top} = 17.8\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ Accept

Top slenderness reduction factor $\Phi_{s,top} = 1 - \dfrac{2e_{top}}{t} = 1 - \dfrac{2 \times 17.8}{140}$ $\Phi_{s,top} = 0.75$

Mid-height eccentricity $e_{mid} = \dfrac{M_{mid}}{N} \pm \dfrac{h_{ef}}{450} = \dfrac{-0.38 \times 10^6}{122.1 \times 10^3} \pm \dfrac{2475}{450}$

$e_{mid} = -3.1 \pm 5.5$. Take the larger actual value $e_{mid} = 8.6\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ Accept

Mid-height slenderness reduction factor $\Phi_{s,mid}$ from Figure 4.17 with $h_{ef}/t_{ef} = 15.9$ and $e_{mid}/t = 8.6/140 = 0.06$ $\Phi_{s,mid} = 0.70$

Bottom eccentricity $e_{bot} = \dfrac{M_{bot}}{N} + \dfrac{h_{ef}}{450} = \dfrac{-0.75 \times 10^6}{122.1 \times 10^3} + \dfrac{2475}{450}$

$e_{bot} = -6.1 \pm 5.5$. Take the larger actual value $e_{bot} = 11.6\,\text{mm}$

Check that e is between $0.05t = 7\,\text{mm}$ and $0.45t = 63\,\text{mm}$ Accept

Bottom slenderness reduction factor $\Phi_{s,bot} = 1 - \dfrac{2e_{bot}}{t} = 1 - \dfrac{2 \times 11.6}{140}$ $\Phi_{s,bot} = 0.83$

Actual slenderness reduction factor = the smallest of $\Phi_{s,top}$, $\Phi_{s,mid}$ and $\Phi_{s,bot}$ $\Phi_s = 0.70$

Load/area = $N/A = 122.1 \times 10^3/(1000 \times 140) = 0.87\,\text{N/mm}^2$

Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 0.87/(1.0 \times 0.70)$ $f_d = 1.25\,\text{N/mm}^2$

As before $\gamma_m = 2.7$

Required value of $f_k = 1.25 \times 2.7$ $f_k = 3.4\,\text{N/mm}^2$

The units and mortar specified for Wall Panel 2 have $f_k = 4.1\,\text{N/mm}^2$, so these will be adequate for Wall Panel 3 Masonry as Wall Panel 2

4.7 Calculation of Unit Strength and Mortar Grade Required to Carry a Vertical Load Using the Simplified Method of EC6 Part 3

EC6 Part 3 gives a simplified method for calculating the vertical load capacity of a wall, using a single capacity reduction factor Φ_s which takes account of both slenderness and eccentricity effects. Φ_s is larger for intermediate walls (which carry load from both sides) than for end supports (which carry load from only one side) because in the latter case the total load is more eccentric. This is illustrated in Figure 4.19.

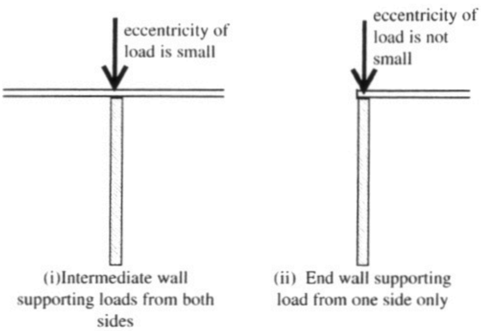


Figure 4.19: Φ_s factors for intermediate and end walls

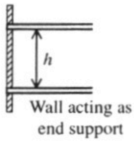
The simplified method of EC6 Part 3 can only be used if a number of conditions are met. The conditions, which will be satisfied by many masonry structures, are listed in Table 4.14.

Table 4.14: Conditions to be satisfied if the simplified method of EC6 Part 3 is to be used

Condition No.	Condition		
1	The height of the building shall not exceed 12.0 m. If the building has a pitched roof, the average of the eaves height and the ridge height shall not exceed 12.0 m		
2	The span of the floors supported on the walls shall not exceed 7.0 m		
3	For walls acting as end supports to floors, the span of the floor shall not exceed the values in the table below		
	Thickness of loadbearing leaf	Maximum span of floor supported by the wall	
		f_d more than 2.5 N/mm^2	f_d = 2.5 N/mm^2 or less
	100 mm 140 mm	5.5 m 5.9 m	5.5 m 5.9 m

(*Continued*)

Table 4.14: (Continued)

<table>
<tr><th>Condition No.</th><th colspan="3">Condition</th></tr>
<tr><td rowspan="2"></td><td>190 mm
200 mm
215 mm</td><td>6.1 m
6.5 m
7.0 m</td><td>6.0 m
6.0 m
6.0 m</td></tr>
<tr><td colspan="3">If the wall is lightly loaded so that
N_{ed} is not more than $0.2Af_d$ (Group 1 masonry units)
or N_{ed} is not more than $0.1Af_d$ (Group 2 masonry units)
then the maximum floor span is 7.0 m in all cases</td></tr>
<tr><td>4</td><td colspan="3">The span of a roof supported by the walls shall not exceed 7.0 m, except that in the case of a lightweight trussed roof system the span shall not exceed 14.0 m</td></tr>
<tr><td>5</td><td colspan="3">The characteristic variable actions on the floors and the roof shall not exceed 5.0 kN/m^2</td></tr>
<tr><td>6</td><td colspan="3">The walls shall be laterally restrained by the floors and roof in a horizontal direction at right angles to the plane of the wall (see Figure 4.8)</td></tr>
<tr><td>7</td><td colspan="3">The walls shall be vertically aligned throughout their height</td></tr>
<tr><td>8</td><td colspan="3">The floors and roof shall have a bearing on the wall of at least $0.4t$, where t is the thickness of the wall, but not less than 75 mm</td></tr>
<tr><td rowspan="4">9</td><td colspan="3">If the wall is an end support to a floor or roof and is subject to wind load, its thickness t shall satisfy
$t >= c_1 q_{Ewd} bh^2/N_{Ed} + c_2 h$


where
h is the clear storey height
q_{Ewd} is the design wind load on the wall per m^2 of wall
N_{Ed} is the design value of vertical load giving the <u>least</u> severe effect on the wall at the top of the storey considered
b is the width over which the vertical load is effective
t is the actual thickness of the wall, or of the loadbearing leaf of a cavity wall
c_1, c_2 are constants derived from this table</td></tr>
<tr><td>Load ratio $\alpha = N_{Ed}/(tbf_d)$</td><td>c_1</td><td>c_2</td></tr>
<tr><td>0.05
0.10
0.20
0.30
0.50</td><td>0.12
0.12
0.14
0.15
0.23</td><td>0.017
0.019
0.022
0.025
0.031</td></tr>
<tr><td colspan="3">Linear interpolation is permitted.
Source: Table 4.1 of EC6 Part 3.
If the wall is lightly loaded so that N_{ed} is not more than $0.2Af_d$ (Group 1 masonry units) or N_{ed} is not more than $0.1Af_d$ (Group 2 masonry units) then the effects of vertical load may be ignored and the wall may be designed for lateral load only. Design of walls for lateral load is not covered by this manual.</td></tr>
</table>

The designer can check conditions 1 to 8 before starting the calculations, but condition 9 can only be investigated when the other calculations have been completed and a value of f_d has been chosen. If at this stage in the design process condition 9 is not met, then the following options should be considered:

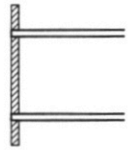
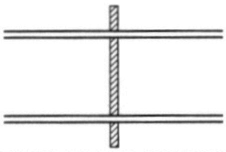
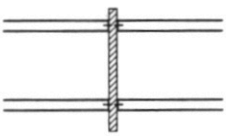
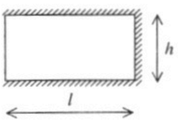
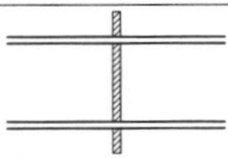
Option 1: Increase the thickness t of the wall.

Option 2: Increase the masonry strength f_d. This will reduce the minimum thickness required by condition 9, but only by a small amount.

Option 3: Abandon the simplified methods of EC6 Part 3 and investigate the wall using the more complex methods of EC6 Part 1-1.

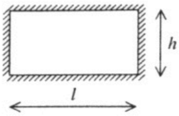
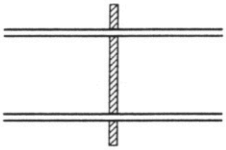
In the simplified method of EC6 Part 3 the effective height of a wall is determined as shown in Table 4.15.

Table 4.15: Effective height reduction factor ρ in the simplified method of EC6 Part 3

Condition	Diagram	Description	ρ
Wall laterally and rotationally restrained at top and bottom by reinforced or pre-stressed concrete floor or roof with a bearing of at least 2/3 of the wall thickness but not less than 85 mm		Wall acting as end support to a floor	1.0
		All other walls	0.75
Wall laterally restrained at top and bottom (e.g. by ring beams of appropriate stiffness or by timber floors) but not rotationally restrained		All walls	1.0
Wall laterally restrained at top and bottom and at one vertical edge 		If the wall is not an end wall and has rotational restraint	$1.5(l/h)$ but not more than 0.75
		All other walls	$1.5(l/h)$ but not more than 1.0

(*Continued*)

Table 4.15: (Continued)

Condition	Diagram	Description	ρ
Wall laterally restrained at top and bottom and at two vertical edges 		If the wall is not an end wall and has rotational restraint	$0.5(l/h)$ but not more than 0.75
		All other walls	$0.5(l/h)$ but not more than 1.0

Source: EC6 Part 3 Clause 4.2.2.4.

The ρ factor, determined from Table 4.15 includes the effect of vertical edge restraints on the slenderness of walls.

Capacity reduction factor for vertical load Φ_s (in the simplified method of EC6 Part 3)

Table 4.16 shows the equations from EC6 Part 3 for finding the capacity reduction factor Φ_s based on the slenderness ratio of the wall and the manner in which it is loaded. For a given wall all the relevant equations should be evaluated and the minimum value should be taken. This is illustrated in Figure 4.20, which shows part of a multi-storey masonry building and states which equations are relevant for each wall.

Table 4.16: Values of the capacity reduction factor for vertical load Φ_s (in the simplified method of EC6 Part 3)

Condition	Formula	Eq. No.	Notes
For intermediate walls that support floors or roofs on both sides	$\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2$	(i)	
For walls acting as end supports to a floor or roof	$\Phi_s = 1.3 - l_{f,ef}/8$	(ii)	$l_{f,ef}$ is the effective span of the floor in metres. Conservatively, the effective span can be taken as equal to the actual span. EC6 Part 3 gives methods for calculating lesser values for continuous or two-way spanning floors
For walls at the highest level acting as end supports to the top floor or roof	$\Phi_s = 0.40$	(iii)	

Source: EC6 Part 3 Clause 4.2.2.3.

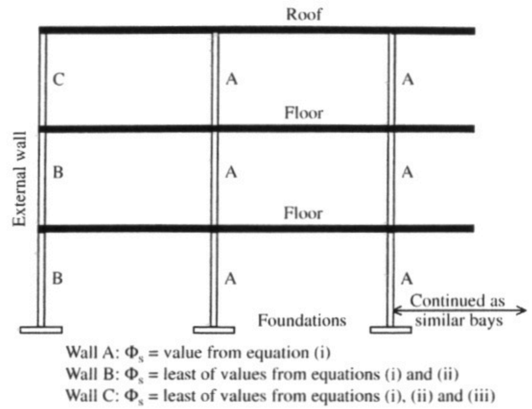


Figure 4.20: Equations for the capacity reduction factor Φ_s in the simplified method of EC6 Part 3

Example 4.2

A cavity wall, effective height $h_{ef} = 2.7\,m$, acts as end support to a roof with a span $l_{f,ef}$ of 5.0 m. The inner leaf is 100 mm thick, the outer leaf is 102.5 mm thick. Using the simplified method of EC6 Part 3 determine Φ_s for the wall.

Calculations for Example 4.2 *(using the simplified method of EC6 Part 3)*	
h_{ef}	$h_{ef} = 2700\,mm$
$t_{ef} = {}^3\sqrt{(t_1^3 + t_2^3)} = {}^3\sqrt{(100^3 + 102.5^3)}$	$t_{ef} = 127.6\,mm$
Eq. (i), $\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2 = 0.85 - 0.0011(2700/127.6)^2 = 0.36$	
Eq. (ii), $\Phi_s = 1.3 - l_{f,ef}/8 = 1.3 - 5/8 = 0.675$	
Eq. (iii), $\Phi_s = 0.40$	
All three equations are applicable, so take the minimum value	$\Phi_s = 0.36$

Design summary for a vertically loaded wall or column (using the simplified method of EC6 Part 3)

(a) Check that the structure meets conditions 1 to 8 for the use of EC6 Part 3 (Table 4.14).

(b) Calculate the effective height h_{ef} using Table 4.15.

(c) If the wall is part of a cavity wall or is stiffened by piers, calculate the effective thickness t_{ef}.

(d) Calculate the slenderness ratio h_{ef}/t_{ef} and check that it does not exceed 27.

(e) If the cross-section area of the wall A is less than 0.1 m^2, find $\Phi_A = (0.7 + 3A)$. Otherwise $\Phi_A = 1.0$.

(f) Obtain the capacity reduction factor Φ_s using equations (i), (ii) and/or (iii) as appropriate from Table 4.16.

(g) Find the ultimate applied load F and the area of the wall A.

(h) Calculate the masonry design strength f_d required from $f_d = (F/A)/(\Phi_A \Phi_s)$.

(i) Select the material partial safety factor γ_m from Table 4.7.

(j) Calculate the masonry characteristic strength f_k required from $f_k = f_d \times \gamma_m$.

(k) Using Table 4.6, choose a type of unit, normalised unit strength f_b and a grade of mortar to provide at least this strength.

(l) If required, find the actual mean strength $= f_b/\delta$, where δ is from Table 4.4.

(m) If the wall is an end support to a floor or roof and carries wind load, check that the wall thickness meets condition 9 for the use of EC6 Part 3 (Table 4.14).

Example 4.3

A 102.5 mm thick single skin wall of standard-format bricks, shown in Figure 4.21, is built between the concrete floors of a multi-storey building. It supports an ultimate axial load, including an allowance for self-weight, of 150 kN per metre run. Control categories are:

Manufacturing control	Category II
Execution (site) control	Class 2

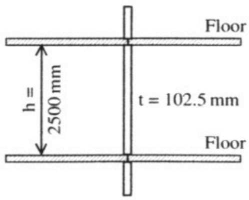


Figure 4.21: Wall in Example 4.3

The structure meets the requirements for the use of EC6 Part 3, and a general-purpose M4 mortar is to be used.

Using the simplified method of EC6 Part 3, find the required compressive strength of the masonry units.

Calculations for Example 4.3 (using the simplified method of EC6 Part 3)	
Data given	
Design vertical load	$F = 150\,\text{kN/m}$
End conditions: concrete floors give rotation restraint as well as lateral support (from Table 4.15)	$\rho = 0.75$

Overall height	$h = 2500\,\text{mm}$
Actual thickness	$t = 102.5\,\text{mm}$
Calculations	
As no information on the length of the wall is given we assume no beneficial effects from edge restraints	
Effective height $h_{ef} = \rho \times h = 0.75 \times 2500$	$h_{ef} = 1875\,\text{mm}$
Effective thickness $t_{ef} = t$	$t_{ef} = 102.5\,\text{mm}$
Slenderness ratio $= h_{ef}/t_{ef} = 1875/102.5$	Slenderness ratio = 18.3
Check that the slenderness ratio is not greater than 27	Accept
From eq. (i) of Table 4.16, Capacity reduction factor $\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2 = 0.85 - 0.0011 \times 18.3^2$ (Note that equations (ii) and (iii) do not apply to this wall)	$\Phi_s = 0.48$
The area of the wall is more than $0.1\,\text{m}^2$	$\Phi_A = 1.0$
load/area $= F/A = 150 \times 10^3/(1000 \times 102.5) = 1.46\,\text{N/mm}^2$ Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 1.46/(1.0 \times 0.48)$	$f_d = 3.04\,\text{N/mm}^2$
From Table 4.7 with manufacturing category II and execution control class 2,	$\gamma_m = 3.0$
Required value of $f_k = 3.04 \times 3.0$	$f_k = 9.1\,\text{N/mm}^2$
From Table 4.6 with M4 mortar, use Group 1 clay units with $f_b = 36\,\text{N/mm}^2$, giving $f_k = 9.25\,\text{N/mm}^2$ (by interpolation)	Use Group 1 clay units with normalised compressive strength $f_b = 36\,\text{N/mm}^2$
From Table 4.4, for a standard-format brick the normalised compressive strength $f_b = 0.85 \times$ actual compressive strength	Actual compressive strength of unit $= 42\,\text{N/mm}^2$
Required actual compressive strength = 36/0.85	

Example 4.4

Using the simplified method of EC6 Part 3, choose a block strengths and mortar grades for the wall panels noted as 1, 2 and 3 in the building shown in Figure 4.22. Note this is similar to Example 4.1 but the spans have been reduced to be within the limits allowed for use of the simplified method.

Structure data

Internal walls: 215 mm solid blockwork

External walls: 102.5 mm brickwork + cavity + 140 mm solid blockwork

Weight of 140 mm solid block walls: $2.3\,\text{kN/m}^2$

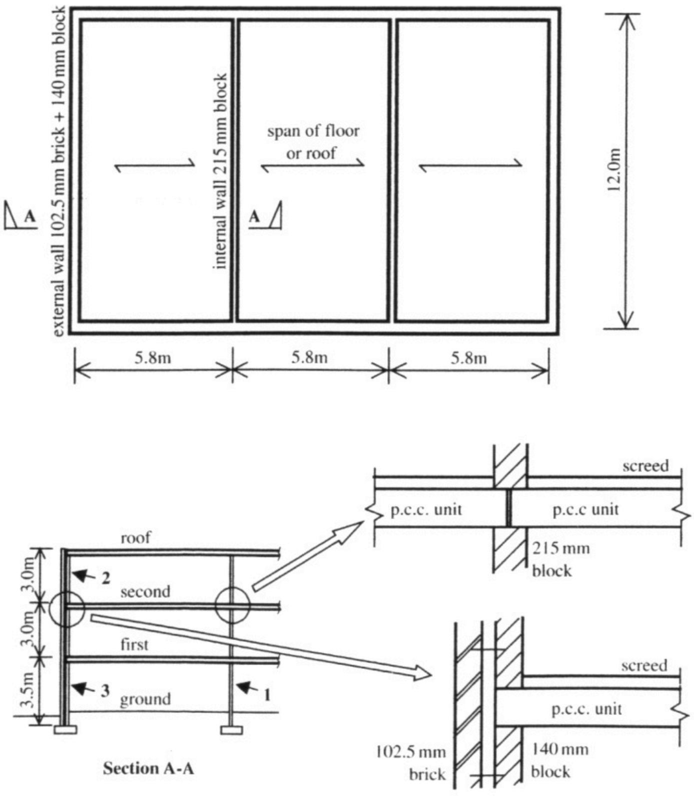
Weight of 215 mm solid block walls: $2.7\,\text{kN/m}^2$

Roof and floors:	150 mm thick pre-cast concrete units,	weight 2.5 kN/m^2
	50 mm concrete screed,	weight 1.2 kN/m^2
Imposed loads:		roof 1.5 kN/m^2
		floors 4.5 kN/m^2

In order to use the simplified method of EC6 Part 3, wind load on external walls must be taken into account. As previously stated this manual does not cover the calculation of wind loads, and for the purpose of this example the wind load is given as:

Wind load on external walls 0.6 kN/m^2

Partial safety factor for materials $\gamma_m = 3.0$



Note: **1**, **2** and **3** are the critical wall panels for vertical load design.

Figure 4.22: Building for Example 4.4

Calculations for Example 4.4 (using the simplified method of EC6 Part 3)

Conditions for use of EC6 Part 3

By inspection, the structure complies with conditions 1 to 8 in Table 4.14.

Condition 9, which applies to the external wall panels 2 and 3 only, is to be checked later

Wall Panel 1: supports 5.8 m spans of two floors and a roof

Loads per metre length of wall		Dead loads	Imposed loads
pcc units	$6 \times (5.8/2) \times 2.5 =$	43.5 kN/m	–
Screed	$6 \times (5.8/2) \times 1.2 =$	20.9 kN/m	–
Imposed load: roof	$2 \times (5.8/2) \times 1.5 =$	–	8.7 kN/m
Imposed load: floors	$4 \times (5.8/2) \times 4.5 =$	–	52.2 kN/m
Self-weight of blockwork	$(3.0 + 3.0 + 3.5/2) \times 2.7 =$	20.9 kN/m	–
	Totals	85.3 kN/m	60.9 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$
$= 1.35 \times 85.3 + 1.50 \times 60.9$ — $F = 206.5$ kN/m

Clear height $h = 3500 - 150$ — $h = 3350$ mm

The wall

- is not an end support to a floor
- has rotation restraint from the pcc units at top and bottom
- has lateral restraint at both edges with length $l = 12000$ mm

Hence, from Table 4.15 the effective height reduction factor $\rho = 0.5(l/h)$ but not more than 0.75. $0.5l/h = 0.5 \times 12000/3350 = 1.8$ so — $\rho = 0.75$

Effective height $h_{ef} = \rho h = 0.75 \times 3350$ — $h_{ef} = 2513$ mm

Effective thickness $t_{ef} = t$ — $t_{ef} = 215$ mm

Slenderness ratio $h_{ef}/t_{ef} = 2513/215$ — $h_{ef}/t_{ef} = 11.7$

Check that slenderness ratio is not more than 27 — Accept

The area of the wall is more than 0.1 m^2 — $\Phi_A = 1.0$

The wall is an intermediate support, so the capacity reduction factor Φ_s is given by equation (i) from Table 4.16

$\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2 = 0.85 - 0.0011 \times 11.7^2$ — $\Phi_s = 0.70$

Load/area $= F/A = 206.5 \times 10^3/(1000 \times 215) = 0.96$ N/mm^2

Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 0.96/(1.0 \times 0.70)$ — $f_d = 1.37$ N/mm^2

γ_m value given in structure data. — $\gamma_m = 3.0$

Required value of $f_k = 1.37 \times 3.0$ — $f_k = 4.1$ N/mm^2

From Table 4.6 use Group 1 concrete blocks with $f_b = 8.3$ N/mm^2 in M6 mortar giving $f_k = 4.1$ N/mm^2 — Use Group 1 concrete blocks with normalized compressive strength $f_b = 8.3$ N/mm^2 in M6 mortar

Continued on next page

Calculations for Example 4.4 (Continued from previous page)

From Table 4.4, for a 215 mm high × 215-mm-wide block, the normalised compressive strength $f_b = 1.168 \times$ actual compressive strength.
Required actual compressive strength = 8.3/1.168 — Actual compressive strength of unit = 7.1 N/mm^2

Wall Panel 2: supports 5.8/2 m span of roof
Load per metre length of wall on inner leaf only
Note that the outer leaf of brickwork is self-supporting so its weight is not included

		Dead loads	Imposed loads
pcc units	(5.8/2) × 2.5 =	7.3 kN/m	–
Screed	(5.8/2) × 1.2 =	3.5 kN/m	–
Imposed load: roof	(5.8/2) × 1.5 =	–	4.4 kN/m
Self-weight of blockwork	(3.0/2) × 2.3 =	3.5 kN/m	–
	Totals	14.3 kN/m	4.4 kN/m

Total load for ultimate limit states = $1.35g_k + 1.50q_k$
$= 1.35 \times 14.3 + 1.50 \times 4.4$ — $F = 25.9$ kN/m

Clear height h = 3000 − 150 — $h = 2850$ mm

The wall

- is an end support to a roof
- has rotation restraint from the pcc units at top and bottom
- has lateral restraint at both edges with length $l = 12000$ mm

Hence, from Table 4.15 the effective height reduction factor $\rho = 0.5(l/h)$
but not more than 1.0. $0.5l/h = 0.5 \times 12000/2850 = 2.1$ so — $\rho = 1.00$

Effective height $h_{ef} = \rho h = 1.00 \times 2850$ — $h_{ef} = 2850$ mm

Effective thickness $t_{ef} = \sqrt[3]{(t_1^3 + t_2^3)} = \sqrt[3]{(140^3 + 102.5^3)}$ — $t_{ef} = 156$ mm

Slenderness ratio $h_{ef}/t_{ef} = 2850/156$ — $h_{ef}/t_{ef} = 18.3$

Check that slenderness ratio is not more than 27 — Accept

The area of the wall is more than 0.1 m^2 — $\Phi_A = 1.0$

The wall is an end support at highest level, so from Figure 4.20 the equations (i), (ii) and (iii) of Table 4.16 all apply:
eq. (i) $\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2 = 0.85 - 0.0011 \times 18.3^2 = 0.48$
eq. (ii) $\Phi_s = 1.3 - l_{f,ef}/8 = 1.3 - 5.8/8 = 0.58$
eq. (iii) $\Phi_s = 0.40$
Adopt lowest value — $\Phi_s = 0.40$

Load/area = $F/A = 25.9 \times 10^3/(1000 \times 140) = 0.19$ N/mm^2
Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 0.19/(1.0 \times 0.40)$ — $f_d = 0.48$ N/mm^2

γ_m value given in structure data — $\gamma_m = 3.0$

Required value of $f_k = 0.48 \times 3.0$ — $f_k = 1.44$ N/mm^2

From Table 4.6, by interpolation, use Group 1 concrete blocks with $f_b = 2.1\,\text{N/mm}^2$ in M4 mortar giving $f_k = 1.44\,\text{N/mm}^2$ (To avoid confusion on site M6 mortar will be used to be consistent with wall panel 1)	Use Group 1 concrete blocks with normalised compressive strength $= 2.1\,\text{N/mm}^2$ in M6 mortar
From Table 4.4, for a 215-mm-high × 140-mm-wide block, the normalised compressive strength $f_b = 1.30 \times$ actual compressive strength. Required actual compressive strength = 2.1/1.30	Actual compressive strength of unit = $1.6\,\text{N/mm}^2$
Check compliance of wall panel 2 with condition 9 from Table 4.14	
Wind load q_{Ewd}	$q_{Ewd} = 0.6\,\text{kN/m}^2$
The minimum load on the wall will occur when the actual (unfactored) dead load is applied but none of the imposed load. Thus	
Minimum wall load $N_{Ed} = 1.0g_k + 0.0q_k = 1.0 \times 14.3$	$N_{Ed} = 14.3\,\text{kN/m}$
$\alpha = N_{Ed}/(tbf_d) = 14.3 \times 10^3/(140 \times 1000 \times 0.48)$	$\alpha = 0.21$
From Table 4.14	$c_1 = 0.141$, $c_2 = 0.0223$
Minimum thickness $= c_1 q_{Ewd} bh^2/N_{Ed} + c_2 h$ $= 0.141 \times 0.6 \times 1.0 \times 2.85^2/14.3 + 0.0223 \times 2.85 = 0.112\,\text{m}$	Minimum thickness = 112 mm Accept

Wall Panel 3: supports 5.8/2 = 2.9 m spans of two floors and one roof
Loads per metre length of wall on inner leaf only
Note that the outer leaf of brickwork is self-supporting so its weight is not included

		Dead loads	Imposed loads
pcc units	3 × (5.8/2) × 2.5 =	21.8 kN/m	–
Screed	3 × (5.8/2) × 1.2 =	10.4 kN/m	–
Imposed load: roof	(5.8/2) × 1.5 =	–	4.4 kN/m
Imposed load: floors	2 × (5.8/2) × 4.5 =	–	26.1 kN/m
Self-weight of blockwork	(3.0 + 3.0 + 3.5/2) × 2.3 =	17.8 kN/m	–
	Totals	50.0 kN/n	30.5 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$ $= 1.35 \times 50.0 + 1.50 \times 30.5$	$F = 113.3\,\text{kN/m}$
Clear height $h = 3500 - 150$	$h = 3350\,\text{mm}$

The wall

- is an end wall
- has rotation restraint from the pcc units at top and bottom
- has lateral restraint at both edges with length $l = 12\,000\,\text{mm}$

Hence from Table 4.15 the effective height reduction factor $\rho = 0.5(l/h)$ but not more than 1.00 $0.5l/h = 0.5 \times 12000/3350 = 1.8$ so	$\rho = 1.00$
Effective height $h_{ef} = \rho h = 1.00 \times 3350$	$h_{ef} = 3350\,\text{mm}$
Effective thickness $t_{ef} = \sqrt[3]{(t_1^3 + t_2^3)} = \sqrt[3]{(140^3 + 102.5^3)}$	$t_{ef} = 156\,\text{mm}$
Slenderness ratio $h_{ef}/t_{ef} = 3350/156$	$h_{ef}/t_{ef} = 21.5$

Continued on next page

Calculations for Example 4.4 (Continued from previous page)		
The area of the wall is more than $0.1\,m^2$		$\Phi_A = 1.0$
The wall is an end support, so equations (i) and (ii) of Table 4.16 apply: eq. (i) $\Phi_s = 0.85 - 0.0011(h_{ef}/t_{ef})^2 = 0.85 - 0.0011 \times 21.5^2$ eq. (ii) $\Phi_s = 1.3 - l_{f,ef}/8 = 1.3 - 5.8/8$ Adopt lower value	 $= 0.34$ $= 0.58$	 $\Phi_s = 0.34$
Load/area $= F/A = 113.3 \times 10^3/(1000 \times 140) = 0.81\,N/mm^2$		
Required value of $f_d = (F/A)\,(\Phi_A\Phi_s) = 0.81/(1.0 \times 0.34)$		$f_d = 2.38\,N/mm^2$
γ_m value given in structure data		$\gamma_m = 3.0$
Required value of $f_k = 2.38 \times 3.0$		$f_k = 7.1\,N/mm^2$
From Table 4.6 by interpolation, use Group 1 concrete blocks with $f_b = 18\,N/mm^2$ in M6 mortar giving $f_k = 7.1\,N/mm^2$		Use Group 1 concrete blocks with normalised compressive strength $f_b = 18\,N/mm^2$ in M6 mortar
From Table 4.4, for a 215-mm-high × 140-mm-wide block, the normalised compressive strength $f_b = 1.30 \times$ actual compressive strength. Required actual compressive strength $= 18.0/1.30$		Actual compressive strength of unit $= 14\,N/mm^2$
Check compliance of wall panel 2 with condition 9 from Table 4.14		
Wind load q_{Ewd}		$q_{Ewd} = 0.6\,kN/m^2$
Minimum wall load $N_{Ed} = 1.0g_k + 0.0q_k = 1.0 \times 50.0$		$N_{Ed} = 50.0\,kN/m$
$\alpha = N_{Ed}/(tbf_d) = 50.0 \times 10^3/(140 \times 1000 \times 2.38)$		$\alpha = 0.15$
From Table 4.14		$c_1 = 0.13,\ c_2 = 0.0205$
Minimum thickness $= c_1 q_{Ewd} bh^2/N_{Ed} + c_2 h$ $= 0.13 \times 0.6 \times 1.0 \times 3.35^2/50.0 + 0.0205 \times 3.35 = 0.086\,m$		 Minimum thickness $= 86\,mm$ Accept

4.8 Concentrated Loads

Concentrated loads can occur at beam, truss or lintel bearings. Whilst these produce relatively high stress concentrations over a small plan area, the stress is usually dispersed rapidly through the wall construction below and capacity is not limited by slenderness factors. Subject to the conditions noted, Clause 4.3 of EC6 Part 3 allows normal design stresses to be exceeded by up to 50%, as shown in this formula for the load capacity of a bearing N_{Rdc}.

For masonry made with Group 1 units

$$N_{Rdc} = (1.2 + 0.4(a_1/h_c))f_d A_b \text{ but not more than } 1.5f_d A_b$$

For masonry made with Group 2 units

$$N_{Rdc} = f_d A_b$$

where

- f_d is the design compressive strength of the masonry
- A_b is the loaded area
- a_1 is the distance from the end of the wall to the concentrated load, as shown in Figure 4.23
- h_c is the height from floor level to the load, as shown in Figure 4.23.

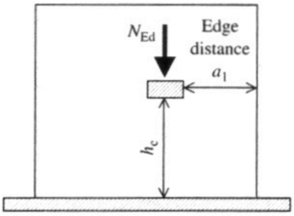


Figure 4.23: Elevation of wall with concentrated load

Provided that:

- the bearing area is not more than ¼ of the area of the wall
- the bearing area is not more than $2t^2$, where t is the thickness of the wall
- the eccentricity of the load from the centre plane of the wall is not more than $t/4$
- the normal requirements for vertical load design should be met at the mid-height of the wall below the bearing assuming the concentrated load spreads at an angle of 60°.

Example 4.5

A 1.2 m wide opening is required in one of the internal walls at ground floor level in the building used in Example 4.3, as shown in Figure 4.24. A 215-mm-deep lintel is to be used. Check whether 425-mm-long lintel bearings will be adequate.

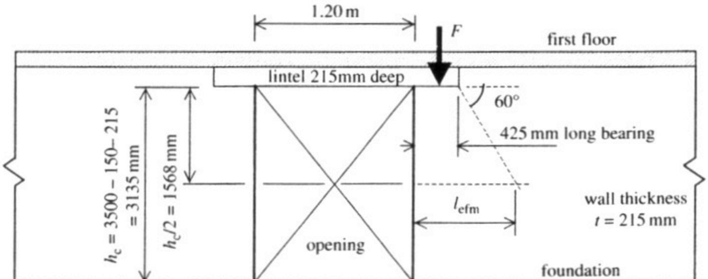


Figure 4.24: Arrangement of opening, lintel and bearings for Example 4.5

Calculations for Example 4.5

Calculations for one lintel bearing

		Dead loads	Imposed loads
pcc units	$6 \times (5.8/2) \times 2.5 =$	43.5 kN/m	–
Screed	$6 \times (5.8/2) \times 1.2 =$	20.9 kN/m	–
Imposed load: roof	$2 \times (5.8/2) \times 1.5 =$	–	8.7 kN/m
Imposed load: floors	$4 \times (5.8/2) \times 4.5 =$	–	52.2 kN/m
Self-weight of blockwork	$(3.0 + 3.0) \times 2.7 =$	16.2 kN/m	–
Self-weight of concrete lintel	$0.215 \times 0.215 \times 25 =$	1.2 kN/m	
	Totals	81.8 kN/m	60.9 kN/m

Total load for ultimate limit states $= 1.35g_k + 1.50q_k$
$= 1.35 \times 81.8 + 1.50 \times 60.9$ $= 201.8$ kN/m

Total length of lintel $= 0.425 + 1.20 + 0.425 = 2.05$ m

Total ultimate load on lintel $= 201.8 \times 2.05 = 413.7$ kN

Ultimate load on one bearing $F = 413.7/2$ kN $F = 206.9$ kN

The capacity of the bearing $N_{Rdc} = (1.2 + 0.4(a_1/h_c))\, f_d A_b$ but not more than $1.5\, f_d A_b$

The bearing is at the end of the supporting wall so the edge distance $a_1 = 0$, so $N_{Rdc} = 1.2 f_d A_b$

Required value of f_d is $206.9 \times 10^3/(1.2 \times 215 \times 425)$ Required $f_d = 1.89$

Check the wall for vertical load at a distance $h_c/2$ below the bearing, assuming the load spreads at an angle of 60°

At this level the load is carried on a length l_{efm} of wall, as shown in Figure 4.24
$l_{efm} = 425 + 1568/\tan(60°)$ $l_{efm} = 1330$ mm

The ultimate load on the length of wall l_{efm} is the sum of:

- The load F on the bearing 206.9 kN
- The weight of an additional area of wall
  1.568 m $\times$ 0.425 m and a unit weight of 2.7 kN/m^2
  with $\gamma_m = 1.35$
  $1.568 \times 0.425 \times 2.7 \times 1.35 =$ 2.4 kN
- An additional $(1.330 - 0.425) = 0.905$ m of the design load used in Example 4.4 wall panel 1
  $0.905 \times 206.5 =$ 186.9 kN

The total ultimate design load F on length l_{efm}

$= 206.9 + 2.4 + 186.9$ $F = 396.2$ kN

From the calculation for Example 4.4 wall panel 1 $\Phi_A = 1.0$
$\Phi_s = 0.70$

Load/area $= F/A = 396.2 \times 10^3/(1330 \times 215) = 1.38$ N/mm^2

Required value of $f_d = (F/A)/(\Phi_A \Phi_s) = 1.38/(1.0 \times 0.70)$ $f_d = 1.97$ N/mm^2

From the calculations for Example 4.4 wall panel 1, $f_d = 1.37$ N/mm^2

Thus the lintel bearing is not adequate. A longer bearing is not permitted because the bearing area would exceed $2t^2 = 2 \times 215^2 = 92\,450\,\text{mm}^2$	Bearing is not adequate
A satisfactory design could be found either by using a block pier under the bearing using stronger masonry. Investigate the latter option	
γ_m value given in structure data	$\gamma_m = 3.0$
Required value of $f_k = 1.97 \times 3.0$	$f_k = 5.9\,\text{N/mm}^2$
From Table 4.6, by interpolation, use Group 1 concrete blocks with $f_b = 14\,\text{N/mm}^2$ in M6 mortar giving $f_k = 6.0\,\text{N/mm}^2$	Use Group 1 concrete blocks with normalised compressive strength $f_b = 14\,\text{N/mm}^2$ in M6 mortar
From Table 4.4 for a 215-mm-high $\times$ 215-mm-wide block the normalised compressive strength $f_b = 1.168 \times$ actual compressive strength. Required actual compressive strength = 14/1.168	Actual compressive strength of unit $= 12\text{N/mm}^2$
Conclusion: The opening and lintel bearings described can be used if the block strength is increased from 7.1N/mm^2 (see Example 4.4) to 12N/mm^2	

4.9 References

BS EN 771 Specification for masonry units.

BS EN 772 Methods of test for masonry units.

BS 4729: 2005 Clay and calcium silicate bricks of special shapes and sizes – recommendations.

BS EN 1996: 2005 Eurocode 6: Design of masonry structures – Part 1-1: General rules for reinforced and unreinforced masonry structures, with UK National Annex.

BS EN 1996: 2006 Eurocode 6: Design of masonry structures – Part 3: Simplified calculation methods for unreinforced masonry structures, with UK National Annex.

Manual for the design of plain masonry in building structures to Eurocode 6, IStructE, February 2008.

CHAPTER 5

Steel Elements

Contents

5.1 Structural Design of Steel Elements

Guidance on the structural use of steelwork is given in BS EN 1993, Eurocode 3. This is published in several sections as shown in Table 5.1.

The organizations listed below, who are associated with structural steelwork, have guidance available on the use of EC3. Much useful information including guidance on UK practice is available from their websites.

(i) The Steel Construction Institute (SCI), www.steel-sci.org

(ii) The British Constructional Steelwork Association, www.steelconstruction.org

(iii) CORUS, www.corusconstruction.com

Additional information including sample calculations, case studies and scheme development guidance can be found on www.access-steel.com. More guidance on the use of EC3 in the UK is issued as Non-Contradictory Complementary Information (NCCI) and can be found on www.steel-ncci.co.uk and on www.access-steel.com.

Limit state design methods as set out in Chapter 1 of this manual are used for structural steelwork. Generally the following load combinations are used:

For checking ultimate limit states (ULS) $\gamma_G G_k + \gamma_Q Q_k = 1.35G_k + 1.50Q_k$
For checking serviceability limit states (SLS) $\gamma_G G_k + \gamma_Q Q_k = 1.00\,G_k + 1.00Q_k$

Table 5.1: Codes relating to the design of structural steelwork

BSI reference		Title
BS EN 1993 Part 1 (Eurocode 3)	**Part 1–1**	**General rules and rules for buildings**
	Part 1–2	Structural fire design
	Part 1–3	Cold-formed thin-gauge members and sheeting
	Part 1–4	Stainless steels
	Part 1–5	**Plated structural elements**
	Part 1–6	Strength and stability of shell structures
	Part 1–7	Strength and stability of planar plated structures transversely loaded
	Part 1–8	Design of joints
	Part 1–9	Fatigue strength
	Part 1–10	Selection of steel for fracture toughness and through-thickness properties
	Part 1–11	Tension components made of steel
	Part 1–12	Supplementary rules for high-strength steel
BS EN 1993 Part 2		Steel bridges
BS EN 1993 Part 3		Towers, masts and chimneys
BS EN 1993 Part 4		Silos, tanks and pipelines
BS EN 1993 Part 5		Piling
BS EN 1993 Part 6		Crane-supporting structures

Each code should be read with the appropriate National Annex. Titles shown in **bold** are relevant to this manual.

The material partial safety factors used for steel in buildings are:

For checking ULS	For resistance of cross-sections generally	$\gamma_{m1} = 1.0$
	For resistance of members to instability	$\gamma_{m2} = 1.0$
	For resistance of cross-sections in tension to fracture	$\gamma_{m3} = 1.1$

As this manual does not cover resistance of cross-sections in tension to fracture, a single value can be used throughout $\gamma_m = 1.0$

For checking SLS $\gamma_m = 1.0$

Ultimate limit states

Strength: Individual structural elements should be checked to ensure that they will not yield, rupture or buckle under the ultimate forces and moments.

- Beams should be checked for the ULS of bending and shear.
- Columns should be checked for the ULS of compression and, where applicable, also bending.

Stability: The building or structural framework as a whole should be checked to ensure that lateral loads do not induce excessive sway or cause overturning. EC3 Part 1-1 gives guidance on which structures should be designed for secondary forces arising from sway deflections.

Fracture due to fatigue: Repeated reversal of stress can cause steel to fail at stresses below static strength values, and connections are particularly prone to such failure. In the majority of building structures changes in stress are gradual and fatigue is not critical. Structures such as bridges and crane supports are subject to dynamic loading and the risk of fatigue failure should be considered. EC3 Part 2 gives guidance on the design of bridges and EC3 Part 6 on crane-supporting structures.

Brittle failure: Sudden failure due to brittle fracture can occur in steelwork exposed to low temperatures, and welded structures are particularly susceptible. Since the steel members in most enclosed building frames are protected from the weather, they are not exposed to low temperatures and usually brittle fracture need not be considered. It is more likely to occur in large welded structures, such as bridges, which are exposed to the extremes of winter temperature. In such circumstances it is necessary to select a suitable grade of steel and to devise details that avoid high-stress concentrations. Guidance on steel grades for use in buildings is given in Section 5.3 and especially in Table 5.5.

Fire resistance: Steel sections can heat up rapidly in a fire and lose significant strength when their temperature exceeds 550°C. EC3 Part 1-2 gives three ways of ensuring adequate fire resistance:

1. Protecting the steel with an adequate thickness of fire-resistant board or sprayed-on coating.
2. Coating the steel with an intumescent paint which will swell in a fire to provide insulation.
3. Calculating the temperature that the unprotected steel will reach in the event of a fire, and then showing that the residual strength of the structure is adequate. Reduced values of γ_f are used, recognising that it is very unlikely that the structure will be fully loaded at the time of the fire.

Serviceability limit states

Deflection and vibration: The structure should not suffer deflections or vibrations that could:

- damage the building fabric (e.g. crack plaster finishes)
- impair the function of the building (e.g. operation of machinery)
- be unsightly or unpleasant for the occupants.

Corrosion: Steel inside a heated building will not suffer significant corrosion in the lifetime of the building. Steel exposed to the elements will corrode, and a degree of protection is normally necessary for permanent structures. Alternatively, steels that are inherently corrosion resistant can be used. Some protection measures, listed in order of increasing cost, are:

- shop- and site-applied paint systems
- galvanizing (zinc coating)
- use of weathering steels
- use of stainless steels.

Since this manual is concerned with the design of individual structural elements, only the strength ULS and the deflection SLS are covered further.

Analysis and design

The design process begins by envisaging a conceptual model of the overall structure which is then analysed to determine the forces and moments in the individual structural members. Two types of conceptual model can be used by the designers:

1. Simple design. This method applies to structures in which the connections between members will not develop any significant restraint moments. Member forces and moments are calculated on the basis of the following assumptions:
   a) All beams are simply supported
   b) All connections are designed to resist only resultant reactions at the appropriate eccentricity
   c) Columns are subjected to vertical loads applied at the appropriate eccentricity
   d) Resistance to sway forces, such as those resulting from wind loads, is provided by bracing, shear walls or core walls.
2. Rigid or semi-rigid design. The structure is considered to be connected by rigid or semi-rigid joints. Member forces and moments are calculated by analysis of frames, and the connections are designed for both forces and bending moments.

The design of steel elements in this manual uses the principles of simple design.

The design of a steel structure may be divided into two stages:

1. Sizing of the individual members for strength and stiffness
2. Design of the bolted or welded connections to transmit forces and bending moments as required.

This manual is concerned with the first of these, and the design of connections is covered in EC3 Part 1-8.

It is important to appreciate that an economic steel design is not necessarily that which uses the least weight of steel. Fabrication and erection costs may exceed the basic material cost of the steel sections, and it will often be more economical to use a heavier section with simple details than a lighter section with more complex details.

5.2 Symbols

Table 5.2 lists some of the symbols used in the design process. Where relevant the units commonly used for the quantity are shown.

Table 5.2: Symbols used in the design of structural steelwork

Symbol	Normal units	Meaning	Comment
A	cm^2	Area of section	
b	mm	Breadth of section	
C_1		Factor for shape of bending moment diagram	
c	mm	Outstand of a flange	
d	mm	Depth of straight portion of a web	
E	kN/mm^2	Modulus of elasticity of steel	For hot-rolled structural steels $E = 210\,kN/mm^2$
f		Enhancement factor for LTB	
f_y	N/mm^2	Yield strength of steel	Grade S275 steel up to 16 mm thick, $f_y = 275\,N/mm^2$ 16 to 40 mm thick, $f_y = 265\,N/mm^2$ over 40 mm thick, $f_y = 255\,N/mm^2$ Grade S355 steel up to 16 mm thick, $f_y = 355\,N/mm^2$ 16 to 40 mm thick, $f_y = 345\,N/mm^2$ over 40 mm thick, $f_y = 335\,N/mm^2$
g_k, G_k	kN, kN/m or kN/m^2	Characteristic permanent action, e.g. Dead Load	
G	kN/mm^2	Shear modulus of steel	For hot-rolled structural steels $G = 81\,kN/mm^2$
h	mm	Depth of section	
h_w	mm	Clear height of web between flanges	$h_w = h - 2t_f$
I_y, I_z	cm^4	Second moment of area of section	
I_T	cm^4	Torsion constant	
I_w	dm^6	Warping constant	
i	cm	Radius of gyration of section	
k		Interaction factor	
k_c		Factor for stanchion baseplate outstand	

(Continued)

Table 5.2: (Continued)

Symbol	Normal units	Meaning	Comment
k_t		Factor for stanchion baseplate thickness	
LTB, LT		Lateral-torsional buckling	
$M_{b,Rd}$	kNm	Design buckling resistance moment	
M_{cr}	kNm	Elastic critical buckling moment	
$M_{c,Rd}$	kNm	Design bending resistance	
$M_{pl,Rd}$	kNm	Plastic bending resistance	
q_k, Q_k	kN, kN/m or kN/m^2	Characteristic variable action, e.g. Imposed Load	
r	mm	Radius of root fillet	
SLS		Serviceability limit state	
t_f	mm	Flange thickness	
t_w	mm	Web thickness	
ULS		Ultimate limit state	
V	kN	Shear force	
V_{Ed}	kN	Design shear force	
$V_{pl,Rd}$	kN	Design plastic shear resistance	
v	N/mm^2	Shear stress	
W_{el}	cm^3	Elastic section modulus	
W_{pl}	cm^3	Plastic section modulus	
w		Vertical deflection	
α		Imperfection factor	
β		Buckling correction factor	
ε		Coefficient depending on f_y for determining section class	$\varepsilon = \surd(235/f_y)$
γ_f		Partial factor of safety for load	At ULS these are normally 1.35 for permanent loads and 1.5 for variable loads
γ_m		Partial factor of safety for strength of material	At ULS for steel generally $\gamma_m = 1.0$
λ		Slenderness ratio	λ = effective length/radius of gyration
$\bar{\lambda}$		Non-dimensional slenderness ratio	
$\bar{\lambda}_0$		Limiting slenderness ratio	
χ		Strength reduction factor for buckling	
σ	N/mm^2	Stress	
ψ		Ratio of moments in a segment of a beam	
Φ		Buckling parameter	
	kN/m^3	Unit weight of steel	Normal value 77 kN/m^3

EC3 makes extensive use of suffixes in symbols. The meaning of a suffix may depend upon its context, for example the suffix 'y' means 'yield' when applied to a stress or '*y-y* axis' when applied to a section property. Particular sets of suffixes that may cause confusion are:

Ed and Rd:	Ed denotes the design value of a force or moment, as caused by the loads or other effects on the structure Rd denotes the resistance of the structure or member to force or moment Thus, the normal criterion for structural adequacy is $X_{Ed} \leq X_{Rd}$, where X stands for a force or a moment
pl and el:	pl stands for 'plastic', el stands for 'elastic'

5.3 Material Properties

The actual stress/strain relationship for steel is shown in Figure 5.1(a) and applies equally to tensile or compressive stress. Note that the steel behaves elastically up to its yield strength f_y and then deforms plastically to reach an ultimate strength which is higher than the yield strength. Figure 5.1(b) shows the idealized stress/strain relationship which is used for design purposes.

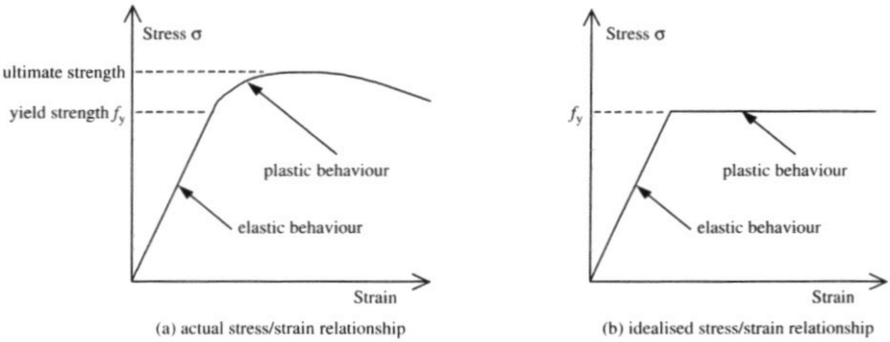


Figure 5.1: Stress/strain relationships for steel

Steel sections are produced by rolling the steel, while it is hot, into various standard profiles. Hot-rolled sections are available in steels of grade S275 and S355 to BS EN 10025: Part 2. The nominal strength values of these steels depend on their thickness, as shown in Table 5.3. Properties are specified as characteristic values, and it is expected that 95% of samples tested will equal or exceed the characteristic value while 5% are permitted to fall below it.

In all cases the standard also gives a required ultimate strength which is always at least 10% higher than the yield strength.

All steel structures should be designed to be ductile so that they deflect without catastrophic failure if they are overstressed. At low temperatures steels may fail by brittle fracture instead

Table 5.3: Nominal values of yield strength f_y (N/mm^2)

Steel grade	S275	S355
Thickness up to 16 mm	275	355
Thickness over 16 mm and up to 40 mm	265	345
Thickness over 40 mm and up to 63 mm	255	335

Source: Table 7 of BS EN 10025-2.

of ductile straining, and steel grades S275 and S355 have sub-grades to indicate their fracture resistance. Table 5.4 summarizes the grades available.

Table 5.4: Steel grades for resistance to brittle fracture at low temperatures

	Increasing resistance to brittle fracture at low temperatures →		
Steel grade S275	S275 JR	S275 J0	S275 J2
Steel grade S355	S355 JR	S355 J0	S355 J2

Clause NA.2.6 of the UK National Annex to EC3 states that for buildings the minimum service temperature in the steel may be taken as −10°C for internal steelwork and −20°C for external steelwork unless a lower value is given by the UK National Annex to BS EN 1991-1-5. Alternatively where the location of the building is known the more exact temperatures given in the UK National Annex to BS EN 1991-1-5 may be used.

Thick sections are more susceptible to brittle fracture than thin sections, and Table 5.5 shows how to choose a suitable grade of steel to suit the expected ambient temperature and the section thickness. The table shows that the basic steel grade S275 JR is suitable for

Table 5.5: Maximum thickness of steels for use in buildings (mm)

Location	Internal	External
Minimum service temperature	**−10°C**	**−20°C**
Grade S275 JR	35	30
Grade S275 J0	55	45
Grade S275 J2	75	65
Grade S355 JR	25	20
Grade S355 J0	40	35
Grade S355 J2	60	50

Source: Table 2.1 of EC3 Part 1–10 with $\sigma_{Ed} = 0.75 f_y(t)$.

internal use if the section thickness does not exceed 35 mm and for external use if the section thickness does not exceed 30 mm. If thicker sections are required then a higher grade of steel (S275 J0 or S275 J2) will be required. Welding of steel tends to make it more brittle, and the grades and thicknesses given in Table 5.5 allow for welding provided appropriate precautions are taken.

Note that the values in Table 5.5 are conservative and that the more detailed methods of EC3 Part 1-10, which involve calculation of the service stresses in the steel, may allow use of thicker sections.

The modulus of elasticity E is taken as 210 kN/mm^2 for all hot-rolled structural steels.

5.4 Section Properties

Dimensions and geometric properties for the hot-rolled steel sections commonly used in the UK for beams and columns are given in BS4 Part 1, and additional sizes beyond those in BS4 are available from CORUS under the trademarked UKB and UKC designations. Channels, tees, angles, asymmetric I beams and hollow sections are also available. Figure 5.2 shows the member axes, typical cross-sections are shown in Figure 5.3, and Figure 5.4 shows the defining dimensions of Universal Beam (UB) and Universal Column (UC) sections.

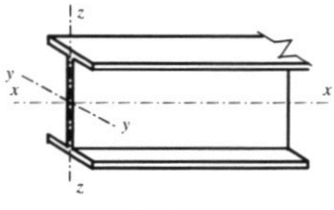
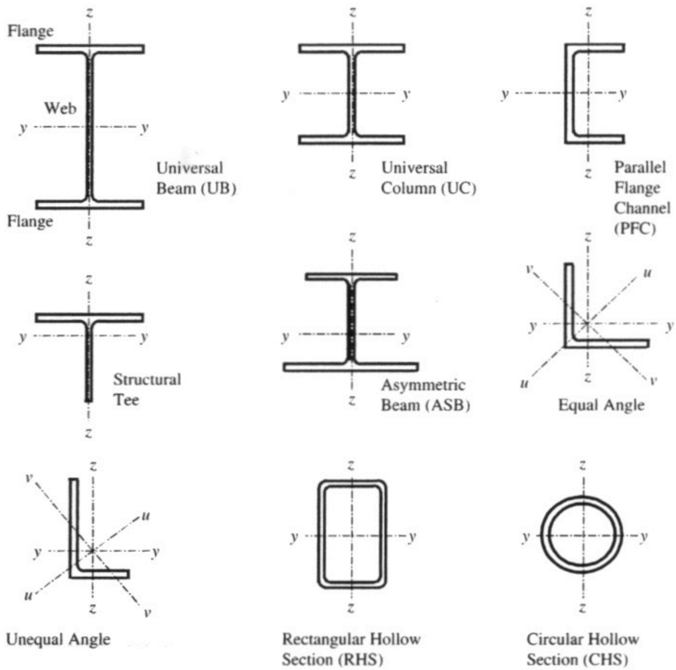


Figure 5.2: Member axes

Many useful section properties can be calculated from the dimensions of the section, and these are given in published tables. Dimensions and properties for selected UB sections are given in Tables 5.6 and 5.7, and for UC sections in Tables 5.8 and 5.9. A full set is available in the CORUS Advance Section brochure which can be downloaded from www.corusconstruction.com.

Cold-formed steel sections produced from light-gauge plate, sheet or strip, usually less than 3 mm thick, are also available. These are widely used for purlins and sheeting rails, and complete structures of cold-formed sections are sometimes used for small buildings such as houses. Guidance on design using cold-formed sections is given in EC3 Part 1.3 and extensive information can be obtained from the manufacturers of the sections.



Note that in all cases the x-x axis is along the length of the member.

Figure 5.3: Rolled steel sections: typical profiles and axes

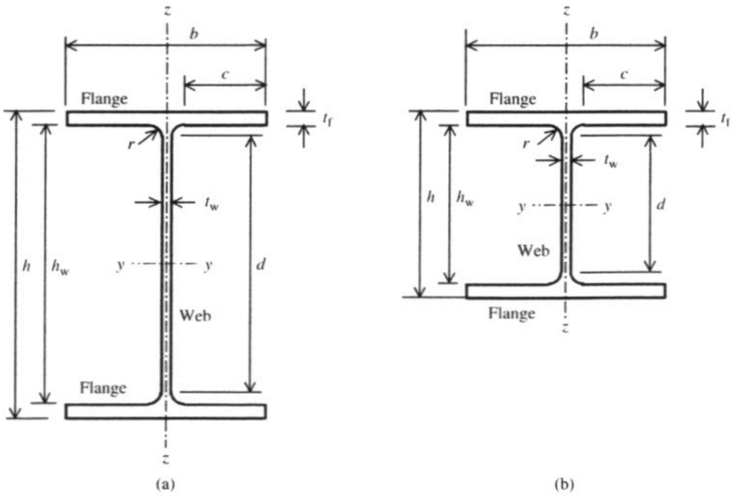


Figure 5.4: Universal beam (a) and universal column (b) sections: dimensions

Table 5.6: Universal Beams: dimensions

Serial size	Mass per metre (kg)	Depth h (mm)	Breadth b (mm)	Web thickness t_w (mm)	Flange thickness t_f (mm)	Root radius r (mm)	Depth between fillets d (mm)	Flange outstand c (mm)	Web slenderness d/t_w	Flange slenderness c/t_f
1016 × 305	487*	1036.3	308.5	30.0	54.1	30.0	868.1	109.3	28.9	2.02
	437*	1026.1	305.4	26.9	49.0	30.0	868.1	109.3	32.3	2.23
	393*	1015.9	303.0	24.4	43.9	30.0	868.1	109.3	35.6	2.49
	349*	1008.1	302.0	21.1	40.0	30.0	868.1	110.5	41.1	2.76
	314*	999.9	300.0	19.1	35.9	30.0	868.1	110.5	45.5	3.08
	272*	990.1	300.0	16.5	31.0	30.0	868.1	111.8	52.6	3.60
	249*	980.1	300.0	16.5	26.0	30.0	868.1	111.8	52.6	4.30
	222*	970.3	300.0	16.0	21.1	30.0	868.1	112.0	54.3	5.31
914 × 419	388	921.0	420.5	21.4	36.6	24.1	799.6	175.5	37.4	4.79
	343	911.8	418.5	19.4	32.0	24.1	799.6	175.5	41.2	5.48
914 × 305	289	926.6	307.7	19.5	32.0	19.1	824.4	125.0	42.3	3.91
	253	918.4	305.5	17.3	27.9	19.1	824.4	125.0	47.7	4.48
	224	910.4	304.1	15.9	23.9	19.1	824.4	125.0	51.8	5.23
	201	903.0	303.3	15.1	20.2	19.1	824.4	125.0	54.6	6.19
838 × 292	226	850.9	293.8	16.1	26.8	17.8	761.7	121.1	47.3	4.52
	194	840.7	292.4	14.7	21.7	17.8	761.7	121.1	51.8	5.58
	176	834.9	291.7	14.0	18.8	17.8	761.7	121.1	54.4	6.44
762 × 267	197	769.8	268.0	15.6	25.4	16.5	686.0	109.7	44.0	4.32
	173	762.2	266.7	14.3	21.6	16.5	686.0	109.7	48.0	5.08
	147	754.0	265.2	12.8	17.5	16.5	686.0	109.7	53.6	6.27
	134	750.0	264.4	12.0	15.5	16.5	686.0	109.7	57.2	7.08
686 × 254	170	692.9	255.8	14.5	23.7	15.2	615.1	105.5	42.4	4.45
	152	687.5	254.5	13.2	21.0	15.2	615.1	105.5	46.6	5.02
	140	683.5	253.7	12.4	19.0	15.2	615.1	105.5	49.6	5.55
	125	677.9	253.0	11.7	16.2	15.2	615.1	105.5	52.6	6.51
610 × 305	238	635.8	311.4	18.4	31.4	16.5	540.0	130.0	29.3	4.14
	179	620.2	307.1	14.1	23.6	16.5	540.0	130.0	38.3	5.51
	149	612.4	304.8	11.8	19.7	16.5	540.0	130.0	45.8	6.60

(Continued)

Table 5.6: Universal Beams: dimensions (Continued)

Serial size	Mass per metre (kg)	Depth h (mm)	Breadth b (mm)	Web thickness t_w (mm)	Flange thickness t_f (mm)	Root radius r (mm)	Depth between fillets d (mm)	Flange outstand c (mm)	Web slenderness d/t_w	Flange slenderness c/t_f
610 × 229	140	617.2	230.2	13.1	22.1	12.7	547.6	95.9	41.8	4.34
	125	612.2	229.0	11.9	19.6	12.7	547.6	95.9	46.0	4.89
	113	607.6	228.2	11.1	17.3	12.7	547.6	95.9	49.3	5.54
	101	602.6	227.6	10.5	14.8	12.7	547.6	95.9	52.2	6.48
610 × 178	100*	607.4	179.2	11.3	17.2	12.7	547.6	71.3	48.5	4.14
	92*	603.0	178.8	10.9	15.0	12.7	547.6	71.3	50.2	4.75
	82*	598.6	177.9	10.0	12.8	12.7	547.6	71.3	54.8	5.57
533 × 312	272	577.1	320.2	21.1	37.6	12.7	476.5	136.9	22.6	3.64
	219	560.3	317.4	18.3	29.2	12.7	476.5	136.9	26.0	4.69
	182	550.7	314.5	15.2	24.4	12.7	476.5	137.0	31.3	5.61
	150	542.5	312.0	12.7	20.3	12.7	476.5	137.0	37.5	6.75
533 × 210	138*	549.1	213.9	14.7	23.6	12.7	476.5	86.9	32.4	3.68
	122	544.5	211.9	12.7	21.3	12.7	476.5	86.9	37.5	4.08
	109	539.5	210.8	11.6	18.8	12.7	476.5	86.9	41.1	4.62
	101	536.7	210.0	10.8	17.4	12.7	476.5	86.9	44.1	4.99
	92	533.1	209.3	10.1	15.6	12.7	476.5	86.9	47.2	5.57
	82	528.3	208.8	9.6	13.2	12.7	476.5	86.9	49.6	6.58
533 × 165	85*	534.9	166.5	10.3	16.5	12.7	476.5	65.4	46.3	3.96
	74*	529.1	165.9	9.7	13.6	12.7	476.5	65.4	49.1	4.81
	66*	524.7	165.1	8.9	11.4	12.7	476.5	65.4	53.5	5.74
457 × 191	161*	492.0	199.4	18.0	32.0	10.2	407.6	80.5	22.6	2.52
	133*	480.6	196.7	15.3	26.3	10.2	407.6	80.5	26.6	3.06
	106*	469.2	194.0	12.6	20.6	10.2	407.6	80.5	32.3	3.91
	98	467.2	192.8	11.4	19.6	10.2	407.6	80.5	35.8	4.11
	89	463.4	191.9	10.5	17.7	10.2	407.6	80.5	38.8	4.55
	82	460.0	191.3	9.9	16.0	10.2	407.6	80.5	41.2	5.03
	74	457.0	190.4	9.0	14.5	10.2	407.6	80.5	45.3	5.55
	67	453.4	189.9	8.5	12.7	10.2	407.6	80.5	48.0	6.34
457 × 152	82	465.8	155.3	10.5	18.9	10.2	407.6	62.2	38.8	3.29
	74	462.0	154.4	9.6	17.0	10.2	407.6	62.2	42.5	3.66
	67	458.0	153.8	9.0	15.0	10.2	407.6	62.2	45.3	4.15
	60	454.6	152.9	8.1	13.3	10.2	407.6	62.2	50.3	4.68
	52	449.8	152.4	7.6	10.9	10.2	407.6	62.2	53.6	5.71

406 × 178	85*	417.2	181.9	10.9	18.2	10.2	360.4	75.3	33.1	4.14
	74	412.8	179.5	9.5	16.0	10.2	360.4	74.8	37.9	4.68
	67	409.4	178.8	8.8	14.3	10.2	360.4	74.8	41.0	5.23
	60	406.4	177.9	7.9	12.8	10.2	360.4	74.8	45.6	5.84
	54	402.6	177.7	7.7	10.9	10.2	360.4	74.8	46.8	6.86
406 × 140	53*	406.6	143.3	7.9	12.9	10.2	360.4	57.5	45.6	4.46
	46	403.2	142.2	6.8	11.2	10.2	360.4	57.5	53.0	5.13
	39	398.0	141.8	6.4	8.6	10.2	360.4	57.5	56.3	6.69
356 × 171	67	363.4	173.2	9.1	15.7	10.2	311.6	71.9	34.2	4.58
	57	358.0	172.2	8.1	13.0	10.2	311.6	71.9	38.5	5.53
	51	355.0	171.5	7.4	11.5	10.2	311.6	71.9	42.1	6.25
	45	351.4	171.1	7.0	9.7	10.2	311.6	71.9	44.5	7.41
356 × 127	39	353.4	126.0	6.6	10.7	10.2	311.6	49.5	47.2	4.63
	33	349.0	125.4	6.0	8.5	10.2	311.6	49.5	51.9	5.82
305 × 165	54	310.4	166.9	7.9	13.7	8.9	265.2	70.6	33.6	5.15
	46	306.6	165.7	6.7	11.8	8.9	265.2	70.6	39.6	5.98
	40	303.4	165.0	6.0	10.2	8.9	265.2	70.6	44.2	6.92
305 × 127	48	311.0	125.3	9.0	14.0	8.9	265.2	49.3	29.5	3.52
	42	307.2	124.3	8.0	12.1	8.9	265.2	49.3	33.2	4.07
	37	304.4	123.4	7.1	10.7	8.9	265.2	49.3	37.4	4.60
305 × 102	33	312.7	102.4	6.6	10.8	7.6	275.9	40.3	41.8	3.73
	28	308.7	101.8	6.0	8.8	7.6	275.9	40.3	46.0	4.58
	25	305.1	101.6	5.8	7.0	7.6	275.9	40.3	47.6	5.76
254 × 146	43	259.6	147.3	7.2	12.7	7.6	219.0	62.5	30.4	4.92
	37	256.0	146.4	6.3	10.9	7.6	219.0	62.5	34.8	5.73
	31	251.4	146.1	6.0	8.6	7.6	219.0	62.5	36.5	7.26
254 × 102	28	260.4	102.2	6.3	10.0	7.6	225.2	40.4	35.7	4.04
	25	257.2	101.9	6.0	8.4	7.6	225.2	40.4	37.5	4.80
	22	254.0	101.6	5.7	6.8	7.6	225.2	40.4	39.5	5.93
203 × 133	30	206.8	133.9	6.4	9.6	7.6	172.4	56.2	26.9	5.85
	25	203.2	133.2	5.7	7.8	7.6	172.4	56.2	30.2	7.20
203 × 102	23	203.2	101.8	5.4	9.3	7.6	169.4	40.6	31.4	4.37
178 × 102	19	177.8	101.2	4.8	7.9	7.6	146.8	40.6	30.6	5.14
152 × 89	16	152.4	88.7	4.5	7.7	7.6	121.8	34.5	27.1	4.48
127 × 76	13	127.0	76.0	4.0	7.6	7.6	96.6	28.4	24.2	3.74

Note: Sections marked * are not in BS4, but are marketed by Corus under the trademarked UKB designation.

Table 5.7: Universal Beams: properties

Serial size	Mass per metre (kg)	Second moment of area I_y (cm^4)	Second moment of area I_z (cm^4)	Radius of gyration i_y (cm)	Radius of gyration i_z (cm)	Elastic modulus $W_{el,y}$ (cm^3)	Elastic modulus $W_{el,z}$ (cm^3)	Plastic modulus $W_{pl,y}$ (cm^3)	Plastic modulus $W_{pl,z}$ (cm^3)	Warping constant I_w (dm^6)	Torsion constant I_T (cm^4)	Shear area A_v (cm^2)	Area A (cm^2)
1016 × 305	487*	1021884	26721	40.6	6.57	19722	1732	23208	2799	64.4	4299	335	620
	437*	910322	23447	40.4	6.49	17743	1535	20769	2467	56.0	3185	300	557
	393*	807503	20496	40.2	6.40	15897	1353	18538	2167	48.4	2330	271	500
	349*	723131	18460	40.3	6.44	14346	1223	16593	1940	43.3	1718	236	445
	314*	644063	16232	40.1	6.37	12883	1082	14850	1712	37.7	1264	213	400
	272*	553974	14004	40.0	6.35	11190	934	12827	1469	32.2	835	185	347
	249*	481192	11754	39.0	6.09	9819	784	11350	1244	26.8	582	181	317
	222*	407961	9546	38.0	5.81	8409	636	9808	1019	21.5	390	172	283
914 × 419	388	719635	45438	38.2	9.59	15627	2161	17666	3340	88.9	1734	212	494
	343	625780	39156	37.8	9.46	13726	1871	15478	2889	75.8	1193	191	437
914 × 305	289	504187	15597	37.0	6.51	10883	1014	12570	1601	31.2	926	190	368
	253	436305	13301	36.8	6.42	9501	871	10942	1370	26.4	626	168	323
	224	376414	11236	36.3	6.27	8269	739	9535	1163	22.1	422	153	286
	201	325254	9423	35.7	6.07	7204	621	8352	982	18.4	291	144	256
838 × 292	226	339704	11360	34.3	6.27	7985	773	9155	1211	19.3	514	145	289
	194	279175	9066	33.6	6.06	6641	620	7640	974	15.2	306	131	247
	176	246021	7799	33.1	5.90	5893	535	6808	842	13.0	221	124	224
762 × 267	197	239957	8175	30.9	5.71	6234	610	7167	958	11.3	404	127	251
	173	205282	6850	30.5	5.58	5387	514	6198	807	9.39	267	115	220
	147	168502	5455	30.0	5.40	4470	411	5156	647	7.40	159	102	187
	134	150692	4788	29.7	5.30	4018	362	4644	570	6.46	119	95.6	171
686 × 254	170	170326	6630	28.0	5.53	4916	518	5631	811	7.42	308	106	217
	152	150355	5784	27.8	5.46	4374	455	5001	710	6.42	220	96.3	194
	140	136267	5183	27.6	5.39	3987	409	4558	638	5.72	169	90.2	178
	125	117992	4383	27.2	5.24	3481	346	3994	542	4.80	116	84.3	159
610 × 305	238	209471	15837	26.3	7.23	6589	1017	7486	1574	14.5	785	124	303
	179	153024	11408	25.9	7.07	4935	743	5548	1144	10.2	340	94.2	228
	149	125876	9308	25.7	7.00	4111	611	4594	937	8.17	200	78.8	190
610 × 229	140	111777	4505	25.0	5.03	3622	391	4142	611	3.99	216	85.0	178
	125	98610	3932	24.9	4.97	3221	343	3676	535	3.45	154	76.9	159
	113	87318	3434	24.6	4.88	2874	301	3281	469	2.99	111	71.3	144
	101	75780	2915	24.2	4.75	2515	256	2881	400	2.52	77	66.9	129

610 × 178	100*	72528	1658	23.8	3.60	2388	185	2786	296	1.44	95	72.4	128
	92*	64577	1436	23.4	3.50	2142	161	2511	258	1.24	71	69.3	117
	82*	55869	1207	23.2	3.40	1867	136	2194	218	1.04	48.8	63.2	104
533 × 312	272	198578	20615	23.9	7.70	6882	1288	7859	1985	15	1288	125	348
	219	150976	15589	23.3	7.48	5389	982	6109	1514	11	642	106	279
	182	123222	12667	23.1	7.40	4475	806	5030	1237	8.77	373	87.6	231
	150	100633	10285	22.9	7.32	3710	659	4142	1009	7.01	216	72.9	192
533 × 210	138*	86088	3864	22.1	4.68	3136	361	3613	568	2.67	250	84.6	176
	122	76043	3388	22.1	4.67	2793	320	3196	500	2.32	178	73.2	155
	109	66822	2943	21.9	4.60	2477	279	2829	436	1.99	126	66.6	139
	101	61519	2692	21.9	4.57	2292	256	2612	399	1.81	101	61.9	129
	92	55227	2389	21.7	4.51	2072	228	2360	355	1.6	75.7	57.6	117
	82	47539	2007	21.3	4.38	1800	192	2059	300	1.33	51.5	54.2	105
533 × 165	85*	48631	1275	21.2	3.44	1818	153	2107	243	0.857	73.8	59.0	108
	74*	41058	1040	20.8	3.30	1552	125	1808	200	0.691	47.9	54.8	95.2
	66*	35028	859	20.5	3.20	1335	104	1561	166	0.566	32	50.0	83.7
457 × 191	161*	79779	4250	19.7	4.55	3243	426	3778	672	2.25	515	90.2	206
	133*	63841	3350	19.4	4.44	2657	341	3070	535	1.73	292	75.8	170
	106*	48873	2515	19.0	4.32	2083	259	2389	405	1.27	146	61.6	135
	98	45727	2347	19.1	4.33	1957	243	2232	379	1.18	121	55.9	125
	89	41015	2089	19.0	4.29	1770	218	2014	338	1.04	90.7	51.3	114
	82	37051	1871	18.8	4.23	1611	196	1831	304	0.922	69.2	48.1	104
	74	33319	1671	18.8	4.20	1458	176	1653	272	0.818	51.8	43.7	94.6
	67	29380	1452	18.5	4.12	1296	153	1471	237	0.705	37.1	40.9	85.5
457 × 152	82	36589	1185	18.7	3.37	1571	153	1812	240	0.591	89.2	51.7	105
	74	32674	1047	18.6	3.33	1414	136	1627	213	0.518	65.9	47.1	94.5
	67	28927	913	18.4	3.27	1263	119	1453	187	0.448	47.7	43.8	85.6
	60	25500	795	18.3	3.23	1122	104	1287	163	0.387	33.8	39.4	76.2
	52	21369	645	17.9	3.11	950	85	1096	133	0.311	21.4	36.5	66.6
406 × 178	85*	31703	1830	17.1	4.11	1520	201	1733	313	0.728	93	48.1	109
	74	27310	1545	17.0	4.04	1323	172	1501	267	0.608	62.8	41.9	94.5
	67	24331	1365	16.9	3.99	1189	153	1346	237	0.533	46.1	38.6	85.5
	60	21596	1203	16.8	3.97	1063	135	1200	209	0.466	33.3	34.6	76.5
	54	18722	1021	16.5	3.85	930	115	1055	178	0.392	23.1	33.3	69
406 × 140	53*	18283	635	16.4	3.06	899	89	1031	139	0.246	29	34.6	67.9
	46	15685	538	16.4	3.03	778	76	888	118	0.207	19	29.8	58.6
	39	12508	410	15.9	2.87	629	58	724	91	0.155	10.7	27.6	49.7

(Continued)

Table 5.7: Universal Beams: properties (Continued)

Serial size	Mass per metre (kg)	Second moment of area		Radius of gyration		Elastic modulus		Plastic modulus		Warping constant	Torsion constant	Shear area	Area
		I_y (cm^4)	I_z (cm^4)	i_y (cm)	i_z (cm)	$W_{el,y}$ (cm^3)	$W_{el,z}$ (cm^3)	$W_{pl,y}$ (cm^3)	$W_{pl,z}$ (cm^3)	I_w (dm^6)	I_T (cm^4)	A_v (cm^2)	A (cm^2)
356 × 171	67	19463	1362	15.1	3.99	1071	157	1211	243	0.412	55.7	35.7	85.5
	57	16038	1108	14.9	3.91	896	129	1010	199	0.33	33.4	31.5	72.6
	51	14136	968	14.8	3.86	796	113	896	174	0.286	23.8	28.7	64.9
	45	12066	811	14.5	3.76	687	95	775	147	0.237	15.8	26.8	57.3
356 × 127	39	10172	358	14.3	2.68	576	57	659	89	0.105	15.1	25.7	49.8
	33	8249	280	14.0	2.58	473	45	543	70	0.0812	8.79	23.1	42.1
305 × 165	54	11696	1063	13.0	3.93	754	127	846	196	0.234	34.8	26.6	68.8
	46	9899	896	13.0	3.90	646	108	720	166	0.195	22.2	22.5	58.7
	40	8503	764	12.9	3.86	560	93	623	142	0.164	14.7	20.1	51.3
305 × 127	48	9575	461	12.5	2.74	616	74	711	116	0.102	31.8	29.9	61.2
	42	8196	389	12.4	2.70	534	63	614	98	0.0846	21.1	26.4	53.4
	37	7171	336	12.3	2.67	471	54	539	85	0.0725	14.8	23.4	47.2
305 × 102	33	6501	194	12.5	2.15	416	38	481	60	0.0442	12.2	22.1	41.8
	28	5366	155	12.2	2.08	348	31	403	48	0.0349	7.4	19.8	35.9
	25	4455	123	11.9	1.97	292	24	342	39	0.0273	4.77	18.8	31.6
254 × 146	43	6544	677	10.9	3.52	504	92	566	141	0.103	23.9	20.2	54.8
	37	5537	571	10.8	3.48	433	78	483	119	0.0857	15.3	17.6	47.2
	31	4413	448	10.5	3.36	351	61	393	94	0.0660	8.55	16.4	39.7
254 × 102	28	4005	179	10.5	2.22	308	35	353	55	0.0280	9.57	17.8	36.1
	25	3415	149	10.3	2.15	266	29	306	46	0.0230	6.42	16.7	32
	22	2841	119	10.1	2.06	224	23	259	37	0.0182	4.15	15.6	28
203 × 133	30	2896	385	8.7	3.17	280	57	314	88	0.0374	10.3	14.6	38.2
	25	2340	308	8.6	3.10	230	46	258	71	0.0294	5.96	12.8	32
203 × 102	23	2105	164	8.5	2.36	207	32	234	50	0.0154	7.02	12.4	29.4
178 × 102	19	1356	137	7.5	2.37	153	27	171	42	0.0099	4.41	9.85	24.3
152 × 89	16	834	90	6.4	2.10	109	20	123	31	0.0047	3.56	8.18	20.3
127 × 76	13	473	56	5.4	1.84	75	15	84	23	0.0020	2.85	6.43	16.5

*See note to Table 5.6.

Table 5.8: Universal Columns: dimensions

Serial size	Mass per metre (kg)	Depth h (mm)	Breadth b (mm)	Web thickness t_w (mm)	Flange thickness t_f (mm)	Root radius r (mm)	Depth between fillets d (mm)	Flange outstand c (mm)	Web slenderness d/t_w	Flange slenderness c/t_f
356 × 406	634	474.6	424.0	47.6	77.0	15.2	290.2	173.0	2.25	6.10
	551	455.6	418.5	42.1	67.5	15.2	290.2	173.0	2.56	6.89
	467	436.6	412.2	35.8	58.0	15.2	290.2	173.0	2.98	8.11
	393	419.0	407.0	30.6	49.2	15.2	290.2	173.0	3.52	9.48
	340	406.4	403.0	26.6	42.9	15.2	290.2	173.0	4.03	10.91
	287	393.6	399.0	22.6	36.5	15.2	290.2	173.0	4.74	12.84
	235	381.0	394.8	18.4	30.2	15.2	290.2	173.0	5.73	15.77
356 × 368	202	374.6	374.7	16.5	27.0	15.2	290.2	163.9	6.07	17.59
	177	368.2	372.6	14.4	23.8	15.2	290.2	163.9	6.89	20.15
	153	362.0	370.5	12.3	20.7	15.2	290.2	163.9	7.92	23.59
	129	355.6	368.6	10.4	17.5	15.2	290.2	163.9	9.37	27.90
305 × 305	283	365.3	322.2	26.8	44.1	15.2	246.7	132.5	3.00	9.21
	240	352.5	318.4	23.0	37.7	15.2	246.7	132.5	3.51	10.73
	198	339.9	314.5	19.1	31.4	15.2	246.7	132.5	4.22	12.92
	158	327.1	311.2	15.8	25.0	15.2	246.7	132.5	5.30	15.61
	137	320.5	309.2	13.8	21.7	15.2	246.7	132.5	6.11	17.88
	118	314.5	307.4	12.0	18.7	15.2	246.7	132.5	7.09	20.56
	97	307.9	305.3	9.9	15.4	15.2	246.7	132.5	8.60	24.92
254 × 254	167	289.1	265.2	19.2	31.7	12.7	200.3	110.3	3.48	10.43
	132	276.3	261.3	15.3	25.3	12.7	200.3	110.3	4.36	13.09
	107	266.7	258.8	12.8	20.5	12.7	200.3	110.3	5.38	15.65
	89	260.3	256.3	10.3	17.3	12.7	200.3	110.3	6.38	19.45
	73	254.1	254.6	8.6	14.2	12.7	200.3	110.3	7.77	23.29

(Continued)

Table 5.8: Universal Columns: dimensions (Continued)

Serial size	Mass per metre (kg)	Depth h (mm)	Breadth b (mm)	Web thickness t_w (mm)	Flange thickness t_f (mm)	Root radius r (mm)	Depth between fillets d (mm)	Flange outstand c (mm)	Web slenderness d/t_w	Flange slenderness c/t_f
203 × 203	127*	241.4	213.9	18.1	30.1	10.2	160.8	87.7	2.91	8.88
	113*	235.0	212.1	16.3	26.9	10.2	160.8	87.7	3.26	9.87
	100*	228.6	210.3	14.5	23.7	10.2	160.8	87.7	3.70	11.09
	86	222.2	209.1	12.7	20.5	10.2	160.8	88.0	4.29	12.66
	71	215.8	206.4	10.0	17.3	10.2	160.8	88.0	5.09	16.08
	60	209.6	205.8	9.4	14.2	10.2	160.8	88.0	6.20	17.11
	52	206.2	204.3	7.9	12.5	10.2	160.8	88.0	7.04	20.35
	46	203.2	203.6	7.2	11.0	10.2	160.8	88.0	8.00	22.33
152 × 152	51*	170.2	157.4	11.0	15.7	7.6	123.6	65.6	4.18	11.24
	44*	166.0	155.9	9.5	13.6	7.6	123.6	65.6	4.82	13.01
	37	161.8	154.4	8.0	11.5	7.6	123.6	65.6	5.70	15.45
	30	157.6	152.9	6.5	9.4	7.6	123.6	65.6	6.98	19.02
	23	152.4	152.2	5.8	6.8	7.6	123.6	65.6	9.65	21.31

Note: Sections marked * are not in BS4, but are marketed by Corus under the trademarked UKC designation.

Table 5.9: Universal Columns: properties

Serial Size	Mass per metre (kg)	Second moment of area		Radius of gyration		Elastic modulus		Plastic modulus		Warping constant	Torsion constant	Shear area	Area
		I_y (cm^4)	I_z (cm^4)	i_y (cm)	i_z (cm)	$W_{el,y}$ (cm^3)	$W_{el,z}$ (cm^3)	$W_{pl,y}$ (cm^3)	$W_{pl,z}$ (cm^3)	I_w (dm^6)	I_T (cm^4)	A_v (cm^2)	A (cm^2)
356 × 406	634	274845	98125	18.4	11.0	11582	4629	14235	7108	38.800	13720	215	808
	551	226938	82671	18.0	10.9	9962	3951	12076	6058	31.100	9240	186	702
	467	183003	67834	17.5	10.7	8383	3291	10003	5034	24.300	5809	155	595
	393	146618	55367	17.1	10.5	6998	2721	8223	4154	18.900	3545	130	501
	340	122543	46853	16.8	10.4	6031	2325	6999	3544	15.500	2343	112	433
	287	99875	38677	16.5	10.3	5075	1939	5813	2949	12.300	1441	93.8	366
	235	79085	30993	16.3	10.2	4151	1570	4687	2383	9.540	812	75.7	299
356 × 368	202	66261	23688	16.1	9.6	3538	1264	3972	1919	7.160	558	67.5	257
	177	57118	20529	15.9	9.5	3103	1102	3455	1671	6.090	381	58.8	226
	153	48589	17553	15.8	9.5	2684	948	2965	1435	5.110	251	50.3	195
	129	40246	14611	15.6	9.4	2264	793	2479	1199	4.180	153	42.5	164
305 × 305	283	78872	24635	14.8	8.3	4318	1529	5105	2342	6.350	2034	101	360
	240	64203	20315	14.5	8.2	3643	1276	4247	1950	5.030	1271	85.8	306
	198	50904	16299	14.2	8.0	2995	1037	3440	1581	3.880	734	70.5	252
	158	38747	12569	13.9	7.9	2369	808	2681	1230	2.870	378	57.3	201
	137	32814	10700	13.7	7.8	2048	692	2297	1052	2.390	249	49.8	174
	118	27672	9059	13.6	7.8	1760	589	1958	895	1.980	161	43.2	150
	97	22249	7308	13.4	7.7	1445	479	1592	726	1.560	91.2	35.6	123

(Continued)

Table 5.9: Universal Columns: properties (Continued)

Serial Size	Mass per metre (kg)	Second moment of area		Radius of gyration		Elastic modulus		Plastic modulus		Warping constant	Torsion constant	Shear area	Area
		I_y (cm^4)	I_z (cm^4)	i_y (cm)	i_z (cm)	$W_{el,y}$ (cm^3)	$W_{el,z}$ (cm^3)	$W_{pl,y}$ (cm^3)	$W_{pl,z}$ (cm^3)	I_w (dm^6)	I_T (cm^4)	A_v (cm^2)	A (cm^2)
254 × 254	167	29998	9870	11.9	6.8	2075	744	2424	1137	1.630	626	58.9	213
	132	22529	7531	11.6	6.7	1631	576	1869	878	1.190	319	46.2	168
	107	17510	5928	11.3	6.6	1313	458	1485	697	0.898	172	38.1	136
	89	14268	4857	11.2	6.6	1096	379	1224	575	0.717	102	30.8	113
	73	11407	3908	11.1	6.5	898	307	992	465	0.562	57.6	25.6	93.1
203 × 203	127*	15437	4920	9.8	5.5	1279	460	1517	704	0.549	427	45.3	162
	113*	13301	4285	9.6	5.5	1132	404	1329	618	0.464	305	40.3	145
	100*	11298	3679	9.4	5.4	988	350	1148	534	0.386	210	35.4	127
	86	9449	3127	9.3	5.3	850	299	977	456	0.318	137	30.7	110
	71	7618	2537	9.2	5.3	706	246	799	374	0.250	80.2	24.3	90.4
	60	6125	2065	9.0	5.2	584	201	656	305	0.197	47.2	22.2	76.4
	52	5259	1778	8.9	5.2	510	174	567	264	0.167	31.8	18.7	66.3
	46	4568	1548	8.8	5.1	450	152	497	231	0.143	22.2	17.0	58.7
152 × 152	51*	3227	1022	7.0	4.0	379	130	438	199	0.061	48.8	19.9	65.2
	44*	2703	860	6.9	3.9	326	110	372	169	0.050	31.7	17.0	56.1
	37	2210	706	6.9	3.9	273	91	309	140	0.040	19.2	14.3	47.1
	30	1748	560	6.8	3.8	222	73	248	112	0.031	10.5	11.6	38.3
	23	1250	400	6.5	3.7	164	53	182	80	0.021	4.63	9.97	29.2

*See note to Table 5.8.

Local buckling and the classification of cross-sections

The web and flanges of rolled steel sections are comparatively slender in relation to their length and breadth. Consequently the compressive force induced in a beam by bending or compression could cause local buckling of the web or flange before the full plastic stress is developed as illustrated in Figure 5.5. This is to be distinguished from:

- Lateral torsional buckling (LTB) of the whole beam, see Section 5.5.1.1
- Web shear buckling, see Section 5.5.2
- Web buckling at supports, see Section 5.5.3.

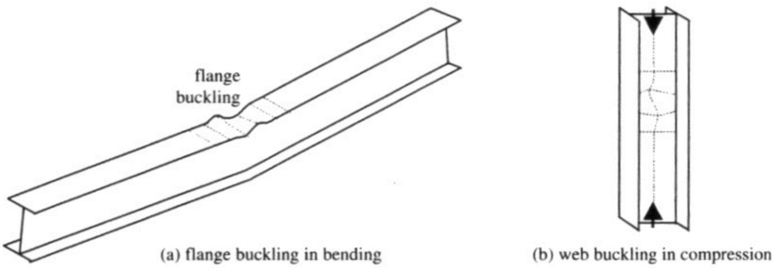


Figure 5.5: Flange and web buckling

Local buckling may be avoided by reducing the stress capacity of the section, and hence its bending moment capacity, if the slenderness of the web or the flange exceeds certain limits. For this purpose sections are divided into four classes, with Class 1 being least susceptible to local buckling and Class 4 most susceptible.

Class 1 Plastic
Class 2 Compact
Class 3 Semi-compact
Class 4 Slender

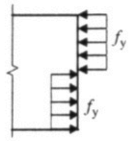
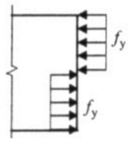
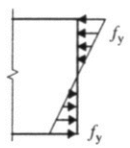
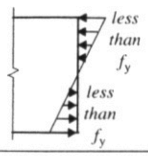
Note that EC3 defines the four numbered classes, while the terms *plastic, compact, semi-compact* and *slender* are not in EC3 but are commonly used in the UK.

If a high-strength steel is used then the stress needed to develop the full plastic strength is higher so local buckling failure is more likely to occur before plastic failure. For this reason, the limits between the classes depend on f_y, the yield strength of the steel, and are determined using the ε factor which is defined as

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

The slenderness limits for the four classes are given in Table 5.10, which also indicates stress distributions at maximum bending moment. The bending and compression limits for flanges are the same because in both cases the whole of a flange is in compression. When the section is subject to bending only part of the web is in compression, while when the section is subject to compression the whole of the web is in compression. For this reason the bending limits for webs are higher than the compression limits for webs. Example 5.1 shows how the class is calculated for two sample sections.

Table 5.10: Classification of cross-sections for bending and compression

Class	**Definition and limits**	**Stress distribution at maximum bending moment**
Class 1: Plastic. Design bending resistance $M_{c,Rd} = M_{pl,Rd} = W_{pl}f_y/\gamma_m$	Sections which can form a plastic hinge with the rotation required from plastic analysis without reduction of the resistance Flange outstand $c/t_f = <9\varepsilon$ Web in bending $d/t_w = <72\varepsilon$ Web in compression $d/t_w = <33\varepsilon$	
Class 2: Compact. Design bending resistance $M_{c,Rd} = M_{pl,Rd} = W_{pl}f_y/\gamma_m$	Sections which can develop their plastic moment of resistance but have limited rotation capacity because of local buckling Flange outstand $c/t_f = <10\varepsilon$ Web in bending $d/t_w = <83\varepsilon$ Web in compression $d/tw = <38\varepsilon$	
Class 3: Semi-compact. Design bending resistance $M_{c,Rd} = M_{el,Rd} = W_{el,min}f_y/\gamma_m$	Sections in which the extreme fibre can reach the yield strength but local buckling may prevent development of the plastic moment of resistance Flange outstand $c/t_f = <14\varepsilon$ Web in bending $d/t_w = <124\varepsilon$ Web in compression $d/t_w = <42\varepsilon$	
Class 4: Slender. Design bending resistance $M_{c,Rd} = W_{eff,min}f_y/\gamma_m$	Sections in which local buckling will occur before the yield strength is attained Flange outstand $c/t_f > 14\varepsilon$ Web in bending $d/t_w > 124\varepsilon$ Web in compression $d/t_w > 42\varepsilon$	

$\varepsilon = \sqrt{\dfrac{235}{f_y}}$	f_y	255	265	275	335	345	355
	ε	0.96	0.94	0.92	0.84	0.83	0.81

Source: EC3 Clause 5.5.2(1). Formulas for $M_{c,Rd}$ are from EC3 Clause 6.2.5(2).

Example 5.1

Determine the section classifications for the following steel sections:

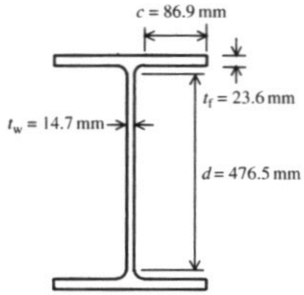
(a) 533 × 210 × 138 kg/m UB in grade S275 steel

(b) 356 × 171 × 45 kg/m UB section in grade S355 steel.

Calculations for Example 5.1

(a)

533 × 210 × 138 kg/m UB in grade S275 steel



16 mm < t_f < 40 mm so, from Table 5.3, $f_y = 265\,\text{N/mm}^2$ and $\varepsilon = 0.94$

Bending

Flange check

$c/t_f = 86.9/23.6 = 3.68$

$9\varepsilon = 9 \times 0.94 = 8.46$

so $c/t_f < 9\varepsilon$

and flange class is **Class 1 plastic**

Web check

$d/t_w = 476.5/14.7 = 32.4$

$72\varepsilon = 72 \times 0.94 = 67.7$

so $d/t_w < 72\varepsilon$ and web class is **Class 1 plastic**

Overall section class for bending is **Class 1 plastic.**

Compression

Flange is **Class 1 plastic** (as above)

Web check

$d/t_w = 476.5/14.7 = 32.4$ (as above)

$33\varepsilon = 33 \times 0.94 = 31.0$

$38\varepsilon = 38 \times 0.94 = 35.7$

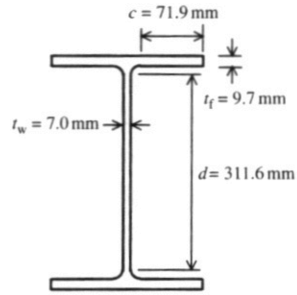
so $33\varepsilon < d/t_w < 38\varepsilon$

and web class is **Class 2 compact**

Overall class for compression is **Class 2 compact**

(b)

356 × 171 × 45 kg/m UB section in grade S355 steel



t_f < 16 mm so, from Table 5.3, $f_y = 355\,\text{N/mm}^2$ and $\varepsilon = 0.81$

Bending

Flange check

$c/t_f = 71.9/9.7 = 7.41$

$9\varepsilon = 9 \times 0.81 = 7.29$, $10\varepsilon = 10 \times 0.81 = 8.1$

so $9\varepsilon < c/t_f < 10\varepsilon$

and flange class is **Class 2 compact**

Web check

$d/t_w = 311.6/7.0 = 44.5$

$72\varepsilon = 72 \times 0.81 = 58.3$

so $d/t_w < 72\varepsilon$ and web class is **Class 1 plastic**

Overall section class for bending is **Class 2 compact.**

Compression

Flange is **Class 2 compact** (as above)

Web check

$d/t_w = 311.6/7.0 = 44.5$ (as above)

$42\varepsilon = 42 \times 0.81 = 34$

so $42\varepsilon < d/t_w$

and web class is **Class 4 slender**

Overall class for compression is **Class 4 slender**

Similar calculations can be carried out for all the UB and UC sections, and some of the results are given in Table 5.11. It is worth noting that:

- In Grade S275 steel all of the UB sections are Class 1 plastic for bending but they may be Class 1, 2, 3 or 4 for compression. In Grade S355 steel one of the UB sections is Class 2 compact for bending.
- Most of the UC sections are Class 1 plastic for bending and for compression but there are exceptions.

Table 5.11: Section classes for UB and UC sections in bending and compression

	Section class in bending				
	Steel grade	**Class 1 Plastic**	**Class 2 Compact**	**Class 3 Semi-compact**	**Class 4 Slender**
Universal Beam (UB) sections	S275	All UB sections	–	–	–
	S355	All other UB sections	356 × 171 × 45 UB	–	–
Universal Column (UC) sections	S275	All other UC sections	356 × 368 × 129 UC 305 × 305 × 97 UC	152 × 152 × 23 UC	–
	S355	All other UC sections	356 × 368 × 153 UC 254 × 254 × 73 UC 203 × 203 × 46 UC	356 × 368 × 129 UC 305 × 305 × 97 UC 152 × 152 × 23 UC	–
	Section class in compression				
	Steel grade	**Class 1 Plastic**	**Class 2 Compact**	**Class 3 Semi-compact**	**Class 4 Slender**
Universal Beam (UB) Sections	S275	Class varies			
	S355	Class varies			
Universal Column (UC) Sections	S275	All other UC sections	356 × 368 × 129 UC 305 × 305 × 97 UC	152 × 152 × 23 UC	–
	S355	All other UC sections	356 × 368 × 153 UC 254 × 254 × 73 UC 203 × 203 × 46 UC	356 × 368 × 129 UC 305 × 305 × 97 UC 152 × 152 × 23 UC	–

5.5 Beams

Steel beams in simple design should be checked against the following limit states:

1. Bending strength at ULS, including
   a) Local buckling of flange or web

b) LTB if the beam is not fully restrained

c) Plastic moment capacity

2. Shear strength at ULS
3. Resistance to transverse forces at ULS (web bearing and web buckling)
4. Deflection SLS.

These are described in the following sections of this manual.

5.5.1 Beam Bending ULS

5.5.1.1 Lateral-Torsional Buckling

When a simply-supported steel I beam bends under downward loads the extreme fibres at the top of the beam are in compression. This puts the top flange of the beam into compression, and the bottom flange into tension. The top flange therefore acts as a strut, and may buckle under the compressive force. Generally the web of the steel section will prevent the flange from buckling by deflecting vertically, but the flange can move horizontally by a combination of lateral movement and twisting. This is termed lateral-torsional buckling (often abbreviated to LT or LTB) and is illustrated in Figure 5.6.

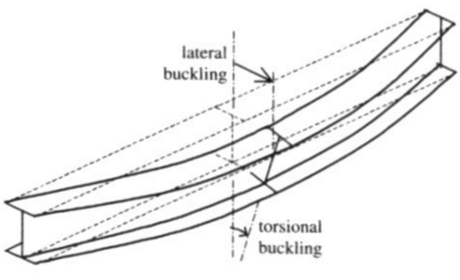


Figure 5.6: LTB of an unrestrained beam

LTB can be prevented by restraining the compression flange along its entire length, and methods of achieving this are shown in Figure 5.7. Alternatively, discrete transverse restraints can be provided at intervals along the beam, as shown in Figure 5.8, but it may still be possible for LTB to occur between the restraints as shown in Figure 5.9.

A beam may fail by LTB before it reaches its plastic moment capacity. Thus it is necessary to investigate the bending strength ULS of beams in two ways:

- laterally restrained
- laterally unrestrained.

These are discussed in the next two sections.

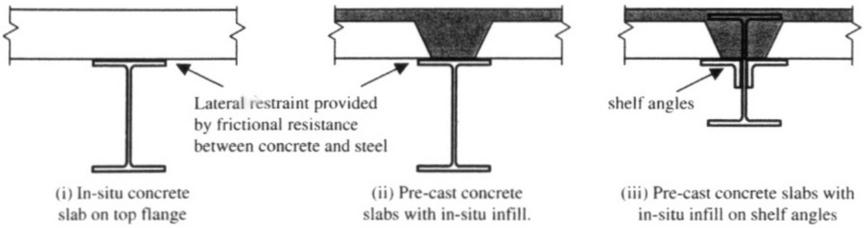


Figure 5.7: Cross-sections through beams with full restraint to compression flange

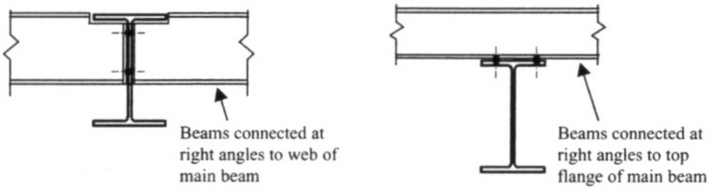


Figure 5.8: Cross-sections through beams with restraint to compression flange at intervals along the length

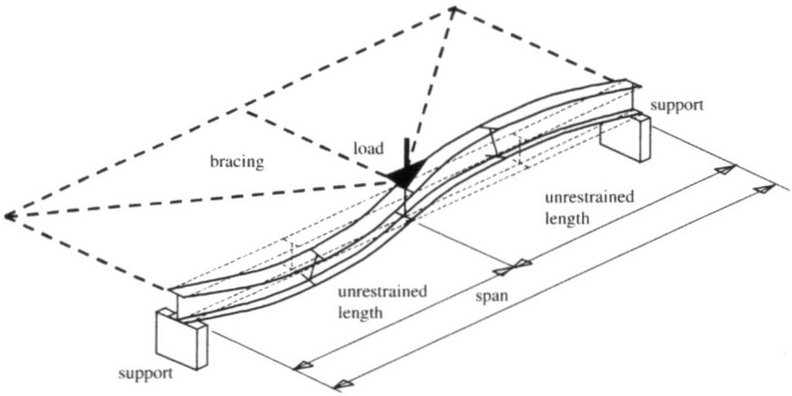


Figure 5.9: LTB of a beam with central restraint and central point load

5.5.1.2 Bending Strength ULS of Laterally Restrained Beams

Laterally restrained beams are not subject to LTB, so their bending strength is determined from the properties of the cross-section and the strength of the steel. Table 5.10 gives formulas for determining the design bending resistance $M_{c,Rd}$ for members, and it can be seen that for both Class 1 plastic and Class 2 compact members the resistance is given by:

$$M_{c,Rd} = M_{pl,Rd} = f_y W_{pl}/\gamma_m$$

or, since generally $\gamma_m = 1.0$,

$$M_{c,Rd} = f_y W_{pl}$$

Table 5.11 shows that all UB sections are plastic or compact in bending, so we can use this formula for all UB sections without checking their class. Example 5.2 shows how a UB section can be chosen to have sufficient bending strength for a particular application.

Example 5.2 Laterally restrained steel floor beam

Steel floor beams arranged as shown in Figure 5.10 support a reinforced concrete slab with a screed finish. Other design data:

- slab thickness 150 mm
- screed weight 1.2 kN/m^2
- imposed load on slabs 5.0 kN/m^2
- assumed mass of steel beams 75 kg/m

Choose a UB section in S275 steel that has sufficient bending capacity at ULS.

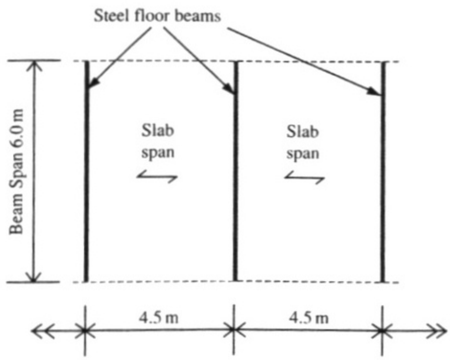


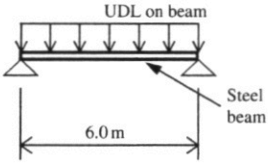
Figure 5.10: Structure for Example 5.2

Calculations for Example 5.2

Unit weight of reinforced concrete $= 25\,kN/m^2$

Concrete slab self-weight $= 0.15 \times 25$ $= 3.75\,kN/m^2$

Steel beam self-weight $= 75 \times 0.0098$ $= 0.74\,kN/m$



Loads on steel beam		Dead loads (self-weight)	Imposed loads
Screed	$1.2 \times 6.0 \times 4.5 =$	32.4 kN	–
Concrete slab	$3.75 \times 6.0 \times 4.5 =$	101.3 kN	–
Steel beam	$0.74 \times 6.0 =$	4.4 kN	–
Imposed load	$5.0 \times 6.0 \times 4.5 =$	–	135.0 kN
Totals		$G_k = 138.1\,kN$	$Q_k = 135.0\,kN$

Total UD load for ultimate limit states

$= \gamma_G G_k + \gamma_Q Q_k = 1.35 \times 138.1 + 1.50 \times 135.0$ $F = 388.9\,kN$

$M = FL/8 = 338.9 \times 6.0/8$ $M = 254.2\,kNm = 254.2 \times 10^6\,Nmm$

The beam supports a concrete slab so it is laterally restrained

Assume that the steel thickness does not exceed 16 mm:
From Table 5.3, the design strength f_y for steel grade S275 is $f_y = 275\,N/mm^2$

The design bending resistance of a plastic or compact UB section is given by

$M_{c,Rd} = f_y W_{pl}$

So the plastic section modulus W_{pl} required

$= M\gamma_m/f_y = 254.2 \times 10^6 \times 1.0/275$ W_{pl} required $= 924.4 \times 10^3\,mm^3 = 924.4\,cm^3$

From Table 5.7, the lightest UB section with a plastic modulus not less than $924.4\,cm^3$
is a $406 \times 140 \times 53\,kg/m$ UB with $W_{pl} = 1031\,cm^3$

The section mass is 53 kg/m, which is less than the assumed mass of 75 kg/m

No adjustment to the design is required

The flange thickness is 12.9 mm which is less than 16 mm
so it was correct to adopt $f_y = 275\,N/mm^2$

Subject to checks on shear and deflection,
adopt $406 \times 140 \times 53\,kg/m$ UB

5.5.1.3 Bending Strength ULS of Laterally Unrestrained Beams

If the beam shown in Figure 5.6 is perfectly straight, unloaded along its length and simply supported at the ends, then theoretically it will buckle under a uniform bending moment M_{cr} given by

$$M_{cr} = \frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}\right]} \quad \text{(Source: Document SN003)}$$

where L is the effective length of the beam, and the following properties can be found from Table 5.7 for UB sections or Table 5.9 for UC sections:

I_T = torsion constant
I_w = warping constant
I_z = second moment of area about the z-z axis.

M_{cr} is called the elastic critical buckling moment.

If the bending moment along the beam (or along the segment of beam between lateral restraints) is not uniform, then M_{cr} is modified by the C_1 factor from Table 5.12 and the C_2 factor discussed below to give

$$M_{cr} = C_1\frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z} + (C_2 z_g)^2\right]} - C_2 z_g$$

(Source: Document SN003a for the common case with $k = k_w = 1.0$)

The C_2 factor relates to destabilising loads, which exist when the load applied to the compression flange can move laterally with the beam as illustrated in Figure 5.11(a). This may be prevented by introducing stabilising members such as the secondary beams shown in Figure 5.11(b).

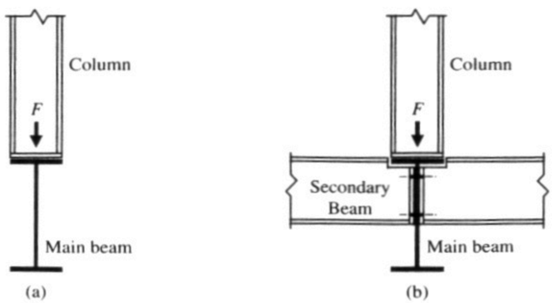


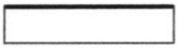
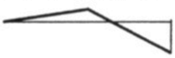
Figure 5.11: (a) Destabilising load and (b) stabilised detail

Methods of designing for destabilising loads are given in EC3 but are not covered by this manual. If the loads are not destabilising then $C_2 = 0$ and M_{cr} is given by

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}\right]} \text{ (Source: Document SN003 with } C_2=0 \text{ and } k=k_w=1.0)$$

Note that values of C_1 from Table 5.12 are never less than 1.0. If the bending moment is uniform along the beam then $C_1 = 1.0$ and M_{cr} is given by the earlier expression, and it will always be conservative to take $C_1 = 1.0$ for any beam.

Table 5.12: Values of C_1 for different bending moment diagrams

Shape of bending moment (BM) diagram	**Value of C_1**
M  M	$C_1 = 1.0$
M  ψM	$\psi = 0.8$, $C_1 = 1.11$ $\psi = 0.60$, $C_1 = 1.24$ $\psi = 0.40$, $C_1 = 1.39$ $\psi = 0.20$, $C_1 = 1.57$
M  0	$C_1 = 1.77$
M  ψM	$\psi = -0.20$, $C_1 = 1.99$ $\psi = -0.40$, $C_1 = 2.22$ $\psi = -0.60$, $C_1 = 2.43$ $\psi = -0.80$, $C_1 = 2.57$
M  -M	$C_1 = 2.55$
	BM from UD load with two pinned supports $C_1 = 1.13$
	BM from UD load with one fixed and one pinned support $C_1 = 2.88$
	BM from UD load with two fixed supports $C_1 = 2.58$
	BM from central point load with two pinned supports $C_1 = 1.35$
	BM from central point load with one fixed and one pinned support $C_1 = 1.38$
	BM from central point load with two fixed supports $C_1 = 1.68$

Source: Document SN003: NCCI elastic critical moment for lateral-torsional buckling, from www.access-steel.com.

It is worth noting that M_{cr} depends on the elastic modulus E and the shear modulus G of the steel but not on the yield strength f_y of the steel.

We now have two ways of looking at the bending strength of an unrestrained UB section:

1. If the section had full lateral restraint its strength would be $M_{c,Rd} = f_y W_{pl}/\gamma_m$; see Section 5.5.1.2.
2. If the section was perfectly straight but unrestrained its buckling strength would be M_{cr}, see Section 5.5.13.

Both of these are optimistic approximations, so the actual strength of the beam will be less than $M_{c,Rd}$ and less than M_{cr}.

Any real beam will not be perfectly straight and its buckling moment will be less than M_{cr}. The amount of imperfection is represented by the α_{LT} factor given in Table 5.13. Higher values of α_{LT} represent greater deviations from the ideal of a straight member.

Table 5.13: Values of the imperfection factor α_{LT} for LTB

	Limits	α_{LT}
UB and UC sections, hot-finished hollow sections	$h/b \le 2$ $2 < h/b \le 3.1$ $3.1 < h/b$	$\alpha_{LT} = 0.34$ $\alpha_{LT} = 0.49$ $\alpha_{LT} = 0.76$
Angles (for moments in major principal plane)	–	$\alpha_{LT} = 0.76$
All other hot-rolled sections	–	$\alpha_{LT} = 0.76$
Welded doubly symmetric sections, cold-rolled sections	$h/b \le 2$ $2 < h/b \le 3.1$	$\alpha_{LT} = 0.49$ $\alpha_{LT} = 0.76$

Source: UK National Annex to EC3, table in clause NA. 2.17.

The method of calculating the design buckling resistance moment $M_{b,Rd}$ for an unrestrained UB segment makes use of the imperfection factor α_{LT} and is shown in Table 5.14.

Table 5.14: Method of calculating the design buckling resistance moment $M_{b,Rd}$ for an unrestrained UB segment from Clause 6.3.2.3(1) of EC3

Determine the effective length L of the beam segment Determine the shape of the bending moment diagram for the beam segment Find the C_1 factor from Table 5.12 Using section properties from Table 5.7, determine the elastic critical buckling moment $M_{cr} = C_1 \dfrac{\pi^2 E I_z}{L^2} \sqrt{\left[\dfrac{I_w}{I_z} + \dfrac{L^2 G I_T}{\pi^2 E I_z}\right]}$
From Table 5.3 find f_y. From Table 5.7 find $W_{pl,}$ and hence find the plastic moment capacity $f_y W_{pl,y}$
(i) Calculate the non-dimensional slenderness $\bar{\lambda}_{LT} = \sqrt{\dfrac{f_y W_{pl,y}}{M_{cr}}}$

(*Continued*)

Table 5.14: (Continued)

(ii)	Calculate h/b for the section. From Table 5.13 select the correct value for the imperfection factor α_{LT}
(iii)	Take the limiting slenderness $\bar{\lambda}_{LT,0} = 0.4$ and the correction factor $\beta = 0.75$ (from EC3 Part 1–1 Clause NA.2.17)
(iv)	Calculate the buckling parameter $\Phi_{LT} = 0.5(1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2)$
(v)	Calculate the reduction factor $\chi_{LT} = \dfrac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}}$ but not more than 1.0 and not more than $\dfrac{1}{\bar{\lambda}_{LT}^2}$
(vi)	If $C_1 = 1.0$ then the enhancement factor $f = 1.0$, otherwise $f = 1 - 0.5\left(1 - \dfrac{1}{\sqrt{C_1}}\right)(1 - 2(\bar{\lambda}_{LT} - 0.8)^2)$ but not more than 1.0
(vii)	Calculate the modified reduction factor $\chi_{LT,mod} = \chi_{LT}/f$ but not more than 1.0
(viii)	Calculate the design buckling resistance moment $M_{b,Rd} = \chi_{LT,mod} f_y W_{pl,y}$

Example 5.3 Laterally unrestrained steel beam

Determine the design buckling resistance moment of an unrestrained 686 × 254 × 170 kg/m UB segment in S275 steel with an effective length of 5.0 m if the bending moment is constant along the segment.

Calculations for Example 5.3, using the method in Table 5.14

The effective length L is given	$L = 5000$ mm
The bending moment is constant along the segment, so from Table 5.12	$C_1 = 1.0$
From Table 5.2	$E = 210$ kN/mm^2
	$G = 81$ kN/mm^2
From Table 5.6, for 686 × 254 × 170 kg/m UB	$h = 629.2$ mm
	$b = 255.8$ mm
	$t_f = 23.7$ mm
From Table 5.7, for 686 × 254 × 170 kg/m UB	$I_z = 6630$ cm^4
	$W_{pl,y} = 5631$ cm^3
	$I_w = 7.42$ dm^6
	$I_T = 308$ cm^4
For the calculation of M_{cr} it is convenient to convert all the parameters into N and mm units	
Note that:	
1 kN/mm^2 = 10^3 N/mm^2	
1 cm^3 = 10^3 mm^3	

1 cm^4 $= 10^4$ mm^4

1 dm^6 $= 10^{12}$ mm^6

10^6 Nmm $= 1$ kNm

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

$$= 1.0 \times \frac{\pi^2 \times 210 \times 10^3 \times 6\,630 \times 10^4}{5\,000^2}\sqrt{\frac{7.42 \times 10^{12}}{6\,630 \times 10^4} + \frac{5\,000^2 \times 81 \times 10^3 \times 308 \times 10^4}{\pi^2 \times 210 \times 10^3 \times 6\,630 \times 10^4}}$$

$= 2.18 \times 10^9$ Nmm — $M_{cr} = 2180$ kNm

The steel is grade S275 and $t_f = 23.7$ mm, so from Table 5.3 — $f_y = 265$ N/mm^2

$f_y W_{pl,y} = 265 \times 5631 \times 10^3 = 1.492 \times 10^9$ Nmm — $f_y W_{pl,y} = 1492$ kNm

$$\bar{\lambda}_{LT} = \sqrt{\frac{f_y W_{pl,y}}{M_{cr}}} = \sqrt{\frac{1\,492}{2\,180}}$$ — $\bar{\lambda}_{LT} = 0.827$

h/b for the section $= 692.9/255.8 = 2.71$, so from Table 5.13 — $\alpha_{LT} = 0.49$

From the method in Table 5.14 — $\bar{\lambda}_{LT,0} = 0.4$

$\beta = 0.75$

$\Phi_{LT} = 0.5(1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}{}^2) = 0.5(1 + 0.49(0.827 - 0.4) + 0.75 \times 0.827^2)$ — $\Phi_{LT} = 0.861$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} = \frac{1}{0.861 + \sqrt{0.861^2 - 0.75 \times 0.827^2}}$$ — $\chi_{LT} = 0.747$

Check that χ_{LT} is not more than 1.0 and is not more than $\dfrac{1}{\bar{\lambda}_{LT}^2} = \dfrac{1}{0.827^2} = 1.46$ — Accept

$C_1 = 1.0$, so $f = 1.0$ — $f = 1.0$

$\chi_{LT,mod} = \chi_{LT}/f = 0.747/1.0$ — $\chi_{LT,mod} = 0.747$

Design buckling resistance moment $M_{b,Rd} = \chi_{LT,mod} f_y W_{pl,y} = 0.747 \times 1492$ kNm — $M_{b,Rd} = 1114$ kNm

Note that the design buckling resistance moment is less than both the 'ideal' strengths $M_{cr} = 2180$ kNm and $f_y W_{pl,y} = 1492$ kNm

Similar calculations can be carried out for other sections and effective lengths. Some results for beams in S275 steel are given in Table 5.15(a) and for S355 steel in Table 5.15(b).

Table 5.15(a): Buckling resistance moment $M_{b,Rd}$ (kNm) for unrestrained UBs of S275 steel bending about the y-y axis,
based on the method in Table 5.14 with $C_1 = 1.0$

Serial Size	Mass (kg/m)	$M_{pl,Rd}$	Effective length L (m)										
			2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
1016 × 305	487*	5918	5918	5918	5817	5509	5224	4716	4279	3906	3588	3315	3080
	437*	5296	5296	5296	5169	4885	4621	4148	3741	3394	3099	2848	2633
	393*	4727	4727	4727	4579	4319	4075	3637	3259	2937	2664	2434	2239
	349*	4397	4397	4397	4220	3969	3734	3309	2942	2630	2368	2149	1964
	314*	3935	3935	3935	3753	3523	3307	2915	2576	2289	2048	1847	1680
	272*	3399	3399	3399	3230	3027	2836	2486	2183	1925	1710	1531	1382
	249*	3008	3008	3008	2820	2635	2460	2140	1863	1629	1435	1276	1145
	222*	2599	2599	2573	2399	2233	2077	1790	1544	1338	1169	1031	919
914 × 419	388	4681	4681	4681	4681	4681	4650	4399	4151	3907	3669	3440	3223
	343	4102	4102	4102	4102	4102	4059	3831	3605	3381	3160	2947	2745
914 × 305	289	3331	3331	3331	3249	3117	2987	2726	2471	2232	2015	1824	1659
	253	2900	2900	2900	2816	2698	2579	2342	2109	1890	1692	1519	1370
	224	2527	2527	2527	2440	2333	2226	2010	1796	1595	1416	1260	1128
	201	2213	2213	2213	2122	2025	1927	1728	1532	1349	1187	1048	932
838 × 292	226	2426	2426	2426	2339	2238	2137	1934	1737	1553	1389	1246	1125
	194	2025	2025	2024	1936	1847	1758	1577	1401	1237	1093	970	867
	176	1804	1804	1795	1714	1633	1550	1383	1219	1069	937	826	733
762 × 267	197	1899	1899	1873	1786	1700	1614	1444	1283	1138	1013	908	819
	173	1642	1642	1611	1533	1456	1378	1222	1075	943	831	738	660
	147	1366	1366	1330	1263	1195	1126	989	859	744	648	568	504
	134	1277	1277	1232	1167	1100	1032	897	770	660	570	496	437
686 × 254	170	1492	1492	1459	1389	1320	1250	1114	987	874	778	698	631
	152	1325	1325	1291	1227	1163	1099	973	855	751	663	590	530
	140	1208	1208	1173	1113	1054	993	874	763	666	584	517	462
	125	1058	1058	1021	968	913	858	749	647	559	486	426	378
610 × 305	238	1984	1984	1984	1977	1909	1843	1716	1594	1480	1375	1279	1192
	179	1470	1470	1470	1453	1399	1345	1239	1136	1038	947	866	795
	149	1217	1217	1217	1198	1151	1105	1011	920	832	752	680	618
610 × 229	140	1098	1098	1047	991	935	880	774	678	597	530	474	429
	125	974	974	925	874	822	771	672	584	509	448	398	357
	113	869	869	822	774	727	679	587	505	436	380	336	300
	101	792	785	739	694	647	601	512	433	369	319	279	247

610 × 178	100*	738	660	588	524	467	417	339	282	241	210	186	167
	92*	691	604	535	472	418	370	297	244	207	179	157	141
	82*	603	522	460	404	354	312	247	202	169	145	127	113
533 × 312	272	2083	2083	2083	2083	2057	2014	1931	1850	1771	1693	1618	1545
	219	1619	1619	1619	1619	1586	1549	1475	1400	1326	1252	1180	1112
	182	1333	1333	1333	1332	1299	1267	1200	1131	1061	991	924	861
	150	1098	1098	1098	1093	1065	1037	977	915	850	785	724	666
533 × 210	138*	957	951	900	851	802	755	667	591	526	472	428	391
	122	847	839	793	747	702	658	576	505	445	396	357	324
	109	750	739	697	655	613	573	496	430	375	331	296	267
	101	692	681	642	602	562	523	450	388	336	295	263	236
	92	649	634	595	556	516	478	406	345	296	257	227	203
	82	566	549	514	479	443	407	341	286	243	209	183	163
533 × 165	85*	558	487	432	383	341	304	248	207	178	156	139	125
	74*	497	422	371	325	286	253	202	166	141	122	108	–
	66*	429	359	313	273	238	209	164	134	112.4	96.7	84.8	–
457 × 191	161*	1001	995	946	901	859	819	747	683	627	578	536	499
	133*	814	801	758	718	679	642	575	517	468	426	391	361
	106*	633	617	581	546	511	479	419	368	327	293	265	242
	98	591	575	541	507	475	443	386	337	298	265	239	218
	89	534	517	485	454	423	393	339	293	257	227	204	185
	82	504	484	452	421	390	360	306	261	226	199	177	160
	74	455	435	406	377	348	320	269	228	196	171	151	136
	67	405	385	359	332	305	278	232	194	165	143	126	112
457 × 152	82	480	436	401	367	335	306	257	220	192	170	152	138
	74	431	389	356	325	294	267	222	187	162	142	127	–
	67	400	355	323	291	262	235	191	160	136	119	106	–
	60	354	313	283	254	226	201	162	134	113	97.9	86.4	–
	52	301	263	236	209	185	163	128	104	87.2	74.8	65.5	–
406 × 178	85*	459	441	413	387	361	336	292	255	226	202	182	166
	74	413	392	365	339	313	289	246	212	185	163	146	133
	67	370	350	325	301	277	254	214	182	157	138	123	111
	60	330	311	288	265	243	222	185	156	133	116	103	92.2
	54	290	271	250	229	209	189	155	129	109	94.5	83.1	74.1

(Continued)

Table 5.15(a): Buckling resistance moment $M_{b,Rd}$ (kNm) for unrestrained UBs of S275 steel bending about the y-y axis, based on the method in Table 5.14 with $C_1 = 1.0$ (Continued)

Serial Size	Mass (kg/m)	$M_{pl,Rd}$	Effective length L (m)										
			2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
406 × 140	53*	284	245	221	197	175	156	126	105	89.3	77.9	69.1	–
	46	244	210	187	166	146	129	102	83.9	70.8	61.2	54.0	–
	39	199	167	148	129	112	97.5	75.6	60.8	50.6	43.2	–	–
356 × 171	67	333	315	294	273	253	235	202	175	154	137	124	113
	57	278	261	242	223	205	188	159	136	118	104	92.4	83.6
	51	246	230	213	196	179	163	136	115	98.3	85.9	76.3	68.6
	45	213	197	182	166	151	136	112	92.9	78.9	68.3	60.2	53.8
356 × 127	39	181	148	130	113	99.1	87.2	69.4	57.4	48.8	42.6	–	–
	33	149	119	103	88.7	76.4	66.3	51.7	42.0	35.3	–	–	–
305 × 165	54	233	223	212	200	188	176	153	134	118	105	94.2	85.6
	46	198	189	179	168	157	146	125	107	92.7	81.6	72.8	65.7
	40	171	163	153	144	133	123	103	87.0	74.6	65.0	57.5	51.6
305 × 127	48	196	165	149	134	121	110	92.1	79.1	69.2	61.6	–	–
	42	169	140	125	111	99.5	89.4	73.7	62.5	54.3	48.0	–	–
	37	148	122	108	95.0	84.0	74.8	60.8	51.0	44.0	38.7	–	–
305 × 102	33	132	96.6	82.5	70.7	61.3	53.9	43.2	36.1	–	–	–	–
	28	111	78.2	65.6	55.2	47.2	41.0	32.3	26.6	–	–	–	–
	25	94.1	63.8	52.5	43.6	36.7	31.6	24.5	–	–	–	–	–
254 × 146	43	156	146	138	129	121	113	98.0	85.6	75.5	67.5	60.9	55.5
	37	133	124	116	108	100	92.6	78.8	67.7	59.0	52.2	46.8	42.4
	31	108	100	92.7	85.5	78.2	71.1	58.8	49.4	42.3	37.0	32.9	29.6
254 × 102	28	97.1	72.8	63.1	54.8	48.1	42.7	34.8	29.4	–	–	–	–
	25	84.2	61.3	52.4	44.9	38.9	34.2	27.5	22.9	–	–	–	–
	22	71.2	50.1	42.1	35.5	30.4	26.4	20.9	17.3	–	–	–	–
203 × 133	30	86.4	79.0	73.8	68.7	63.6	58.7	50.3	43.5	38.2	34.0	30.6	–
	25	71.0	64.1	59.4	54.6	49.8	45.3	37.8	32.1	27.8	24.5	21.9	–
203 × 102	23	64.4	53.5	48.3	43.3	38.8	35.0	29.0	24.7	21.5	–	–	–
178 × 102	19	47.0	39.1	35.2	31.6	28.3	25.5	21.1	18.0	15.6	–	–	–
152 × 89	16	33.8	27.2	24.4	21.9	19.7	17.8	14.9	12.7	–	–	–	–
127 × 76	13	23.1	18.1	16.3	14.7	13.3	12.1	10.2	–	–	–	–	–

Note: Values are not given for slenderness ratios exceeding 300.
*See note to Table 5.6.

Table 5.15(b): Buckling resistance moment $M_{b,Rd}$ (kNm) for unrestrained UBs of S355 steel bending about the *y-y* axis, based on the method in Table 5.14 with $C_1 = 1.0$

Serial size	Mass (kg/m)	$M_{pl,Rd}$	Effective length L (m)										
			2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
1016 × 305	487*	7775	7775	7727	7241	6793	6378	5646	5032	4521	4097	3743	3444
	437*	6958	6958	6873	6427	6013	5630	4951	4381	3909	3520	3198	2929
	393*	6210	6210	6094	5686	5306	4953	4326	3800	3366	3011	2719	2477
	349*	5725	5725	5581	5195	4835	4498	3899	3397	2986	2651	2378	2154
	314*	5123	5123	4967	4615	4285	3976	3425	2964	2588	2283	2036	1835
	272*	4425	4425	4278	3969	3678	3405	2915	2504	2168	1898	1680	1504
	249*	3916	3916	3740	3458	3193	2943	2497	2125	1825	1585	1393	1239
	222*	3384	3384	3185	2933	2696	2473	2077	1751	1489	1283	1119	989
914 × 419	388	6095	6095	6095	6095	6047	5855	5475	5096	4723	4363	4025	3713
	343	5340	5340	5340	5340	5279	5107	4762	4416	4073	3741	3430	3144
914 × 305	289	4337	4337	4275	4076	3877	3676	3278	2898	2557	2263	2016	1810
	253	3775	3775	3707	3529	3350	3169	2807	2461	2152	1887	1667	1486
	224	3290	3290	3214	3054	2893	2728	2399	2085	1805	1569	1375	1217
	201	2881	2881	2797	2652	2505	2355	2053	1768	1517	1307	1137	999
838 × 292	226	3158	3158	3081	2928	2774	2619	2310	2019	1762	1545	1365	1218
	194	2636	2636	2551	2418	2283	2146	1873	1616	1392	1205	1053	931
	176	2349	2349	2261	2139	2014	1888	1635	1399	1195	1028	892	784
762 × 267	197	2473	2473	2355	2224	2092	1960	1704	1472	1276	1115	985	880
	173	2138	2138	2023	1906	1787	1667	1434	1225	1050	908	796	706
	147	1779	1769	1669	1567	1462	1357	1153	971	821	702	609	535
	134	1649	1628	1532	1433	1332	1230	1033	861	722	613	528	461
686 × 254	170	1943	1938	1832	1726	1619	1513	1308	1127	976	854	756	677
	152	1725	1715	1619	1522	1424	1326	1138	971	833	723	636	565
	140	1573	1559	1470	1380	1288	1196	1019	864	736	635	555	491
	125	1378	1359	1279	1197	1113	1029	868	728	614	525	455	400
610 × 305	238	2583	2583	2583	2491	2389	2288	2093	1908	1738	1586	1453	1337
	179	1914	1914	1910	1828	1746	1664	1502	1346	1202	1076	968	876
	149	1585	1585	1576	1505	1435	1364	1221	1083	957	847	753	675

(*Continued*)

Table 5.15(b): Buckling resistance moment $M_{b,Rd}$ (kNm) for unrestrained UBs of S355 steel bending about the *y-y* axis, based on the method in Table 5.14 with $C_1 = 1.0$ **(Continued)**

Serial size	Mass (kg/m)	$M_{pl,Rd}$	Effective length L (m)										
			2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
610 × 229	140	1429	1394	1308	1222	1137	1052	896	765	659	576	510	457
	125	1268	1232	1154	1076	997	919	775	654	558	484	426	379
	113	1132	1095	1024	952	879	807	673	563	476	409	358	317
	101	1023	980	913	843	774	705	579	477	399	340	294	259
610 × 178	100*	961	803	701	611	534	470	373	305	258	223	196	176
	92*	891	728	630	545	473	413	323	262	220	189	165	147
	82*	779	627	540	464	400	347	268	216	179	153	133	118
533 × 312	272	2711	2711	2711	2679	2612	2545	2413	2281	2152	2026	1907	1795
	219	2108	2108	2108	2067	2010	1952	1833	1711	1590	1473	1364	1265
	182	1735	1735	1735	1695	1645	1593	1486	1374	1261	1153	1053	965
	150	1429	1429	1429	1390	1347	1302	1206	1104	1001	903	814	737
533 × 210	138*	1246	1197	1121	1045	970	899	771	665	581	515	461	418
	122	1103	1056	985	916	847	781	662	565	489	429	382	345
	109	976	930	866	801	738	676	566	478	409	357	315	283
	101	901	856	796	735	675	617	513	430	366	317	279	249
	92	838	790	731	672	613	557	457	378	319	274	240	213
	82	731	684	631	577	523	472	382	312	260	222	193	170
533 × 165	85*	727	590	512	445	388	341	272	224	190	165	146	131
	74*	642	506	434	372	321	280	219	178	149	129	113	–
	66*	554	428	365	311	266	230	178	143	119	102	89	–
457 × 191	161*	1303	1252	1180	1111	1046	985	877	785	708	644	590	544
	133*	1059	1006	942	880	821	765	667	586	521	468	425	389
	106*	824	774	719	665	613	564	479	411	358	317	284	257
	98	770	721	669	618	568	521	439	375	325	286	256	231
	89	695	648	599	551	504	460	384	324	279	244	217	195
	82	650	601	553	506	459	416	342	286	243	212	187	168
	74	587	540	496	452	409	369	300	248	210	181	159	142
	67	522	478	437	397	357	319	257	210	176	151	132	117
457 × 152	82	625	539	485	434	388	347	283	238	204	179	160	144
	74	561	480	430	382	339	301	243	202	172	150	133	–
	67	516	434	385	339	297	261	207	170	144	125	110	–
	60	457	381	336	293	255	223	175	142	119	102	90	–
	52	389	319	279	241	207	179	137	110	91	78	68	–

406 × 178	85*	598	552	509	468	429	393	331	283	246	217	195	176
	74	533	485	445	406	368	333	275	231	199	174	155	139
	67	478	433	396	359	323	291	237	198	168	146	130	116
	60	426	383.9	349.8	315.8	283.2	253.3	204.1	168.1	142.0	122.6	107.8	96.2
	54	375	334.4	303.2	271.9	242.0	214.8	170.5	138.8	116.0	99.4	86.8	77.1
406 × 140	53*	366	297.6	260.6	226.4	196.7	172.0	135.5	110.9	93.8	81.3	71.8	–
	46	315	253.7	220.5	189.8	163.3	141.5	109.6	88.6	74.1	63.7	55.9	–
	39	257	201.3	172.4	146.1	123.8	105.8	80.2	63.7	52.6	44.8	–	–
356 × 171	67	430	389.9	357.9	326.8	297.5	270.7	226.0	192.1	166.5	146.8	131.4	119
	57	359	321.9	293.5	265.7	239.3	215.4	176.0	147.1	125.7	109.7	97.4	87.5
	51	318	283.8	257.8	232.0	207.7	185.6	149.7	123.6	104.8	90.7	80.0	71.7
	45	275	243.1	219.6	196.2	174.1	154.2	122.4	99.7	83.6	71.8	62.9	56.0
356 × 127	39	234	176.4	150.2	127.6	109.1	94.5	73.7	60.2	50.9	44.2	–	–
	33	193	141.2	118.4	98.9	83.4	71.3	54.5	43.9	36.6	–	–	–
305 × 165	54	300	278.7	261.0	242.3	223.3	204.9	172.1	146.2	126.4	111.1	99.2	89.5
	46	256	236.0	219.9	202.8	185.2	168.2	138.4	115.7	98.8	86.0	76.1	68.3
	40	221	203.1	188.4	172.6	156.3	140.6	113.6	93.5	78.9	68.1	59.8	53.4
305 × 127	48	252	198.6	174.6	153.6	136.0	121.5	99.6	84.3	73.2	64.7	–	–
	42	218	168.0	145.9	126.8	110.9	98.0	79.1	66.2	57.0	50.2	–	–
	37	191	145.3	125.0	107.4	93.0	81.5	64.9	53.8	46.0	40.3	–	–
305 × 102	33	171	112.3	92.8	77.6	66.2	57.5	45.5	37.7	–	–	–	–
	28	143	90.2	73.1	60.1	50.5	43.4	33.8	27.6	–	–	–	–
	25	121.4	72.9	58.0	47.0	39.1	33.3	25.5	–	–	–	–	–
254 × 146	43	201	181.4	168.4	155.0	141.9	129.6	108.8	92.8	80.7	71.4	64.0	58.0
	37	171	153.6	141.5	129.0	116.7	105.3	86.5	72.7	62.5	54.8	48.8	44.0
	31	140	123.3	112.4	100.9	89.8	79.7	63.7	52.5	44.5	38.5	34.0	30.5
254 × 102	28	125.3	85.2	71.5	60.7	52.4	45.9	36.8	30.8	–	–	–	–
	25	108.6	71.3	58.9	49.3	42.0	36.5	28.9	23.9	–	–	–	–
	22	91.9	57.8	46.9	38.6	32.6	28.0	21.9	17.9	–	–	–	–
203 × 133	30	111.5	97.6	89.4	81.1	73.4	66.4	55.0	46.7	40.5	35.7	32.0	–
	25	91.6	78.9	71.4	63.8	56.7	50.5	40.8	34.0	29.1	25.5	22.7	–
203 × 102	23	83.1	64.2	56.0	48.8	42.8	37.9	30.8	25.9	22.4	–	–	–
178 × 102	19	60.7	46.9	40.9	35.6	31.2	27.6	22.4	18.8	16.2	–	–	–
152 × 89	16	43.7	32.2	28.0	24.4	21.5	19.2	15.8	13.3	–	–	–	–
127 × 76	13	29.8	21.3	18.6	16.3	14.5	13.0	10.8	–	–	–	–	–

The following example investigates an unrestrained beam segment with non-uniform bending moment.

Example 5.4 Steel beam with central lateral restraint

A 533 × 165 × 66 kg/m UB in S275 steel has a simply supported span of 8.0 m and carries a single central point ULS load of 150 kN which includes an allowance for the self-weight of the beam (see Figure 5.12). The beam is laterally restrained at both ends and at the centre but unrestrained elsewhere. Determine whether the bending strength of the beam is sufficient.

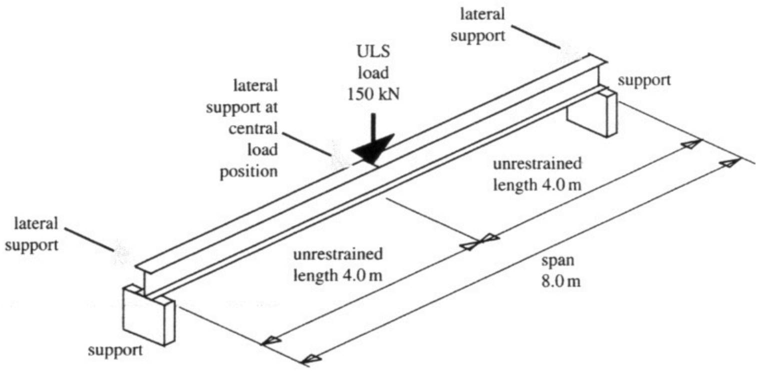
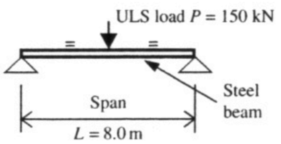
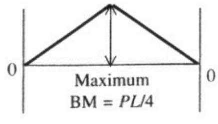


Figure 5.12: Beam in Example 5.4

Calculations for Example 5.4, using the method in Table 5.14





Bending moment diagram

Bending moment from applied ULS load $M_{Ed} = PL/4 = 150 \times 8/4$	$M_{Ed} = 300$ kNm
The effective unrestrained length L is 4.0 m	$L = 4000$ mm
The bending moment along the 4 m segment varies linearly from zero at one end to maximum at the other end, so from Table 5.12	$C_1 = 1.77$
From Table 5.2	$E = 210$ kN/mm^2 $G = 81$ kN/mm^2
From Table 5.6, for 533 × 165 × 66 kg/m UB	$h = 524.7$ mm $b = 165.1$ mm $t_f = 11.4$ mm

From Table 5.7, for 533 × 165 × 66 kg/m UB

$I_z = 859\,\text{cm}^4$
$W_{pl,y} = 1561\,\text{cm}^3$
$I_w = 0.566\,\text{dm}^6$
$I_T = 32\,\text{cm}^4$

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}\right]}$$

$$= 1.77 \times \frac{\pi^2 \times 210 \times 10^3 \times 859 \times 10^4}{4{,}000^2}\sqrt{\left[\frac{0.566 \times 10^{12}}{859 \times 10^4} + \frac{4{,}000^2 \times 81 \times 10^3 \times 32 \times 10^4}{\pi^2 \times 210 \times 10^3 \times 859 \times 10^4}\right]}$$

$= 588 \times 10^6\,\text{Nmm}$ $\qquad M_{cr} = 588\,\text{kNm}$

The steel is grade S275 and $t_f = 11.4\,\text{mm}$, so from Table 5.3 $\qquad f_y = 275\,\text{N/mm}^2$

$f_y W_{pl,y} = 275 \times 1561 \times 10^3 = 429 \times 10^6\,\text{Nmm}$ $\qquad f_y W_{pl,y} = 429\,\text{kNm}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{f_y W_{pl,y}}{M_{cr}}} = \sqrt{\frac{429}{588}}$$ $\qquad \bar{\lambda}_{LT} = 0.854$

h/b for the section = 524.7/165.1 = 3.18, so from Table 5.13 $\qquad \alpha_{LT} = 0.76$

From Table 5.14 $\qquad \bar{\lambda}_{LT,0} = 0.4$

$\beta = 0.75$

$$\Phi_{LT} = 0.5(1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^{\,2}) = 0.5(1 + 0.76(0.854 - 0.4) + 0.75 \times 0.854^2)$$ $\qquad \Phi_{LT} = 0.946$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} = \frac{1}{0.946 + \sqrt{0.946^2 - 0.75 \times 0.854^2}}$$ $\qquad \chi_{LT} = 0.651$

Check that χ_{LT} is not more than 1.0 and is not more than $\frac{1}{\bar{\lambda}_{LT}^2} = \frac{1}{0.854^2} = 1.37$ $\qquad$ Accept

C_1 is not equal to 1.0, so f must be calculated:

$$f = 1 - 0.5\left(1 - \frac{1}{\sqrt{C_1}}\right)(1 - 2(\bar{\lambda}_{LT} - 0.8)^2) = 1 - 0.5\left(1 - \frac{1}{\sqrt{1.77}}\right)(1 - 2(0.857 - 0.8)^2)$$

but not more than 1.0 $\qquad f = 0.877$

$\chi_{LT,mod} = \chi_{LT}/f = 0.651/0.877$ but not more than 1.0 $\qquad \chi_{LT,mod} = 0.742$

Design buckling resistance moment $M_{b,Rd} = \chi_{LT,mod} f_y W_{pl,y} = 0.742 \times 429\,\text{kNm}$ $\qquad M_{c,Rd} = 318\,\text{kNm}$

$M_{Ed}/M_{c,Rd} = 300/318 = 0.94 < 1.0$

so the bending strength of the beam is sufficient

Note that Table 5.15(a) gives $M_{b,Rd} = 209\,\text{kNm}$ for a 533 × 165 × 66 kg/m UB with an effective length of 4.0 m

The difference is because Table 5.15 is based on beam segments with a uniform moment and $C_1 = 1.0$, whereas the current example has a non-uniform moment and $C_1 = 1.77$

5.5.2 Beam Shear Strength ULS

Clause 6.2.6(2) of EC3 gives the design plastic shear resistance of a UB section for shear parallel to the web as

$$V_{pl,Rd} = A_v(f_y/\sqrt{3})/\gamma_m$$

where A_v is the shear area.

As we can always take $\gamma_m = 1.0$, this can be written as

$$V_{pl,Rd} = A_v f_y/\sqrt{3} = 0.577 f_y A_v$$

The shear area $A_v = A - 2bt_f + (t_w + 2r)t_f$ and is the shaded areas shown in Figure 5.13. A_v for UB sections is given in Table 5.7 and for UC sections in Table 5.9.

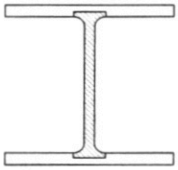


Figure 5.13: Shear area for a rolled section loaded parallel to the web

Web shear buckling

If a section has a slender web then its shear capacity may be limited by web shear buckling as illustrated in Figure 5.14. Web stiffeners may be provided to prevent shear buckling of the web, and these are commonly required in plate girders on bridges.

Clause 6.2.6(6) of EC3 Part 1-1 states that shear buckling of the web does not need to be considered unless h_w/t_w is more than 72ε, where h_w is the height of the web as shown in

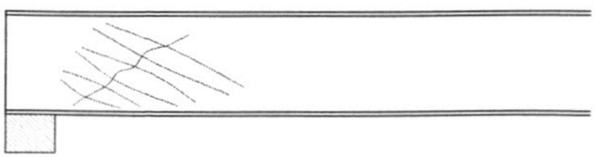


Figure 5.14: Shear buckling of a web

Figure 5.4 and is given by $h_w = d - 2t_f$. Table 5.16 shows that this condition only applies to two of the UB sections when Grade S355 steel is used. For all other UB sections and for

Table 5.16: Web shear buckling check for UB and UC sections in shear parallel to web

	Steel grade	Check required	Check not required
UB sections	S275	–	All UB sections
	S355	762 × 267 × 134 kg/m UB 406 × 140 × 39 kg/m UB	All other UB sections
UC sections	S275	–	All UC sections
	S355	–	All UC sections

all UC sections shear buckling does not need to be considered. Procedures for checking web shear buckling are given in EC3 Part 1–5 but are not covered by this manual.

Reduction of moment capacity in area of high shear force

If the design shear force V_{Ed} is less than half the plastic shear resistance $V_{pl,Rd}$ then there is no reduction in moment capacity.

If the design shear force V_{Ed} is more than half the plastic shear resistance $V_{pl,Rd}$ then the design bending resistance should be calculated using the reduced yield strength $(1 - (2V_{Ed}/V_{pl,Rd} - 1)^2)f_y$, and values of this factor are given in Table 5.17.

Table 5.17: Reduced yield strengths for bending resistance in areas of high shear force

$V_{Ed}/V_{pl,Rd}$	0.5 or less	0.6	0.7	0.8	0.9	1.0
Yield strength for calculating moment resistance	f_y	$0.96f_y$	$0.84f_y$	$0.64f_y$	$0.36f_y$	zero

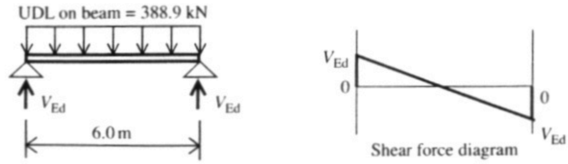
Source: EC3 Part 1–1 Clause 6.2.8, formula $(1\text{-}(2V_{Ed}/V_{pl,Rd} - 1)^2)f_y$.

Example 5.5 Shear capacity of a beam

Check whether the shear capacity of the beam in Example 5.2 is adequate at ULS. The structure is shown in Figure 5.10.

Calculations for Example 5.5

See the data and beam load calculations for Example 5.2



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Calculations for Example 5.5 (Continued from previous page)	
As the beam is symmetric, each reaction $V_{Ed} = 388.9/2$	$V_{Ed} = 194.5\,kN$
From the calculations for Example 5.2, the chosen beam section is a $406 \times 140 \times 53\,kg/m$ UB	
From Table 5.16, a web shear buckling check is not required	
From Table 5.7, the shear area A_v for this section is $34.6\,cm^2$	$A_v = 34.6\,cm^2$
From Table 5.6, web thickness	$t_w = 7.9\,mm$
From Table 5.3 with S275 steel and $t = 7.9\,mm$	$f_y = 275\,N/mm^2$
$V_{pl,Rd} = 0.577 f_y A_v = 0.577 \times 275 \times 34.6 \times 10^2 = 549016\,N$	$V_{pl,Rd} = 549\,kN$
V_{Ed} is less than $V_{pl,Rd}$	Accept
As the maximum bending moment is in the centre of the beam where the shear force is zero, it is not necessary to check for reduced bending moment capacity in an area of high shear force	Section is satisfactory in shear

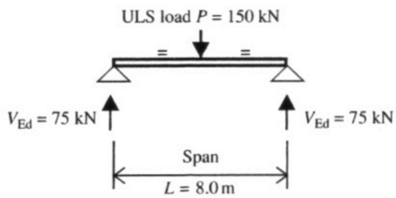
The following example does require a check for reduced bending moment capacity.

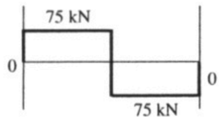
Example 5.6 Shear capacity of a beam

Check whether the shear capacity of the beam in Example 5.4 is adequate at ULS.

Calculations for Example 5.6

See the data for Example 5.4





Shear force diagram

The beam section is a $533 \times 165 \times 66\,kg/m$ UB in S275 steel	
From Table 5.16, a web shear buckling check is not required	
From Table 5.7, the shear area A_v for this section is $50.0\,cm^2$	$A_v = 50.0\,cm^2$
From Table 5.6, web thickness	$t_w = 8.9\,mm$
From Table 5.3 with S275 steel and $t = 8.9\,mm$	$f_y = 275\,N/mm^2$
$V_{pl,Rd} = 0.577 f_y A_v = 0.577 \times 275 \times 50.0 \times 10^2 = 793375\,N$	$V_{pl,Rd} = 793\,kN$
V_{Ed} is less than $V_{pl,Rd}$	Accept
As the maximum bending moment is in the centre of the beam where the shear force is not zero, it is necessary to check for reduced bending moment capacity in an area of high shear force	
$V_{Ed}/V_{pl,Rd} = 75/793 = 0.095$	
From Table 5.17, since $V_{Ed}/V_{pl,Rd}$ is less than 0.5 then no reduction in bending resistance is required	Section is satisfactory in shear

5.5.3 Beam Resistance to Transverse Forces

A beam subject to transverse forces, for instance at a bearing or where a point load is applied, should be checked for web bearing and buckling. Figure 5.15 illustrates that the transverse force is dispersed through the beam flange into the web, so the bearing resistance is the strength of an area of steel l_y long and t_w thick, and web buckling may further limit the resistance. EC3 Part 1-5 Section 6 gives the method of calculating the resistance which takes into account both bearing and buckling. The examples in this manual concern transverse forces from bearings at the beam supports, although similar principles apply wherever concentrated loads are applied to a beam. The relevant dimensions and parameters are shown in Figure 5.16.

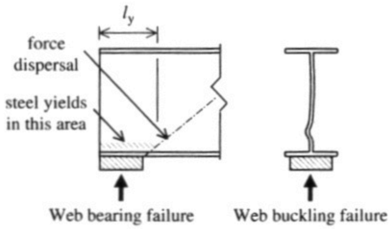


Figure 5.15: Web buckling and web bearing at a support

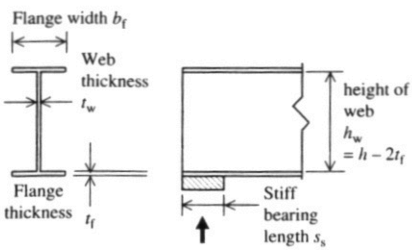


Figure 5.16: Dimensions for check on design resistance to transverse forces

Table 5.18 gives a simplified method for determining the design resistance to transverse forces based on EC3 Part 1-5, and Example 5.7 shows how the method is applied. This simplified method takes the factor $m_2 = 0$ in Equation 6.9 of EC3 Part 1-5 and may give a conservative value for the resistance. If $\overline{\lambda}_F$ is more than 0.5 then the more exact methods of EC3 Part 1-5 Clause 6 may give a higher resistance.

Table 5.18: Method for determining the design resistance to transverse forces at an end bearing

		Reference to EC3 Part 1–5
(i)	Find the dimension parameters shown in Figure 5.15, the steel strength f_y and the E value of the steel	
(ii)	Calculate $k_F = 2 + 6\left(\frac{s_s}{h_w}\right)$ but not more than 6	Figure 6.1(c)
(iii)	Calculate $l_e = \frac{k_F E t_w^2}{2 f_y h_w}$ but not more than s_s	Equation 6.14
(iv)	Calculate $m_1 = \frac{b_f}{t_w}$	Equation 6.8
(v)	Calculate three values for the effective loaded length l_y	
	$l_y = s_s + 2t_f\left(1 + \sqrt{m_1}\right)$	Equation 6.10
	$l_y = l_e + t_f\sqrt{\left[\frac{m_1}{2} + \left(\frac{l_e}{t_f}\right)^2\right]}$	Equation 6.11
	$l_y = l_e + t_f\sqrt{m_1}$	Equation 6.12
(vi)	Take l_y equal to the smallest of the three values	Clause 6.5(3)
(vii)	Calculate the critical force $F_{cr} = 0.9 k_F E \frac{t_w^3}{h_w}$	Equation 6.5
(viii)	Calculate the relative slenderness $\bar{\lambda}_F = \sqrt{\frac{l_y t_w f_y}{F_{cr}}}$	Equation 6.4
(ix)	Calculate the reduction factor due to local buckling $\chi_F = \frac{0.5}{\bar{\lambda}_F}$ but not more than 1.0	Equation 6.3
(x)	Calculate the design resistance $F_{Rd} = f_y \chi_F l_y t_w$	Equations 6.1 and 6.2

Example 5.7 Beam resistance to transverse force

Determine the design resistance to transverse force F_{Rd} of a 356 × 171 × 67 kg/m UB in Grade S275 steel on a 100 mm long stiff end bearing. The relevant dimensions from Table 5.6 are shown in Figure 5.17.

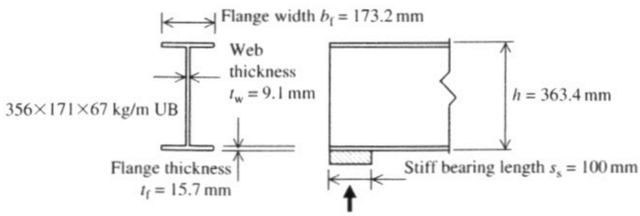


Figure 5.17: Dimensions for Example 5.7

Calculations for Example 5.7

Steel properties and dimensions	Strength $f_y = 275\,\text{N/mm}^2$
	$E = 210\,000\,\text{N/mm}^2$
	Stiff bearing $s_s = 100\,\text{mm}$
	Web thickness $t_w = 9.1\,\text{mm}$
	Flange thickness $t_f = 15.7\,\text{mm}$
	Web height $h_w = 363.4 - 2 \times 15.7 = 332.0\,\text{mm}$
	Flange width $b_f = 173.2\,\text{mm}$
Following the procedure in Table 5.18	
$k_F = 2 + 6\left(\dfrac{s_s}{h_w}\right) = 2 + 6\left(\dfrac{100}{332.0}\right) = 3.81$ but not more than 6	$k_F = 3.81$
$l_e = \dfrac{k_F E t_w^2}{2 f_y h_w} = \dfrac{3.81 \times 210\,000 \times 9.1^2}{2 \times 275 \times 332.0} = 363$ mm but not more than $s_s = 100$ mm	$l_e = 100\,\text{mm}$
$m_1 = \dfrac{b_f}{t_w} = \dfrac{173.2}{9.1} = 19.0$	$m_1 = 19.0$
Determine three values for the effective loaded length l_y	
$l_y = s_s + 2t_f(1 + \sqrt{m_1}) = 100 + 2 \times 15.7(1 + \sqrt{19.0}) = 268.3$ mm	
$l_y = l_e + t_f\sqrt{\left[\dfrac{m_1}{2} + \left(\dfrac{l_e}{t_f}\right)^2\right]} = 100 + 15.7\sqrt{\left[\dfrac{19.0}{2} + \left(\dfrac{100}{15.7}\right)^2\right]} = 211.1\,\text{mm}$	
$l_y = l_e + t_f\sqrt{m_1} = 100 + 15.7\sqrt{19.0} = 168.4\,\text{mm}$	
Take l_y equal to the smallest of the three values	$l_y = 168.4\,\text{mm}$
Critical force $F_{cr} = 0.9 k_F E \dfrac{t_w^3}{h_w} = 0.9 \times 3.81 \times 210\,000 \times \dfrac{9.1^3}{332.0} = 1\,634\,000$ N	$F_{cr} = 1\,635\,000\,\text{N}$
Relative slenderness $\bar{\lambda}_F = \sqrt{\dfrac{l_y t_w f_y}{F_{cr}}} = \sqrt{\dfrac{168.3 \times 9.1 \times 275}{1\,635\,000}} = 0.508$	$\bar{\lambda}_F = 0.508$
Reduction factor due to local buckling	
$\chi_F = \dfrac{0.5}{\bar{\lambda}_F} = \dfrac{0.5}{0.508} = 0.98$ but not more than 1.0	$\chi_F = 0.98$
Design resistance $F_{Rd} = f_y \chi_F l_y t_w = 275 \times 0.98 \times 168.4 \times 9.1 = 413\,000\,\text{N}$	$F_{Rd} = 413\,\text{kN}$

This check is not required if the beam is connected via end plates, fin plates or web cleats.

5.5.4 Beam Deflection SLS

Deflections of flexural members must be limited to avoid damage to finishes, ceilings and partitions, and should be calculated under SLS loads. The formulas for calculating deflections in simply supported beams due to bending were given in Chapter 1:

With uniformly distributed load F, the maximum deflection due to bending $w_m = \dfrac{5FL^3}{384EI}$

With a central point load P, the maximum deflection due to bending $w_m = \dfrac{PL^3}{48EI}$

EC3 states that limits for vertical deflections should be specified for each project and agreed with the client. The UK National Annex to EC3 suggests the limits shown in Table 5.19 for deflections caused by variable loads but does not limit deflections under permanent loads. Upward cambers may be incorporated into steel beams during fabrication to offset deflections caused by permanent loads.

Table 5.19: Suggested limiting vertical deflections of members in bending under variable loads

Type of member	**Suggested limit**
Cantilevers	Length/180
Beams carrying plaster or other brittle finish	Span/360
Other beams (except purlins and sheeting rails)	Span/200
Purlins and sheeting rails	To suit the characteristics of the particular cladding

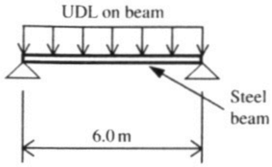
Notes: 1. In some circumstances greater or lesser values would be appropriate.
2. On low-pitch and flat roofs the possibility of rainwater ponding should be investigated.
Source: EC3 Part 1–1 UK National Annex Clause NA.2.23.

Example 5.8 Beam deflection

Check whether the beam in Example 5.2 meets the deflection criteria in Table 5.19 if the floor supports plastered partitions.

Calculations for Example 5.8

A summary of the load calculations from Example 5.2 is repeated here



Loads on steel beam		Dead loads (self-weight)	Imposed loads
Screed	$1.2 \times 6.0 \times 4.5 =$	32.4 kN	–
Concrete slab	$3.75 \times 6.0 \times 4.5 =$	101.3 kN	–
Steel beam	$0.74 \times 6.0 =$	4.4 kN	–
Imposed load	$5.0 \times 6.0 \times 4.5 =$	–	135.0 kN
Totals		$G_k = 138.1$ kN	$Q_k = 135.0$ kN

From the calculations to Example 5.2, the chosen section is a $406 \times 140 \times 53$ kg/m UB

From Table 5.7, for a $406 \times 140 \times 53$ kg/m UB $\quad I_y = 18\,283\,\text{cm}^4$

For all hot-rolled steels $\quad E = 210\,000\,\text{N/mm}^2$

Deflection under dead load $w = \dfrac{5G_kL^3}{384\,EI} = \dfrac{5 \times 138.1 \times 10^3 \times 6\,000^3}{384 \times 210\,000 \times 18\,283 \times 10^4} = 10.1 \quad w = 10.1\,\text{mm}$

Deflection under imposed load $w = \dfrac{5Q_kL^3}{384\,EI} = \dfrac{5 \times 135.0 \times 10^3 \times 6\,000^3}{384 \times 210\,000 \times 18\,283 \times 10^4} = 9.9 \quad w = 9.9\,\text{mm}$

The criteria in Table 5.19 apply only to defection under imposed loads

For beams supporting plastered finishes, deflection limit = span/360 = 6000/360 = 16.7 mm

9.9 mm < 16.7 mm $\quad$ The beam meets the deflection criteria in Table 5.19

The dead-load deflection of 10.1 mm could, if required, be offset by pre-cambering the beam during fabrication. However, pre-cambering is normally only required for deflections significantly greater than 10 mm

Design summary for simply supported steel beams using UB sections

a) Calculate the ULS loads on the beam.

b) Calculate the maximum design bending moment M_{Ed} and the maximum design shear force V_{Ed}.

c) Determine whether the beam has full lateral restraint. If it does not, determine the effective lengths of the unrestrained segments.

d) Choose a steel grade and a UB section.

e) As all UB sections are classified as Class 1 plastic or Class 2 compact for bending, no further checks of the section class are required.

f) If the beam has full lateral restraint, check that $M_{Ed} \leq M_{c,Rd} = M_{pl,Rd} = f_y W_{pl}$.

g) If the beam is laterally unrestrained, calculate $M_{b,Rd}$ and check that $M_{Ed} \leq M_{b,Rd}$.

h) Calculate the plastic shear resistance $V_{pl,Rd}$ and check that $V_{Ed} \leq V_{pl,Rd}$.

i) *If* $V_{Ed}/V_{pl,Rd} > 0.5$, determine whether any reduction in the bending strength is necessary and re-check against M_{Ed} if required.

j) If the beam is supported on bearings check the resistance to transverse forces. This check is not required if the beam is connected via end plates, fin plates or web cleats.

k) Calculate the deflection of the beam under unfactored imposed loads and check against the criteria in Table 5.19 or against the limits specified for the project and agreed with the client.

The final example in this section illustrates the design of a laterally unrestrained UB section with shear and deflection checks. As it is likely that the beam will be supported by bolted connections which are not covered by this manual, the check on resistance to transverse forces is not included.

Example 5.9 Unrestrained beam with checks on bending, shear and deflection
A UB in grade S275 steel is required to carry loads shown in Figure 5.18 over a span of 9.0 m. The beam is laterally restrained at A, B, C and D but unrestrained between these points. The depth of the beam must not exceed 425 mm, and the ceiling under the beam will be plastered. Choose a suitable UB section.

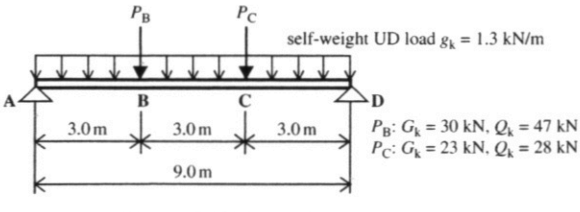


Figure 5.18: Beam for Example 5.9

Calculations for Example 5.9	
First calculate the loads at the ULS	
Point load $P_B = 1.35G_k + 1.50Q_k = 1.35 \times 30 + 1.50 \times 47$	$P_B = 111.0\,kN$
Point load $P_C = 1.35G_k + 1.50Q_k = 1.35 \times 23 + 1.50 \times 28$	$P_C = 73.1\,kN$
Total UD load $= 1.35g_k \times \text{span} = 1.35 \times 1.3 \times 9$	UD load $= 15.8\,kN$
The load diagram at ULS is shown below	
As the loading is not symmetric the reactions and moments are calculated from first principles	

Taking moments about D:

$$9R_A - (111.0 \times 6) - (73.1 \times 3) - (15.8 \times 4.5) = 0$$

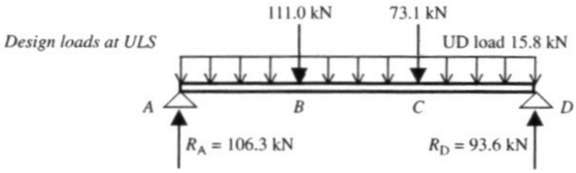
$$R_A = 956.4/9 = 106.3\,\text{kN}$$

Taking moments about A:

$$(111.0 \times 3) + (73.1 \times 6) + (15.8 \times 4.5) - 9R_D - = 0$$

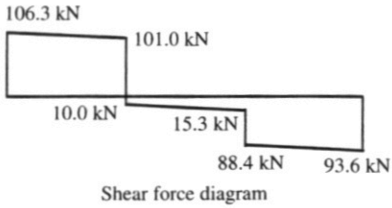
$$R_D = 842.7/9 = 93.6\,\text{kN}$$

Check $106.3 + 93.6 - 111.0 - 73.1 - 15.8 = 0$ Accept

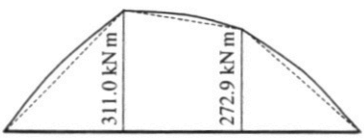


ULS bending moment at B $= (106.3 \times 3) - \left(\frac{15.8}{9}\right)\left(\frac{3^2}{2}\right) = 311.0\,\text{kNm}$ — At B, $M_{Ed} = 311.0\,\text{kNm}$

ULS bending moment at C $= (93.6 \times 3) - \left(\frac{15.8}{9}\right)\left(\frac{3^2}{2}\right) = 272.9\,\text{kNm}$ — At C, $M_{Ed} = 272.9\,\text{kNm}$



Shear force diagram



Bending moment diagram

Bending ULS

We need to choose a section with a buckling resistance moment $M_{b,Rd}$ of at least 311 kNm

Table 5.15(a) gives the following values of $M_{b,Rd}$ for UB sections with an unrestrained segment length of 3.0 m and uniform moment

$457 \times 191 \times 67$ kg/m UB, $M_{b,Rd} = 332\,\text{kNm}$

$406 \times 178 \times 74$ kg/m UB, $M_{b,Rd} = 339\,\text{kNm}$

$406 \times 178 \times 67$ kg/m UB, $M_{b,Rd} = 301\,\text{kNm}$

The first of these is not acceptable because it exceeds the permitted depth of 425 mm

Continued on next page

Calculations for Example 5.9 (Continued from previous page)

As the moment is not uniform and a C_1 factor from Table 5.12 which is greater than 1.0 can be used, investigate whether 406 × 178 × 67 kg/m UB is adequate

From Table 5.6, for 406 × 178 × 67 kg/m UB $h = 409.4\,\text{mm}$

$b = 178.8\,\text{mm}$

$t_w = 8.8\,\text{mm}$

$t_f = 14.3\,\text{mm}$

From Table 5.7, for 406 × 178 × 67 kg/m UB $I_y = 24\,331\,\text{cm}^4$

$I_z = 1365\,\text{cm}^4$

$W_{pl,y} = 1346\,\text{cm}^3$

$I_w = 0.533\,\text{dm}^6$

$I_T = 46.1\,\text{cm}^4$

$A_v = 38.6\,\text{cm}^2$

From Table 5.3 with S275 steel and $t = 14.3\,\text{mm}$ $f_y = 275\,\text{N/mm}^2$

From Table 5.2 $E = 210\,\text{kN/mm}^2$

$G = 81\,\text{kN/mm}^2$

Using the method in Table 5.14

The effective length L of the segment is 3.0 m $L = 3000\,\text{mm}$

From Table 5.12, the ratio of the moments on the segment $\psi = 272.9/311.0 = 0.877$ $\psi = 0.877$

From Table 5.12 If $\psi = 1.00$ then $C_1 = 1.00$

If $\psi = 0.80$ then $C_1 = 1.11$, so by interpolation when $\psi = 0.877$ $C_1 = 1.068$

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_Z}\right]}$$

$$= 1.068 \times \frac{\pi^2 \times 210 \times 10^3 \times 1365 \times 10^4}{3000^2} \sqrt{\left[\frac{0.533 \times 10^{12}}{1365 \times 10^4} + \frac{3000^2 \times 81 \times 10^3 \times 46.1 \times 10^4}{\pi^2 \times 210 \times 10^3 \times 1365 \times 10^4}\right]}$$

$= 758 \times 10^6\,\text{Nmm}$ $M_{cr} = 758\,\text{kNm}$

$f_y W_{pl,y} = 275 \times 1\,346 \times 10^3 = 370 \times 10^6\,\text{Nmm}$ $f_y W_{pl,y} = 370\,\text{kNm}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{f_y W_{pl,y}}{M_{cr}}} = \sqrt{\frac{370}{758}}$$ $\bar{\lambda}_{LT} = 0.699$

h/b for the section $= 409.4/178.8 = 2.29$, so from Table 5.13 $\alpha_{LT} = 0.49$

From Table 5.14 $\bar{\lambda}_{LT,0} = 0.4$

$\beta = 0.75$

$\Phi_{LT} = 0.5(1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2) = 0.5(1 + 0.49(0.699 - 0.4) + 0.75 \times 0.699^2)$ — $\Phi_{LT} = 0.756$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} = \frac{1}{0.756 + \sqrt{0.756^2 - 0.75 \times 0.699^2}}$$

$\chi_{LT} = 0.827$

Check that χ_{LT} is not more than 1.0 and is not more than $\frac{1}{\bar{\lambda}_{LT}^2} = \frac{1}{0.699^2} = 2.05$ — Accept

C_1 is not equal to 1.0, so f must be calculated.

$$f = 1 - 0.5\left(1 - \frac{1}{\sqrt{C_1}}\right)(1 - 2(\bar{\lambda}_{LT} - 0.8)^2) = 1 - 0.5\left(1 - \frac{1}{\sqrt{1.068}}\right)(1 - 2(0.699 - 0.8)^2)$$

but not more than 1.0 — $f = 0.984$

$\chi_{LT,mod} = \chi_{LT}/f = 0.827/0.984$ but not more than 1.0 — $\chi_{LT,mod} = 0.840$

Design buckling resistance moment $M_{b,Rd} = \chi_{LT,mod} f_y W_{pl,y} = 0.840 \times 370$ — $M_{b,Rd} = 311\,\text{kNm}$

From the ULS bending moment diagram, the maximum design bending moment is 311 kNm — $M_{Ed} = 311\,\text{kNm}$

$M_{Ed}/M_{b,Rd} = 311/311$ which is not more than 1.0 — The bending strength of the $406 \times 178 \times 67$ kg/m UB is sufficient

Shear ULS

From the ULS shear force diagram, the maximum design shear force is 106.3 kN — $V_{Ed} = 106.3\,\text{kN}$

The plastic shear resistance $V_{pl,Rd} = 0.577 f_y A_v = 0.577 \times 275 \times 38.6 \times 10^2 = 612\,490\,\text{N}$ — $V_{pl,Rd} = 612\,\text{kN}$

V_{Ed} is less than $V_{pl,Rd}$ — Accept

From Table 5.16, a web shear buckling check is not required

As the maximum bending moment is at B where the shear force is not zero, it is necessary to check for reduced bending moment capacity in an area of high shear force

$V_{Ed}/V_{pl,Rd} = 101/612 = 0.17$

From Table 5.17, since $V_{Ed}/V_{pl,Rd}$ is less than 0.5 then no reduction in bending resistance is required — The $406 \times 178 \times 67$ kg/m UB is satisfactory in shear

Deflection SLS

As the loading on the beam is not symmetric the calculations of the actual deflection are quite complex. A simple approach is to find the distributed load that would cause the same maximum bending moment, then to check whether the deflection caused by that distributed load is acceptable. The deflection under unfactored imposed loads alone is required

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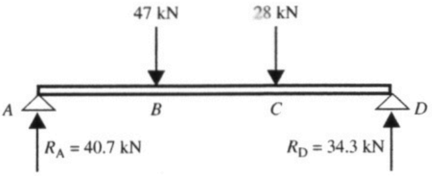
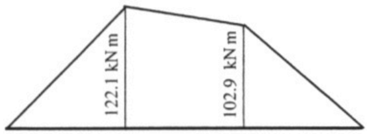
Calculations for Example 5.9 (Continued from previous page)



Loading diagram under unfactored imposed loads



Bending moment diagram

The equivalent UD load F can be found from $122.1 = \dfrac{FL}{8} = \dfrac{9F}{8}$

So $F = \dfrac{8 \times 122.1}{9} = 108.5\ \text{kN}$

Deflection $w = \dfrac{5FL^3}{384EI_y} = \dfrac{5 \times 108.5 \times 10^3 \times 9000^3}{384 \times 210\,000 \times 24\,331 \times 10^4} = 20.2\,\text{mm}$

From Table 5.19 the suggested deflection limit for beam carrying brittle finishes is span/360 = 9000/360 = 25 mm, which is greater than the actual deflection of 20.2 mm

The 406 × 178 × 67 kg/m UB meets the deflection criteria in Table 5.19

5.5.5 Fabricated Beams

Fabricated beams may be used in situations where standard rolled sections are not suitable.

Compound beams and plate girders

Welding additional plates to the flanges can increase the strength and stiffness of standard rolled sections. Some examples are shown in Figure 5.19, including a crane beam formed from a rolled UB section with a channel to assist in resisting lateral loads.

Plate girders are normally formed by welding together three plates, as shown in Figure 5.19. If large holes are required in a beam web then a fabricated plate girder with a thick web plate may be cheaper than a rolled UB section because the latter would require stiffeners round the holes. Tapered and curved plate girders are readily made.

Heavy plate girders are occasionally used in buildings where heavy loads or long spans dictate, but more often they are used for bridges. Whilst plate girders can theoretically be made to any size, for economic reasons their depth is normally between span/8 and span/12.

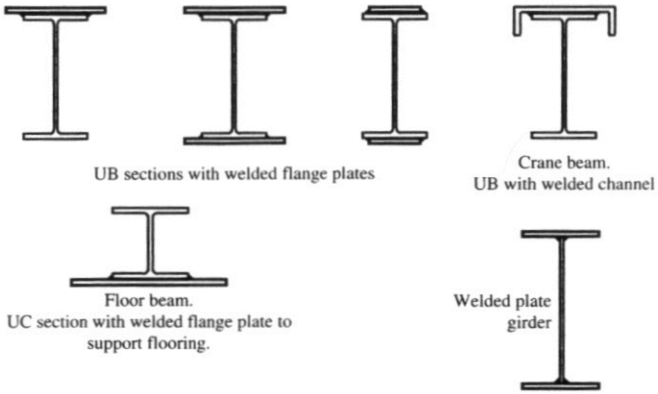


Figure 5.19: Compound beams and plate girders

Cellular beams

While standard rolled sections are ideal for many situations, it is often found that the web is stronger than necessary for beam applications. The standard sections can be converted by cutting and welding into much deeper sections known as cellular beams. These offer greater bending strength and stiffness than the basic rolled sections without using more steel, and the holes can be useful as service routes.

To form a cellular beam the basic rolled section is first cut to a prescribed profile as shown in Figure 5.20(a), and then the two parts are rejoined by welding as shown in Figure 5.20(b). The finished sections is stronger in bending than the original but the shear strength is less and can be enhanced locally if required by welding fitted plates into some of the holes.

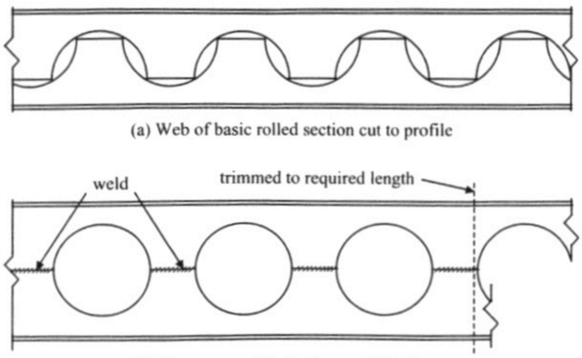
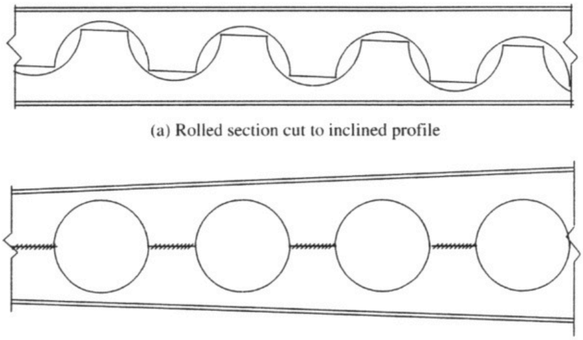


Figure 5.20: Basic cellular beam

A wide variety of cellular beams can be fabricated by varying the standard method. If required the top flange can be cut from one rolled section and the bottom flange from a different section, resulting in an asymmetric cellular beam. The cut sections can be bent slightly to produce cambered cellular beams, and tapered cellular beams can be made by cutting the rolled section on an inclined profile as shown in Figure 5.21.



(a) Rolled section cut to inclined profile

(b) One part reversed and welded to form a tapered cellular beam

Figure 5.21: Tapered cellular beam

Lattice and Vierendeel girders

Lattice girders are a framework of individual members bolted or welded together to form an open-web beam. Most lattice girders have inclined internals, and two common patterns are the N girder and the Warren girder as shown in Figure 5.22(a) and (b). Generally the economical depth is span/10 to span/15, while in heavily loaded trusses a depth of span/6 may be appropriate. A depth of span/20 may suffice for lightly loaded roof girders. The top and bottom booms act to resist the bending while the internal members resist the shear. If the members are reasonably slender it is realistic to assume that the joints transmit no bending, and in this case the girder can be analysed as if all the joints were pinned and the members carry only direct compression or tension force.

If aesthetic considerations or the need for access through the truss mean that inclined internals cannot be used, then a Vierendeel girder as shown in Figure 5.22(c) may be used. A Vierendeel girder requires members and joints that can sustain bending moment.

5.6 Columns

A steel column may be subject to direct compression alone, where the load is applied axially, or to a combination of compressive load and bending. Bending of the column may be caused by vertical load applied eccentrically, by end moments from beams connected via stiff joints or by lateral wind loads, although only the first of these is considered in this manual.

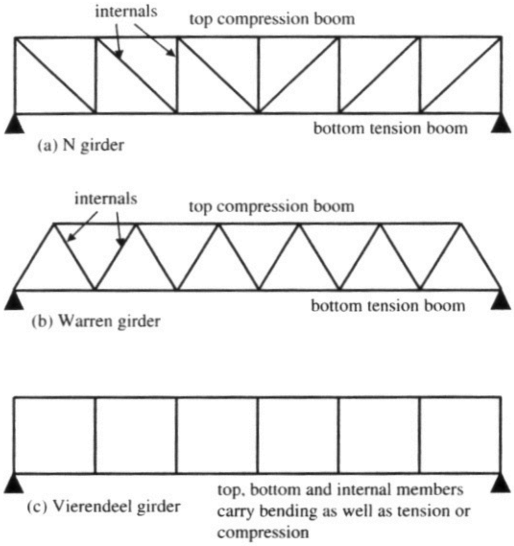


Figure 5.22: Lattice and Vierendeel girders

In this manual the following aspects of column design are considered:

1. Axially loaded columns
2. Axially loaded columns with moments from eccentric loads
3. Column baseplates.

The capacity of sections in compression may be limited by local buckling as explained in Section 5.4. However, Table 5.11 shows that the majority of UC sections are Class 1 Plastic or Class 2 Compact for compression, so local buckling seldom limits the capacity of these sections.

5.6.1 Axially Loaded Columns

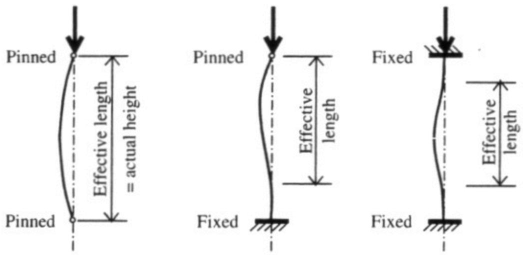
A steel column, because of its slender nature, will tend to buckle laterally under axial compression. For most practical columns this means that the average compressive stress in the column (calculated as axial force divided by area) cannot reach the yield strength of the steel. EC3 gives several methods of determining design compression resistance of a column, and the simplest of these is summarised later (see Table 5.21).

Effective length of columns

As explained in Chapter 1 of this manual the effective length of a column depends on its actual height and also on the degree of fixity at the top and the bottom. Effective length factors are shown in Figure 5.23 and the effective length is given by

$$\text{Effective length} = \text{effective length factor} \times \text{actual height}$$

These factors in Figure 5.23 are conservative, and further analysis using document SN008a may give smaller factors. The figure shows only braced columns; that is, columns in which the top cannot displace laterally relative to the bottom. This will be the case for columns in buildings braced by diagonally braced bays or by shear walls, and the design of unbraced columns (as used in portal frames) is beyond the scope of this manual.



End restraints	Pinned/Pinned	Fixed/Pinned	Fixed/Fixed
Effective length factor	1.0	0.85	0.7

Figure 5.23: Effective lengths of braced columns

Using bracing to reduce the effective length of a column

The design compression resistance of a column can often be significantly increased by providing additional lateral restraint to reduce its effective length, as illustrated in Figure 5.24. Example 5.12 below shows how the design compression resistance can be increased by providing bracing about the weak z-z axis only.

Buckling curves and α values

An ideal column is perfectly straight but real columns always have some imperfections which deviate from this ideal. The amount of imperfection to be expected varies with the type of member and the way that it is used, so that for example a UB section is likely to deviate more about the weak z-z axis than it does about the strong y-y axis. EC3 represents the amount of deviation to be expected by the factor α, and Table 5.20 gives α values to be used for various

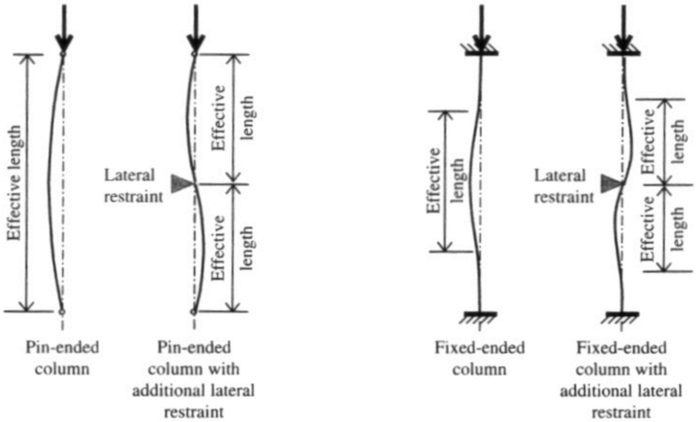


Figure 5.24: Using lateral restraint to reduce the effective length of a column

Table 5.20: Choice of buckling curve for cross-section in compression

		Buckling about *y-y* axis	**Buckling about *z-z* axis**
UB sections, $h/b > 1.2$	Flange ≤ 40 mm Flange > 40 mm	Buckling curve a Buckling curve b	Buckling curve b Buckling curve c
UC sections, $h/b \leq 1.2$		Buckling curve b	Buckling curve c
Hollow sections	Hot finished Cold finished	Buckling curve a Buckling curve c	
Channel and Tee sections		Buckling curve c	
Rolled steel angles		Buckling curve b	

The buckling curves correspond to values of α as follows:

Buckling curve	a	b	c	d
value of α	0.21	0.34	0.49	0.76

Source: EC3 Table 6.2.

sections and orientations. Higher values of α represent greater deviations from the ideal of a straight member and so lower strength when compared to a perfect strut. Figure 5.25 shows how buckling curves from different values of α are related, and see later Tables 5.23(a) and 5.23(b) for the appropriate numerical values.

As mentioned earlier, the simplest method for calculating the design compression resistance of axially loaded columns is given in Table 5.21.

Example 5.10 uses the method in Table 5.21.

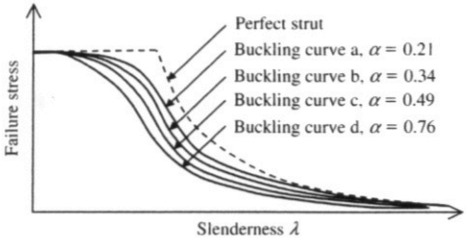


Figure 5.25: Buckling curves

Table 5.21: Method of calculating the design compression resistance of a UC section, using Clause 6.3.1.2 of EC3 Part 1–1

(i)	Determine the effective length L of the column
(ii)	Using section properties from Table 5.9 of this manual, determine the elastic critical buckling load $N_{cr} = \dfrac{\pi^2 EI}{L^2}$
(iii)	From Table 5.3 of this manual find f_y. From Table 5.9 of this manual find the section area A and hence find the plastic axial force capacity $f_y A$
(iv)	Calculate the non-dimensional slenderness $\bar{\lambda} = \sqrt{\dfrac{f_y A}{N_{cr}}}$
(v)	From Table 5.20 of this manual select the correct value for α
(vi)	Calculate the buckling parameter $\Phi = 0.5(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2)$
(vii)	Calculate the reduction factor $\chi = \dfrac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}$ but not more than 1.0
(viii)	Calculate the design compression resistance $N_{Rd} = \chi f_y A$

Example 5.10 Design compression resistance N_{Rd} of a UC section column

Considering only buckling about the z axis, determine the design compression resistance of a 203 × 203 × 52 kg/m UC section in S355 steel with an effective length of 4.0 m.

Calculations for Example 5.10, using the method of Table 5.21	
Effective length L of the column	$L = 4000\,\text{mm}$
From Table 5.8, for 203 × 203 × 52 kg/m UC	$t_f = 12.5\,\text{mm}$
From Table 5.9, for 203 × 203 × 52 kg/m UC	$I_z = 1778\,\text{cm}^4$
	$A = 66.3\,\text{cm}^2$

From Table 5.11, a 203 × 203 × 52 kg/m UC in S355 steel is Class 1 Plastic for compression, so calculations of local buckling resistance are not required	
From Table 5.2	$E = 210\,\text{kN/mm}^2$
The elastic critical buckling load $N_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2 \times 210\,000 \times 1\,778 \times 10^4}{4\,000^2}$	
$= 2.303 \times 10^6\,\text{N}$	$N_{cr} = 2\,303\,\text{kN}$
From Table 5.3 with $t_f = 12.5\,\text{mm}$	$f_y = 355\,\text{N/mm}^2$
Plastic axial force capacity $Af_y = 66.3 \times 10^2 \times 355 = 2.354 \times 10^6\,\text{N}$	$Af_y = 2\,354\,\text{kN}$
Non-dimensional slenderness $\bar{\lambda} = \sqrt{\dfrac{Af_y}{N_{cr}}} = \sqrt{\dfrac{2\,354}{2\,303}} = 1.01$	$\bar{\lambda} = 1.01$
From Table 5.20 for buckling about the z axis	Buckling curve c
	$\alpha = 0.49$
$\Phi = 0.5(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2) = 0.5(1 + 0.49(1.01 - 0.2) + 1.01^2)$	$\Phi = 1.21$
Reduction factor $\chi = \dfrac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = \dfrac{1}{1.21 + \sqrt{1.21^2 - 1.01^2}}$ but not more than 1.0	$\chi = 0.533$
Design compression resistance $N_{Rd} = \chi f_y A = 0.533 \times 355 \times 66.3 \times 10^2 = 1.255 \times 10^3\,\text{N}$	$N_{Rd} = 1\,255\,\text{kN}$

Similar calculations can be carried out as required for other sections and other effective lengths. However, it is often convenient to use the fact that the design compressive stress in a column depends on its slenderness λ and to use Table 5.23(a) or 5.23(b) of this manual to look up these stresses. This alternative method is shown in Table 5.22.

Table 5.22: Alternative method of calculating the design compressive resistance of a UC section, using tables of f_{Rd}

(i)	Determine the effective length L of the column
(ii)	From Tables 5.8 and 5.9 of this manual determine the flange thickness t_f, the area A and the appropriate radius of gyration i
(iii)	Calculate the slenderness $\lambda = L/i$
(iv)	From Table 5.3 of this manual, use the steel grade and the flange thickness to determine f_y
(v)	From Table 5.20 of this manual select the correct buckling curve
(vi)	Using Table 5.23(a) for S275 steel or Table 5.23(b) for S355 steel, find f_{Rd} appropriate to the λ value. Use interpolation if required
(vii)	Calculate the design compression resistance $N_{Rd} = f_{Rd}A$

Example 5.11

Repeat Example 5.10 using the alternative method in Table 5.22.

Calculations for Example 5.11*, using the alternative method in Table 5.22*	
Effective length L of the column	$L = 4000\,mm$
From Table 5.8, for 203 × 203 × 52 kg/m UC	$t_f = 12.5\,mm$
From Table 5.9, for 203 × 203 × 52 kg/m UC	$A = 66.3\,cm^2$
	$i_z = 5.2\,cm$
From Table 5.11, a 203 × 203 × 52 kg/m UC in S355 steel is Class 1 Plastic for compression, so calculations of local buckling resistance are not required	
Find the slenderness $\lambda_z = L/i_z = 4000/(5.2 \times 10)$	$\lambda_z = 77$
From Table 5.3, for S355 steel with $t_f = 12.5\,mm$	$f_y = 355\,N/mm^2$
From Table 5.20, for buckling about the z axis	Buckling curve c
From Table 5.23(b) with $f_y = 355\,N/mm^2$, $\lambda_z = 77$ and buckling curve c, by interpolation	$f_{Rd} = 190\,N/mm^2$
Design compression resistance $N_{Rd} = f_{Rd}A = 190 \times 66.3 \times 10^2 = 1.260 \times 10^3\,N$	$N_{Rd} = 1260\,kN$
	as in the previous calculation

Table 5.23(a): Values of f_{Rd} for members in compression in Grade S275 steel

	S275 steel up to 16 mm thick, $f_y = 275\,N/mm^2$				S275 steel 16 to 40 mm thick, $f_y = 265\,N/mm^2$				S275 steel over 40 mm thick, $f_y = 255\,N/mm^2$			
	Buckling curve				Buckling curve				Buckling curve			
λ	a	b	c	d	a	b	c	d	a	b	c	d
0	275	275	275	275	265	265	265	265	255	255	255	255
10	275	275	275	275	265	265	265	265	255	255	255	255
20	273	272	271	268	263	263	261	260	254	253	252	251
30	266	260	255	245	257	252	246	237	247	243	238	229
40	257	248	238	222	249	240	230	215	240	232	223	209
50	247	233	220	200	239	226	214	194	231	219	207	189
60	234	217	201	178	227	211	196	174	220	205	190	170
70	218	198	181	158	212	193	177	155	206	188	173	152
80	198	178	161	140	194	174	158	137	189	171	155	135
90	176	158	143	123	173	155	140	121	170	153	138	119
100	154	139	126	109	152	137	124	107	150	135	122	106
110	135	122	111	96	133	121	110	95	132	119	108	94
120	117	107	98	86	116	106	97	85	116	105	96	84
130	103	94	87	76	102	94	86	75	101	93	85	75
140	90	84	77	68	90	83	77	68	89	82	76	67
150	80	74	69	61	80	74	69	61	79	74	68	60
160	71	67	62	55	71	66	62	55	71	66	61	55
170	64	60	56	50	64	60	56	50	63	59	55	50
180	57	54	51	46	57	54	50	45	57	54	50	45
190	*52*	*49*	*46*	*42*	*52*	*49*	*46*	*42*	*52*	*49*	*46*	*41*
200	*47*	*45*	*42*	*38*	*47*	*44*	*42*	*38*	*47*	*44*	*42*	*38*

(*Continued*)

Table 5.23(a): (Continued)

	S275 steel up to 16 mm thick, $f_y = 275\,N/mm^2$				S275 steel 16 to 40 mm thick, $f_y = 265\,N/mm^2$				S275 steel over 40 mm thick, $f_y = 255\,N/mm^2$			
	Buckling curve				Buckling curve				Buckling curve			
λ	a	b	c	d	a	b	c	d	a	b	c	d
210	*43*	*41*	*39*	*35*	*43*	*41*	*38*	*35*	*43*	*41*	*38*	*35*
220	*39*	*37*	*36*	*33*	*39*	*37*	*35*	*32*	*39*	*37*	*35*	*32*
230	*36*	*34*	*33*	*30*	*36*	*34*	*33*	*30*	*36*	*34*	*33*	*30*
240	*33*	*32*	*30*	*28*	*33*	*32*	*30*	*28*	*33*	*32*	*30*	*28*
250	*31*	*30*	*28*	*26*	*31*	*29*	*28*	*26*	*31*	*29*	*28*	*26*
260	*29*	*27*	*26*	*24*	*29*	*27*	*26*	*24*	*28*	*27*	*26*	*24*
270	*27*	*26*	*24*	*23*	*27*	*25*	*24*	*23*	*26*	*25*	*24*	*23*
280	*25*	*24*	*23*	*21*	*25*	*24*	*23*	*21*	*25*	*24*	*23*	*21*
290	*23*	*22*	*21*	*20*	*23*	*22*	*21*	*20*	*23*	*22*	*21*	*20*
300	*22*	*21*	*20*	*19*	*22*	*21*	*20*	*19*	*22*	*21*	*20*	*19*
310	*20*	*20*	*19*	*18*	*20*	*20*	*19*	*18*	*20*	*20*	*19*	*18*
320	*19*	*19*	*18*	*17*	*19*	*18*	*18*	*17*	*19*	*18*	*18*	*17*
330	*18*	*17*	*17*	*16*	*18*	*17*	*17*	*16*	*18*	*17*	*17*	*16*
340	*17*	*16*	*16*	*15*	*17*	*16*	*16*	*15*	*17*	*16*	*16*	*15*
350	*16*	*16*	*15*	*14*	*16*	*16*	*15*	*14*	*16*	*16*	*15*	*14*

Source: EC3 Clause 6.3.1.2 using α values from EC3 Table 6.1.
Values in *italics* are for slenderness values exceeding 180.

Table 5.23(b): Values of f_{Rd} for members in compression in Grade S355 steel

	S355 steel up to 16 mm thick, $f_y = 355\,N/mm^2$				S355 steel 16 to 40 mm thick, $f_y = 345\,N/mm^2$				S355 steel over 40 mm thick, $f_y = 335\,N/mm^2$			
	Buckling curve				Buckling curve				Buckling curve			
λ	a	b	c	d	a	b	c	d	a	b	c	d
0	355	355	355	355	345	345	345	345	335	335	335	335
10	355	355	355	355	345	345	345	345	335	335	335	335
20	350	347	344	338	341	338	335	329	331	329	326	321
30	339	330	320	304	330	321	312	297	321	313	304	289
40	325	310	295	271	317	303	288	265	309	295	281	259
50	308	287	267	239	301	281	262	234	293	274	256	230
60	285	260	238	209	280	255	234	206	273	250	230	202
70	257	231	209	182	252	227	206	179	248	224	203	176
80	225	201	182	158	222	199	180	156	219	196	178	154
90	193	174	158	137	191	172	156	135	190	171	155	134
100	165	150	137	119	164	149	136	118	163	148	134	117

(Continued)

Table 5.23(b): (Continued)

λ	S355 steel up to 16 mm thick, $f_y = 355\,N/mm^2$				S355 steel 16 to 40 mm thick, $f_y = 345\,N/mm^2$				S355 steel over 40 mm thick, $f_y = 335\,N/mm^2$			
	Buckling curve				Buckling curve				Buckling curve			
	a	b	c	d	a	b	c	d	a	b	c	d
110	142	130	119	104	141	129	118	103	140	128	117	103
120	122	113	104	92	122	112	103	91	121	112	103	90
130	106	99	91	81	106	98	91	81	105	98	90	80
140	93	87	81	72	93	86	80	72	92	86	80	71
150	82	77	72	65	82	77	72	64	82	76	71	64
160	73	68	64	58	73	68	64	58	72	68	64	58
170	65	61	58	53	65	61	58	52	65	61	57	52
180	58	55	52	48	58	55	52	47	58	55	52	47
190	*53*	*50*	*47*	*43*	*53*	*50*	*47*	*43*	*52*	*50*	*47*	*43*
200	*48*	*46*	*43*	*40*	*48*	*45*	*43*	*40*	*48*	*45*	*43*	*39*
210	*43*	*42*	*40*	*37*	*43*	*42*	*40*	*36*	*43*	*41*	*39*	*36*
220	*40*	*38*	*36*	*34*	*40*	*38*	*36*	*34*	*40*	*38*	*36*	*33*
230	*37*	*35*	*34*	*31*	*36*	*35*	*33*	*31*	*36*	*35*	*33*	*31*
240	*34*	*32*	*31*	*29*	*34*	*32*	*31*	*29*	*34*	*32*	*31*	*29*
250	*31*	*30*	*29*	*27*	*31*	*30*	*29*	*27*	*31*	*30*	*29*	*27*
260	*29*	*28*	*27*	*25*	*29*	*28*	*27*	*25*	*29*	*28*	*27*	*25*
270	*27*	*26*	*25*	*23*	*27*	*26*	*25*	*23*	*27*	*26*	*25*	*23*
280	*25*	*24*	*23*	*22*	*25*	*24*	*23*	*22*	*25*	*24*	*23*	*22*
290	*23*	*23*	*22*	*21*	*23*	*23*	*22*	*20*	*23*	*23*	*22*	*20*
300	*22*	*21*	*20*	*19*	*22*	*21*	*20*	*19*	*22*	*21*	*20*	*19*
310	*20*	*20*	*19*	*18*	*20*	*20*	*19*	*18*	*20*	*20*	*19*	*18*
320	*19*	*19*	*18*	*17*	*19*	*19*	*18*	*17*	*19*	*19*	*18*	*17*
330	*18*	*18*	*17*	*16*	*18*	*18*	*17*	*16*	*18*	*18*	*17*	*16*
340	*17*	*17*	*16*	*15*	*17*	*17*	*16*	*15*	*17*	*17*	*16*	*15*
350	*16*	*16*	*15*	*15*	*16*	*16*	*15*	*14*	*16*	*16*	*15*	*14*

Source: EC3 Part 1–1 Clause 6.3.1.2 using α values from EC3 Part 1–1 Table 6.1.
Values in *italics* are for slenderness values exceeding 180.

Buckling about both y-y and z-z axes

If a column has the same restraint about both axes then its critical buckling mode will always be about the weaker *z-z* axis. It is not necessary to check *y-y* axis buckling because the compression resistance will always be higher than for *z-z* buckling. However, it is often practical and economic to provide additional bracing in the *z-z* direction and so reduce the effective length for this mode of buckling.

The example which follows shows a pair of identical columns in which the effective length for *y-y* buckling is greater than the effective length for *z-z* buckling. This arrangement is often convenient to use where the columns are in a wall, as the bracing is confined to the plane of the wall so it does not intrude into usable space within the building.

Example 5.12 Column with different effective lengths for y-y and z-z axis buckling
Two 6.0 m high parallel columns in S275 steel, each a 203 × 203 × 60 kg/m UC section, are braced together as shown in Figure 5.26. The columns are pinned at the top and bottom, and the bracing provides lateral support in the *z-z* direction only. The effective length of each column is 6.0 m for *y-y* buckling and 2.0 m for *z-z* buckling. Determine the design compression resistance of each column.

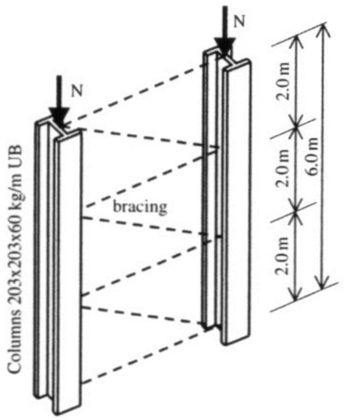


Figure 5.26: Columns in Example 5.12

Calculations for Example 5.12	
From Table 5.8, for 203 × 203 × 60 kg/m UC	$t_f = 14.2$ mm
From Table 5.9, for 203 × 203 × 60 kg/m UC	$A = 76.4$ cm^2
	$i_y = 9.0$ cm
	$i_z = 5.2$ cm
From Table 5.3, for S275 steel with $t_f = 12.5$ mm	$f_y = 275$ N/mm^2
From Table 5.11, a 203 × 203 × 60 kg/m UC in S275 steel is Class 1 Plastic for compression, so calculations of local buckling resistance are not required	
Buckling about the *y-y* axis	
Effective length	$L = 6000$ mm
Slenderness $\lambda = L/i_y = 6000/(9.0 \times 10)$	$\lambda = 67$
From Table 5.20	Buckling curve b
From Table 5.23(a) with $f_y = 275$ N/mm^2, $\lambda = 67$ and buckling curve b, by interpolation	$f_{Rd} = 203$ N/mm^2

Continued on next page

Calculations for Example 5.12 (Continued from previous page)	
Buckling about the *z-z* axis	
Effective length	$L = 2000\,\text{mm}$
Slenderness $\lambda = L/i_z = 2000/(5.2 \times 10)$	$\lambda = 38$
From Table 5.20	Buckling curve c
From Table 5.23(a) with $f_y = 275\,\text{N/mm}^2$, $\lambda = 38$ and buckling curve c, by interpolation	$f_{Rd} = 241\,\text{N/mm}^2$
Design compression resistance of one column	
Take the lower of the two f_{Rd} values	$f_{Rd} = 203\,\text{N/mm}^2$
Design compression resistance $N_{Rd} = f_{Rd}A = 203 \times 76.4 \times 10^2 = 1.551 \times 10^3\,\text{N}$	$N_{Rd} = 1551\,\text{kN}$
Note: It can be shown that if the additional bracing in the *z-z* direction had not been provided the design compression resistance of each column would be only 798 kN.	

5.6.2 Axially Loaded Columns with Moments from Eccentric Loads

When a steel structure is designed by the 'simple design' method explained in Section 5.1, the columns should be designed to carry the end reactions from the beams at a nominal eccentricity. Document SN005a gives a set of simplified rules for determining the eccentricities. Where a beam is supported on a column cap plate the load is assumed to act at the face of the plate. Beams bolted to the flange or web of the section are assumed to apply their reactions 100 mm from the relevant steel face, so generating the following eccentricities:

For a beam bolted to the column flange, eccentricity $e = h/2 + 100\,\text{mm}$
For a beam bolted to the column web, eccentricity $e = t_w/2 + 100\,\text{mm}$

These are illustrated in Figures 5.27 and 5.28.

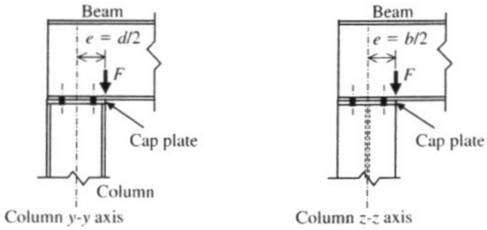
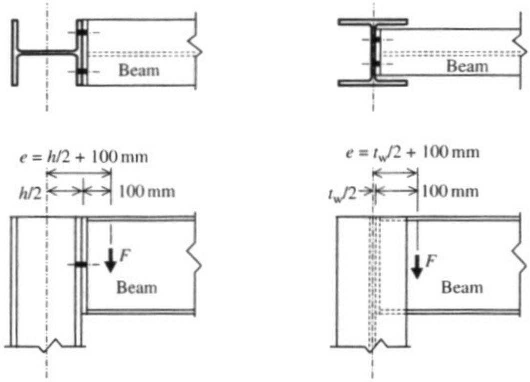


Figure 5.27: Beams supported on a column cap plate

Where equal beams connect to both sides of a column then both beams should be assumed to be fully loaded and the column should be designed for axial compression alone. In practice, the imposed loads on the beams may be unequal so that the resulting compression load is not axial but this effect, which is called pattern loading, may be ignored.



(a) Beam connected to column flange

(b) Beam connected to column web

Figure 5.28: Beams connected to the face of a column

A column loaded eccentrically in this way has to carry both axial force and bending moment. Clearly the steel section cannot carry its full bending capacity at the same time as its full axial load capacity, and acceptable combinations of bending moment and axial load are determined by using an interaction formula. Equations 6.61 and 6.62 of EC3 Part 1-1 are the full interactions formulas for steel sections but these are complicated to use and require the evaluation of numerous parameters. A simpler and more conservative interaction formula from Document SN048b is

$$\frac{N_{\text{Ed}}}{N_{\text{b,Rd}}} + \frac{M_{\text{y,Ed}}}{M_{\text{y,b,Rd}}} + 1.5\frac{M_{\text{z,Ed}}}{f_{\text{y}} W_{\text{pl,z}}} \leq 1.0$$

The following design summary explains how to use this interaction formula, and Example 5.13 shows the method in use.

Design summary for axially loaded steel columns with moments from eccentric loads

a) Calculate the ultimate axial load N_{Ed} applied to the column.

b) Select a trial section.

c) Calculate the nominal eccentricities e and the nominal moments $M_{\text{y,Ed}}$ and $M_{\text{z,Ed}}$.

d) Determine the effective lengths $l_{\text{ef,y}}$ and $l_{\text{ef,z}}$.

e) Calculate the slenderness ratios $\lambda_{\text{y}} = l_{\text{ef,y}}/i_{\text{y}}$ and $\lambda_{\text{z}} = l_{\text{ef,z}}/i_{\text{z}}$. Ensure that neither are greater than 180.

f) For the y and z axes separately:

Determine the correct strut curve from Table 5.20

Using λ values, obtain the compression strength f_{Rd} from Table 5.23(a) or Table 5.23(b).

g) Take f_{Rd} as the lower of the two values found in (f), find the axial load capacity $N_{b,Rd} = f_{Rd}A$

h) Using the method in Table 5.14 and taking $C_1 = 1.0$ find the buckling resistance moment for *y-y* bending $M_{y,b,Rd}$.

i) Noting that LTB does not occur when a UC section is bent about its *z-z* axis, determine the bending resistance moment for *z-z* bending $f_y W_{pl,y}$.

j) Check that the trial section satisfies the interaction formula

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{y,Ed}}{M_{y,b,Rd}} + 1.5\frac{M_{z,Ed}}{f_y W_{pl,z}} \leq 1.0$$

Example 5.13

A steel column is 4.5 m high and carries the beams shown in Figure 5.29. The beam reactions and column self-weight (SW) are also shown. The column is fixed to a substantial foundation at the bottom and is restrained in position but not direction at the top. Choose a UC section in S275 steel and determine whether it is satisfactory.

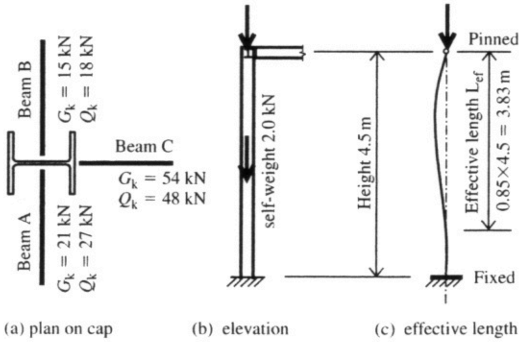


Figure 5.29: Column in Example 5.13

Calculations for Example 5.13

Calculate the forces at ULS

Beam A:	$1.35 \times 21 + 1.50 \times 27$	= 68.9 kN
Beam B:	$1.35 \times 15 + 1.50 \times 18$	= 47.3 kN
Beam C:	$1.35 \times 54 + 1.50 \times 48$	= 144.9 kN
Column SW:	1.35×2.0	= 2.7 kN

Ultimate axial load $N_{Ed} = 68.9 + 47.3 + 144.9 + 2.7$	$N_{Ed} = 263.8\,kN$
Assume a trial section of $152 \times 152 \times 37\,kg/m$ UC	
From Table 5.8, for $152 \times 152 \times 37\,kg/m$ UC	$h = 161.8\,mm$
	$b = 154.4\,mm$
	$t_w = 8.0\,mm$
	$t_f = 11.5\,mm$
From Table 5.9, for $152 \times 152 \times 37\,kg/m$ UC	$I_y = 2210\,cm^4$
	$I_z = 706\,cm^4$
	$i_y = 6.9\,cm$
	$i_z = 3.9\,cm$
	$W_{pl.y} = 309\,cm^3$
	$W_{pl.z} = 140\,cm^3$
	$I_w = 0.040\,dm^6$
	$I_T = 19.2\,cm^4$
	$A = 47.1\,cm^2$
From Table 5.2	$E = 210\,kN/mm^2$
	$G = 81\,kN/mm^2$
From Table 5.11, a $152 \times 152 \times 37\,kg/m$ UC in S275 steel is Class 1 Plastic for compression, so calculations of local buckling resistance are not required	
From Table 5.3 for S275 steel with $t_f = 11.5\,mm$	$f_y = 275\,N/mm^2$
Axial load	
As the effective length is the same about both axes it is only necessary to check for buckling about the weaker *z-z* axis	
Buckling about the *z-z* axis	
Effective length	$L = 3830\,mm$
Slenderness $\lambda = L/i_z = 3830/(3.9 \times 10)$	$\lambda = 98$
Check that slenderness does not exceed 180	Accept
From Table 5.20	Buckling curve c
From Table 5.23(a) with $f_y = 275\,N/mm^2$, $\lambda_z = 98$ and buckling curve c, by interpolation	$f_{Rd} = 129\,N/mm^2$
Design compression resistance $N_{Rd} = f_{Rd}A = 129 \times 47.1 \times 10^2 = 607 \times 10^3\,N$	$N_{Rd} = 607\,kN$
Bending about *y-y* axis	
Nominal eccentricity $e = h/2 + 100 = 161.8/2 + 100$	$e = 180.9\,mm$
Nominal moment $M_{y.Ed} = V_{Ed} \times e = 144.9 \times 0.1809$	$M_{y.Ed} = 26.2\,kNm$
Using the method in Table 5.14	

Continued on next page

Calculations for Example 5.13 (Continued from previous page)

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 G I_T}{\pi^2 E I_z}\right]}$$

$$= 1.0 \times \frac{\pi^2 \times 210 \times 10^3 \times 706 \times 10^4}{3{,}830^2} \sqrt{\left[\frac{0.040 \times 10^{12}}{706 \times 10^4} + \frac{3{,}830^2 \times 81 \times 10^3 \times 19.2 \times 10^4}{\pi^2 \times 210 \times 10^3 \times 706 \times 10^4}\right]}$$

$= 145.4 \times 10^6$ Nmm — $M_{cr} = 145.4$ kNm

$f_y W_{pl,y} = 275 \times 309 \times 10^3 = 85.0 \times 10^6$ Nmm — $f_y W_{pl,y} = 85.0$ kNm

$$\bar{\lambda}_{LT} = \sqrt{\frac{f_y W_{pl,y}}{M_{cr}}} = \sqrt{\frac{85.0}{145.4}} \qquad \bar{\lambda}_{LT} = 0.765$$

h/b for the section $= 161.8/154.4 = 1.05$, so from Table 5.13 — $\alpha_{LT} = 0.34$

From Table 5.14 — $\bar{\lambda}_{LT,0} = 0.4$

$\beta = 0.75$

$$\Phi_{LT} = 0.5(1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2) = 0.5(1 + 0.34(0.765 - 0.4) + 0.75 \times 0.765^2) \qquad \Phi_{LT} = 0.781$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} = \frac{1}{0.781 + \sqrt{0.781^2 - 0.75 \times 0.765^2}} \qquad \chi_{LT} = 0.837$$

Check that χ_{LT} is not more than 1.0 and is not more than $\frac{1}{\bar{\lambda}_{LT}^2} = \frac{1}{0.765^2} = 1.71$ — Accept

$C_1 = 1.0$, so $f = 1.0$ — $f = 1.0$

$\chi_{LT,mod} = \chi_{LT}/f = 0.837/1.0$ — $\chi_{LT,mod} = 0.837$

Design buckling resistance moment $M_{y,b,Rd} = \chi_{LT,mod} f_y W_{pl,y}$

$= 0.837 \times 85.0$ kNm — $M_{y,b,Rd} = 71.1$ kNm

Bending about z-z axis

Nominal eccentricity $e = t_w/2 + 100 = 8/2 + 100$ — $e = 104$ mm

Nominal moment $M_{z,Ed} = V_{Ed} \times e = 68.9 \times 0.104 - 47.3 \times 0.104$ — $M_{z,Ed} = 2.25$ kNm

$f_y W_{pl,z} = 275 \times 140 \times 10^3 = 38.5 \times 10^6$ Nmm — $f_y W_{pl,y} = 38.5$ kNm

Summary of design values and resistances

	Design value	Resistance
Axial force	$N_{Ed} = 263.8$ kN	$N_{b,Rd} = 607$ kN
y-y bending	$M_{y,Ed} = 26.2$ kNm	$M_{y,b,Rd} = 71.1$ kNm
z-z bending	$M_{z,Ed} = 2.25$ kNm	$f_y W_{pl,z} = 38.5$ kNm

Interaction check

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{y,Ed}}{M_{y,b,Rd}} + 1.5\frac{M_{z,Ed}}{f_y W_{pl,z}} = \frac{263.8}{607} + \frac{26.2}{71.1} + 1.5 \times \frac{2.25}{38.5} = 0.435 + 0.368 + 0.088 = 0.891 \leq 1.0$$

Adopt 152 × 152 × 37 kg/m UC

5.6.3 Column Baseplates

Steel columns are normally connected to their concrete foundations by a steel baseplate, and Figure 5.30 shows a typical arrangement. The sleeves around the holding-down bolts and the grout beneath the baseplate give a useful degree of adjustment so that the column can be placed precisely even if the concrete foundation is 10 or 20 mm out of level and the holding-down bolts are similarly out of position. Normal UK practice is to provide at least four holding-down bolts to provide stability during erection, even where it can be shown that two bolts are sufficient when the column is part of a completed frame.

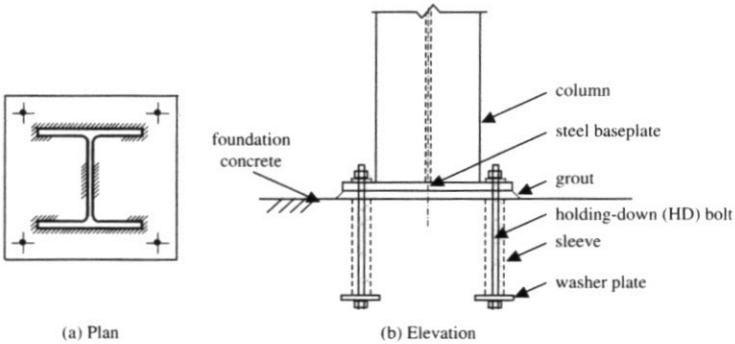


Figure 5.30: Steel baseplate

The purpose of the baseplate is to spread the force from the column onto a sufficient area of concrete. The ultimate bearing strength of the concrete f_{jd} can be taken as $0.67f_{ck}$ where f_{ck} is the cylinder strength of the concrete as explained in Chapter 3. Thus, for grade C30/37 concrete the bearing strength $f_{jd} = 0.67 \times 30 = 20\,\text{N/mm}^2$.

Figure 5.31 shows the way in which load from the column flanges is spread out onto the concrete beneath. The figure illustrates the 'large projection' base in which the spread can take place in all directions as contrasted with the 'short projection' base in which the spread

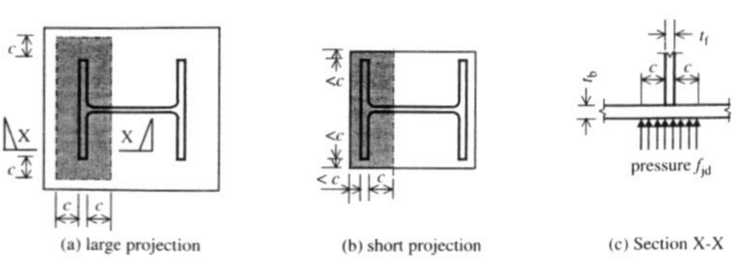


Figure 5.31: Bearing under steel baseplates

is limited by the edge of the baseplate. The dimension c is determined by the requirement that the stress in the concrete should not exceed f_{jd}. The thickness of the plate is then determined by the requirement that the elastic bending stress in the baseplate should not exceed f_y for the steel.

A full design method for large and short projection baseplates is given in EC3, and is further explained in Document SN037a. The following simplified approach, which always gives a 'large projection' baseplate, makes some conservative assumptions. In many cases it will be possible to use the more detailed method of EC3 to show that a smaller baseplate is adequate to carry the axial force, but detailed consideration of the edge spacing and clearance required for the holding-down bolts may mean that the smaller baseplate cannot be used.

First make the conservative assumption that two of the flanges of the column section carry the entire axial load. Considering Figure 5.31(a) and (c), the downward force per millimetre of flange is $N_{ed}/2b$, and this is balanced by an upward force $f_{jd}(2c + t_f)$. Thus, $f_{jd}(2c + t_f) = N_{ed}/2b$, which can be written as

$$c = 0.5\left(\frac{N_{Ed}}{2bf_{jd}} - t_f\right)$$

Section X-X in Figure 5.31 shows that the baseplate carries the pressure f_{jd} as a cantilever of length c. The moment in the baseplate will therefore be $M_{Ed} = f_{jd}c^2/2$ Nmm per millimetre width. The elastic section modulus of the baseplate $W_{el} = t_b^2/6\,\text{mm}^3$ per millimetre width, so the elastic bending stress $= M_{Ed}/W_{el} = f_{jd}c^2/2 \times 6/t_b^2 = 3f_{jd}(c/t_b)^2$. Now the elastic bending stress should not exceed the strength of the steel fy, so $3f_{jd}(c/t_b)^2 \leq fy$. Noting UK practice that the baseplate thickness should not be less than the column flange thickness, we can write

$$t_b = c\sqrt{\frac{3f_{jd}}{f_y}} \quad \text{but not less than } t_f.$$

The use of these two relationships to find the size and thickness of a steel baseplate is shown in Example 5.14.

Example 5.14 Stanchion baseplate thickness and size

A 203 × 203 × 71 kg/m UC stanchion carries a ULS axial load N_{Ed} of 1250 kN. It is to bear on a concrete base with $f_{ck} = 30\,\text{N/mm}^2$ through a steel baseplate. Determine a suitable size and thickness for a baseplate in S275 steel.

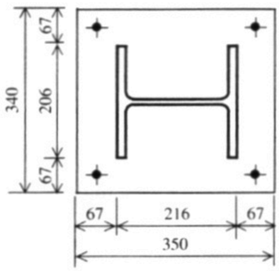
Calculations for Example 5.14

For a 203 × 203 × 71 kg/m UC section, from Table 5.8	$h = 215.8\,\text{mm}$
	$b = 206.4\,\text{mm}$
	$t_f = 17.3\,\text{mm}$
Using Grade S275 steel with baseplate thickness assumed between 16 and 40 mm, from Table 5.3	$f_y = 265\,\text{N/mm}^2$
Bearing strength of the concrete $f_{jd} = 0.67 \times f_{ck} = 0.67 \times 30$	$f_{jd} = 20\,\text{N/mm}^2$
Minimum outstand $c = 0.5\left(\dfrac{N_{Ed}}{2bf_{jd}} - t_f\right) = 0.5\left(\dfrac{1250 \times 10^3}{2 \times 206.4 \times 20} - 17.3\right)$	$c = 67\,\text{mm}$
Minimum baseplate thickness $t_b = c\sqrt{\dfrac{3f_{jd}}{f_y}} = 67\sqrt{\dfrac{3 \times 20}{265}}$ but not less than t_f	$t_b = 32\,\text{mm}$

The minimum baseplate required is shown in the figure on the right

Note that the more detailed design rules in EC3 may show that a smaller baseplate is structurally adequate. However, detailed consideration of the edge spacing and clearance required for the holding-down bolts may show that a smaller baseplate is not practical

From a fabrication point of view it may be more practical to use a 350-mm-square baseplate



32 mm plate

Baseplate for Example 5.14

5.7 Connections

The design of connections normally follows the design of the individual structural members. Where the simple design method has been used the connections are required to carry only forces, while the connections in a rigid or semi-rigid design have to be designed for moments as well as forces.

Connections may be bolted, welded or a combination of both, and should be chosen to facilitate erection of the frame. The actual sizes of fabricated elements will not be exactly the sizes shown in the drawings, so connections must be able to accommodate realistic fabrication and erection tolerances. It is common practice to use bolts in holes which are 2 mm larger than the nominal bolt size, for example 20 mm bolts in 22 mm holes, and this gives sufficient leeway to accommodate dimensional deviations in most small structures. Further precautions are required in large structures or where a closer fit is necessary.

Procedures for the design of connections are given in EC3 Part 1.8 but are not covered by this manual.

5.8 References

BS 4-1: 2005 Structural steel sections – Part 1: Specification for hot-rolled sections.

BS EN 1993-1-1: 2005 Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings.

BS EN 1993-1-5: 2007 Eurocode 3: Design of steel structures – Part 1-5: Plated structural elements.

BS EN 1993-1-8: 2005 Eurocode 3: Design of steel structures – Part 1-8: Design of joints.

BS EN 1993-1-10: 2005 Eurocode 3: Design of steel structures – Part 1-10: Material toughness and through-thickness properties.

BS EN 10025-2: 2004 Hot-rolled products of structural steels – Part 2: Technical delivery conditions for non-alloy structural steels.

Advance Sections brochure: from CORUS, download from www.corusconstruction.com

The following documents are NCCI (Non-Contradictory Complementary Information) to EC3 and can be downloaded from www.access-steel.com

Document SN003a Elastic critical moment for lateral torsional buckling.

Document SN005a Determination of moments on columns in simple construction.

Document SN008a Buckling lengths of columns – rigorous approach.

Document SN037a Design model for simple column bases – axially loaded I section columns.

Document SN048b Verification of columns in simple construction – a simplified interaction criterion.

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CIVIL/STRUCTURAL ENGINEERING

Structural Elements Design Manual: Working With Eurocodes is the structural engineer's 'companion volume' to the fo
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concrete, masonry and steelwork.

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With plenty of worked examples and diagrams, it is a useful textbook not only for students of structural and civil engineering, but also for those on courses in related subjects such as architecture, building and surveying whose studies include the design of structural elements.

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